Imprints of Early Universe Cosmology on Gravitational waves

arXiv: 2409.xxxx [hep-ph]

Mudit Rai

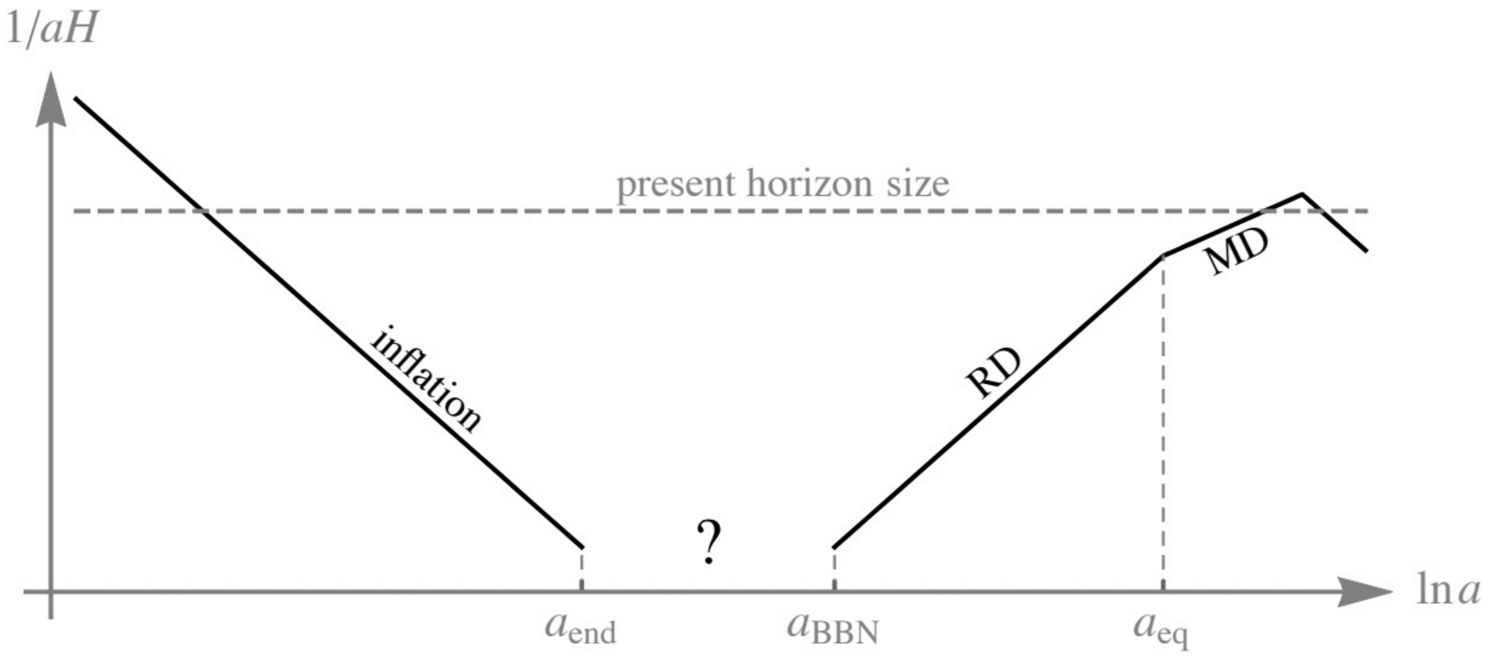
Texas A&M University

Collaborators : Bhaskar Dutta, James B. Dent

muditrai@tamu.edu

Motivation

- Physics before Big Bang Nucleosynthesis (BBN) ($T \sim MeV$) is not well understood due to lack of observational data.
- Gravitational waves can be a natural way to probe this epoch between end of inflation and BBN.



https://arxiv.org/pdf/2006.16182

Cosmological setup

- We consider the scenario where the hidden sector is thermally decoupled to the SM.
- We assume that the SM makes up bulk of the energy density of the RD universe.
- The ratio of hidden sector temperature and that of SM is given by $\xi = \frac{T_h}{T_{SM}} < 1$ which also implies a hierarchy in the energy densities of the two sectors.
- Net energy density of the universe is given as, $\rho_R(T) = \frac{\pi^2}{30}\left(g_h^*(T) + \frac{g_{SM}^*(T_{SM})}{\xi^4}\right)T^4$

Energy injection

- Energy/entropy injection before BBN has been discussed extensively:
  - Fluctuations generated during inflation and later reentry [Carr & Lidsey, ….]
  - Collapse of domain walls [Cai et al, …]
  - **PBH reheating** [Bernal et al, …]
  - Bubble collisions during phase transition [Kodama et al, …]
  - Temperature increase during reheating [Co et al, …]
  - **Moduli decay** [Dutta et al…]
- The rate of energy injection can be either be fast where the field remains stuck as the temperature rises or can be slow where the field tracks its T dependent minima.

Energy injection : Moduli decay

- The amount of energy injection to hidden sector via Moduli decay is given as,

$$\rho_h' = \rho_h + \rho_\chi$$

$$\Longrightarrow T_h' = T_h \left(1 + \frac{30\, m_\chi^2 \chi_i^2}{\pi^2\, g_h^*\, T_h^4}\right)^{1/4}$$

$$T \rightarrow \tilde{T} = T(1+\delta)$$

- For hidden sector of $T_h \approx 100$ GeV and small delta,

$$\delta \approx 0.4 \left(\frac{m_\chi}{2.4\times10^8\ GeV}\ \frac{\chi_i}{4\times10^{-5}\ GeV}\right)^2$$

- Larger initial field value leads to larger injection,

$$\delta \approx 4 \left(\frac{m_\chi}{2.4\times10^8\ GeV}\ \frac{\chi_i}{4.63\times10^{-4}\ GeV}\right)^{\frac{1}{2}} - 1 \approx 3$$

Energy injection : PBH reheating

- Another instance for energy dumping to early universe happens via PBH evaporation.
- Following energy conservation before and after PBH evaporation, we get

$$T'_{SM} = T_{SM}\left(1+\frac{\eta T_0}{T_{SM}}\right)^{1/4}$$

$$T'_h = T_h\left(1+\frac{\eta T_0}{T_h}\right)^{1/4}$$

- For hidden sector of $T_h \approx 100$ GeV and small delta,

$$\delta \approx 0.45 \times \frac{\eta}{10^{-11}} \times \frac{0.1}{\xi} \times \left(\frac{M_{BH0}}{5.3\times 10^4 g}\right)$$

- Larger initial mass fraction leads to larger injection,

$$\delta \approx 4\left(\frac{\eta}{1.39\times 10^{-9}} \times \frac{0.1}{\xi} \times \left(\frac{M_{BH0}}{5.3\times 10^4 g}\right)\right)^{1/4} - 1$$

$\eta \equiv \frac{\rho_{BH}}{\rho_R}|_{T_0}$: Initial PBH mass fraction

Model realization

$$V \approx D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$$

- Initially, at high T, the field is in symmetric phase and there's just 1 minima at $\phi = 0$
- As universe cools, $T < T_1$, there exist a second minima

$$T_1^2 = \frac{T_0^2}{1 - \frac{9E^2}{8\lambda D}}, \quad \phi_1 = \frac{3ET_1}{2\lambda}$$

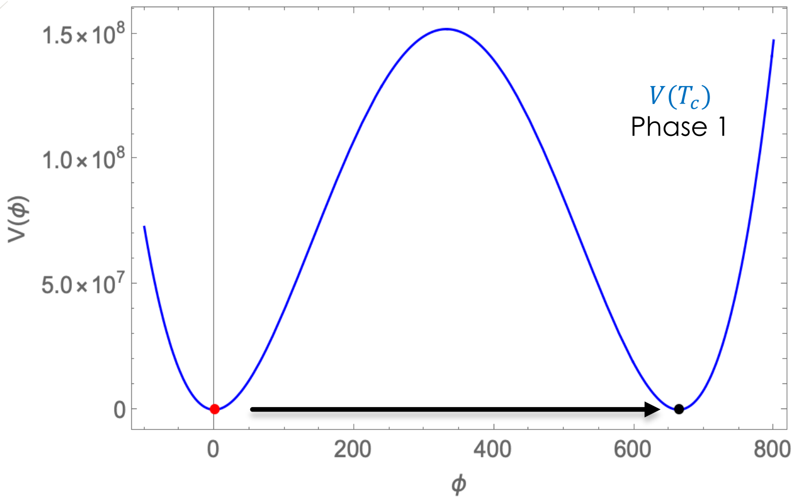
- As it further cools, these two minima become equi-potential and we have an onset of phase transition,

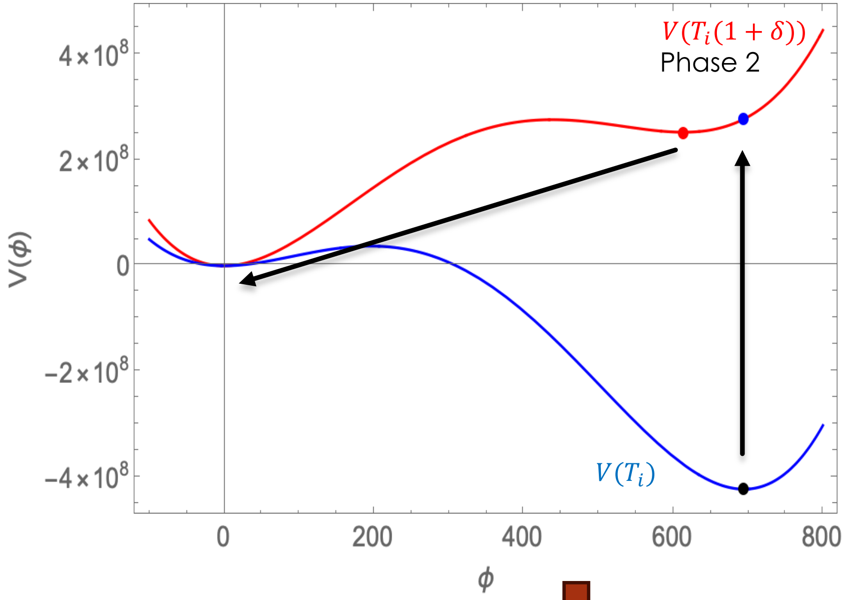
$$V(0, T_c) = V(\phi_c, T_c) \qquad T_c^2 = \frac{T_0^2}{1 - \frac{E^2}{\lambda D}}, \quad \phi_c = \frac{2ET_c}{\lambda}$$

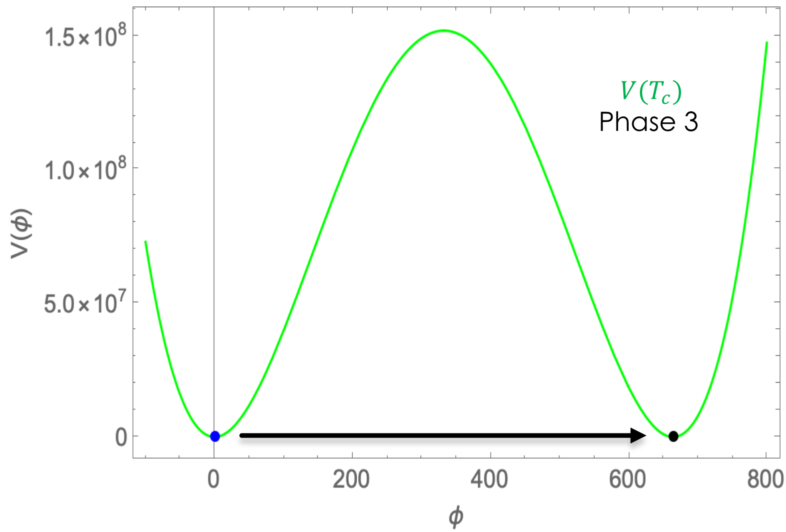
- After $T = T_0$, $\phi = 0$ ceases to be a minima and we are left with,

$$\phi_0 = \frac{3ET_0}{\lambda}$$

Thermal kick effects







- First transition happens at $\mathrm{T} = \mathrm{T_c}$ (Phase 1)
- Due to thermal kick at T_i, $(\mathrm{T_c} > T_i > T_0)$, $T_i \to T_i(1+\delta) > T_c$ whereas the field remains stuck at $\phi_i(T_i)$, leads to PT from $\phi_i \to 0$ (Phase 2)
- As universe cools down, there's another PT from $0 \to \phi_c$, which is like the standard transition but happens at later redshift (Phase 3)

Euclidean Action

- For simple polynomial like potentials, the tunneling action determining the tunneling rate from a false vacuum state to the true vacuum state [Adams]

$$\frac{S_3}{T} = \frac{2\sqrt{2}\,E}{\lambda^{3/2}}\,\frac{8\,\pi\,\kappa^{3/2}(8.2938 - 5.533\kappa + 0.818\kappa^2)}{81(2-\kappa^2)}$$

where

$$\kappa = \frac{2\lambda\,D\,(T^2 - T_0^2)}{E^2\,T^2}, \quad 0 \leq \kappa \leq 2$$

- For Phase II, we can modify the parameters accordingly as

$$2\tilde{D}(T^2 - T_0^2) = 2D(T^2 - T_0^2) + 3\phi_i(2\,E\,T + \lambda\phi_i)$$

$$\tilde{E}\,T = E\,T + \lambda\phi_i,$$

$$\tilde{\kappa} = \frac{\lambda\,(2D(T^2 - T_0^2) + 3\phi_i(2\,E\,T + \lambda\phi_i))}{(E\,T + \lambda\phi_i)^2}$$

Nucleation Temperature and PT rate

- Nucleation temperature can be thought of as the temperature where a true vacuum bubble arise within a Hubble volume, i.e,

$$\Gamma/H^4 \approx 1$$

where,

$$\Gamma = T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} \exp^{-\frac{S_3}{2\pi T}}$$

- Rate of the phase transition can be defined in terms of the Euclidean bounce action as,

$$\frac{\beta}{H_N} = T\frac{d(S_3/T)}{dT}|_{T_N}$$

Strength of PT and wall velocity

- Amplitude of GW signal is controlled by strength parameter α :

$$\alpha = \frac{\Delta(V - \frac{1}{4}\partial_T V)}{\rho_R}\Big|_{T=T_N}$$

where $\Delta X = X_f - X_t$

- For wall velocity, we use analytical approximation[Ellis et al],

$$v_w = \begin{cases} \sqrt{\frac{\Delta V}{\alpha \rho_R}} & \sqrt{\frac{\Delta V}{\alpha \rho_R}} < v_J \\ 1 & \sqrt{\frac{\Delta V}{\alpha \rho_R}} > v_J \end{cases}$$

where

$$v_J = \frac{1}{1+\alpha}\left(\sqrt{\frac{1}{3}} + \sqrt{\alpha\left(\frac{2}{3}+\alpha\right)}\right)$$

Gravitational Waves signal

- Differential GW density parameter characterizes them :

$$\Omega_{GW} = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}, \qquad \rho_c = 3M_{pl}^2 H^2$$

- Semi-analytical parametrizations can be used to describe them,

$$\Omega_{GW}(f) \simeq \sum_i \mathcal{N}_i\,\Delta_i(v_w)\left(\frac{\kappa_i\,\alpha}{1+\alpha}\right)^{p_i}\left(\frac{H}{\beta}\right)^{q_i} s_i(f/f_{p,i})$$

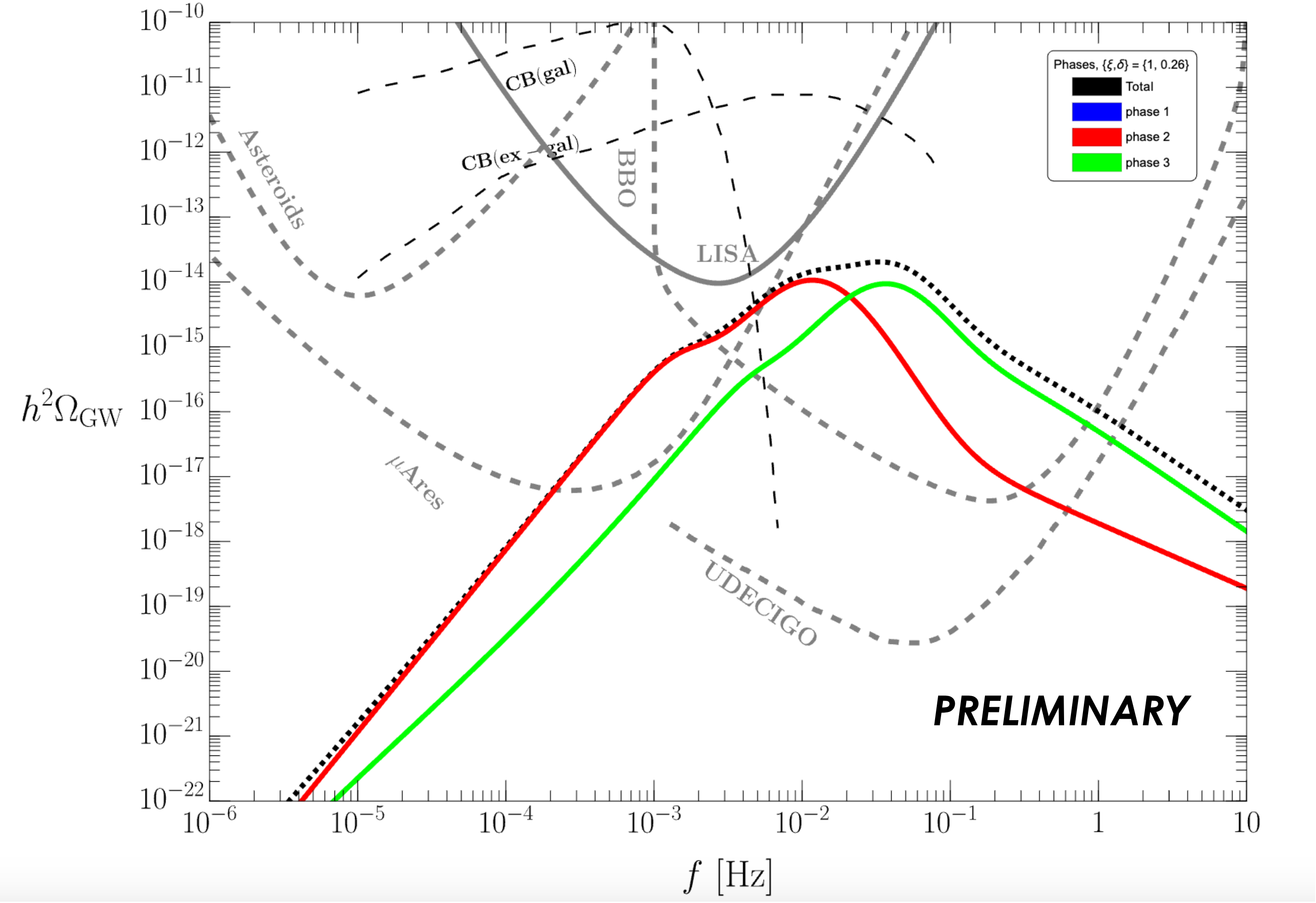
$$\Longrightarrow \quad \Omega^0_{\rm GW}(f) = \mathcal{R}\,\Omega_{\rm GW}\left(\frac{a_0}{a}f\right)$$

$$\mathcal{R} \equiv \left(\frac{a}{a_0}\right)^4\left(\frac{H}{H_0}\right)^2 \simeq 2.473\times 10^{-5}\,h^{-2}\left(\frac{g_s^{\rm EQ}}{g_s}\right)^{4/3}\left(\frac{g_\rho}{2}\right)$$

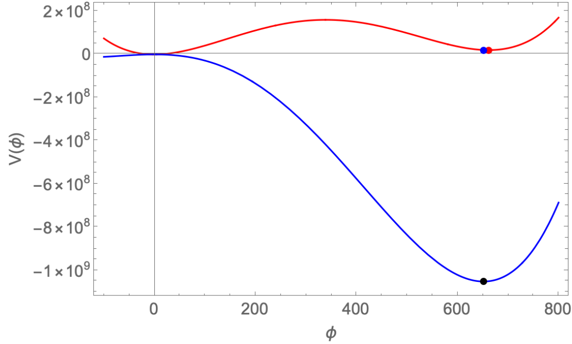
General features of Hidden sector PT

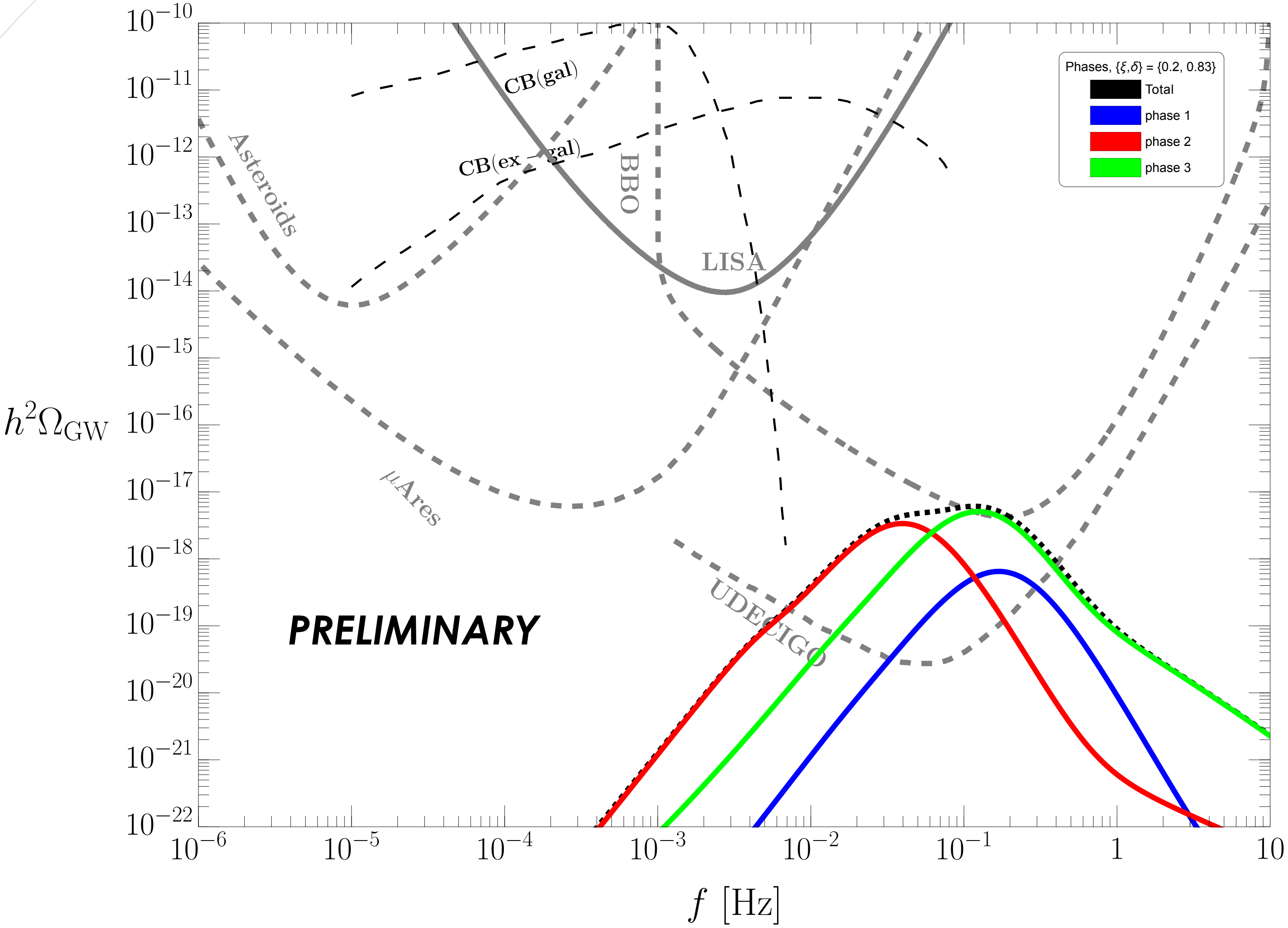
- $\alpha \propto \xi^4$ implies that for strong GW signals, Hidden sector and SM should have comparable temperature.
- Large β means faster transitions, which results in signals that are weaker and peaking at higher frequencies vs slower transitions.
- Larger v_w implies smaller peak frequency and larger GW amplitude.
- Smaller T_N/T_c is an indication of long-lasting phase transitions.

Results : $\xi = 1, \delta = 0.26$



Results : $\xi = 0.2, \delta = 0.83$





Conclusion

Energy injection(χ, PBH) → $T_h \to T_h(1+\delta)$ → 3 GW peaks (broader spectrum)

- Energy injection leads to more than one peak frequencies for GW from FOPT in hidden sector.
- For any reasonable ξ value (small or large), GW spectra have distinctive features due to multiple peaks.
- It is fairly independent w.r.to the mass scale of the hidden sector.
- Hidden sectors with GW can probe a variety of new physics scenario in the pre-BBN era.

THANK YOU!

BACKUP Slides

Hidden sector

- For concreteness, we consider a scalar field with U(1) gauge symmetry and a Yukawa like coupling to fermion field,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi\, D^\mu\phi + i\,\bar{\psi}\not{D}\psi - \frac{y\phi}{\sqrt{2}}\,\bar{\psi}\psi - V(\phi)$$

- The tree and thermal potential are given as

$$V_0 = -\frac{1}{2}\,\mu^2\phi^2 + \frac{1}{4}\,\lambda\phi^4$$

$$V_{th} = \frac{T^4}{2\pi^2}\left(n_\phi\, J_B\left[\frac{m_\phi^2}{T^2}\right] + n_X\, J_B\left[\frac{m_X^2}{T^2}\right] - n_f\, J_F\left[\frac{m_f^2}{T^2}\right]\right)$$

High T Potential

- At high temperatures, the effective potential is given as,

$$V \approx D(T^2 - T_0^2)\phi^2 - E\,T\phi^3 + \frac{\lambda}{4}\phi^4$$

where,

$$D = \frac{\alpha}{2}, \quad \alpha = \frac{\lambda + g^2}{4} + \frac{y^2}{24}$$

$$E = \frac{1}{12\pi}\left(3g^3 + \left(3\lambda + \frac{\alpha\,T^2 - \mu^2}{\phi^2}\right)^{3/2}\right) \approx \frac{g^3}{4\pi}$$

$$T_0^2 = \frac{\mu^2}{2\,D}$$

Impacts of energy injection

- The hierarchy in the two sectors imply that any small change in the energy density of the universe will impact the hidden sector more as compared to SM.
- The energy injection leads to an effective rise in hidden sector temperature

$$T \to \tilde{T} = T(1+\delta)$$

- The energy injection can lead to multiple phase transitions in the hidden sector, and we will show that it can be probed by the associated GW spectrum.
- SM is dominant over the hidden sector implies,

$$\xi(1+\delta) < \left(\frac{g^*_{SM}(T_i/\xi)}{g^*_h(T_i(1+\delta))}\right)^{1/4}$$

Energy injection : Moduli decay

- Consider a scenario where moduli field is coupled to the hidden sector in early universe and undergoes decay as it becomes comparable to Hubble,

$$\Gamma_\chi \approx \frac{m_\chi^3}{M_{pl}^2} \approx H = \frac{\sqrt{\frac{1}{3}\rho_R(T)}}{M_{pl}}$$

- Since we are always in the RD cosmology with SM being the dominant source,

$$\rho_\chi \approx m_\chi^2 \chi_i^2 < \rho_{SM}$$

- Thus the mass of the moduli field correlates with the scale of hidden sector and the field value then gives the amount of energy injection.

Energy injection : PBH reheating

- Setting PBH evaporation temperature to happen roughly around the scale of hidden sector fixes the mass of the PBH and formation temperature

$$T_{ev}^{SM} \approx 10^3 \mathrm{GeV} \times \left(\frac{5.3\times 10^4 \mathrm{g}}{\mathrm{M_{BH0}}}\right)^{3/2} \qquad T_0^{SM} \approx 1.85\times 10^{13}\mathrm{GeV}\left(\frac{5.3\times 10^4 \mathrm{g}}{\mathrm{M_{BH0}}}\right)^{1/2}$$

- Energy conservation :

$$\rho_{SM}(T_{SM}) + \eta\, s(T_{SM})\frac{\rho_R(T_0)}{s(T_0)} = \rho_{SM}(T'_{SM}),$$
$$\rho_h(T_h) + \eta\, s(T_h)\frac{\rho_R(T_0)}{s(T_0)} = \rho_h(T'_H)$$

- The amount of energy injection is then estimated in terms of η, the initial mass fraction of PBH.

Phase 1

- Driving force for the PT in terms of the latent heat is,

$$F_{dr} \approx \alpha\,\rho_R = \Delta(V - \frac{T}{4}\frac{dV}{dT})$$
$$= \frac{T_N^2\,\phi_c^2}{2\lambda}\left(\lambda\,D\frac{T_N}{T_c} - E^2\right) > 0$$

- The friction force is due to particles gaining mass as they go from the false vacua(symmetric phase) to the true vacua (broken phase).
- For runaway transition ($v_w \to c$), $F_{dr} > \Delta p_{LO}^{\gamma_w \to \infty}$ where,

$$\Delta p_{LO}^{\gamma_w \to \infty} \approx \sum_i \frac{c_i\,g_i\,(m_{t,i}^2 - m_{b,i}^2)}{24} > 0$$

- For most of our parameter space, we find that this happens to be a non-runaway transition, i.e, where the bubble wall never reaches the speed of light.

Phase 2

- The pressure difference due to mass difference is negative, since the particles loose mass as they pass from the false vacua (broken phase) to the true vacua (symmetrical phase).

$$\Delta p_{LO}^{\gamma_w \to \infty} \approx \sum_i \frac{c_i\, g_i\, (m_{t,i}^2 - m_{b,i}^2)}{24} < 0$$

- The force due to latent heat difference is also negative, and it acts as an effective friction.

$$F_{opp} \approx -T_i(1+\delta)\,\eta\,(4\,D\,T_i(1+\delta) - 3\,E\,\phi_i) - 2\,D\,T_0^2 < 0$$

- This transition happens to be runaway[1], with the condition being

$$|\Delta p_{LO}^{\gamma_w \to \infty}| > |F_{opp}|$$

[1] For models without any production of soft vector bosons at the boundary

GW signal parametrization

- Normalization factors and exponents :

$$(N_{\rm BW}, N_{\rm SW}, N_{\rm turb}) = (1, 0.159, 20.1)$$

$$(p_{\rm BW}, p_{\rm SW}, p_{\rm turb}) = (2, 2, 3/2) \qquad (q_{\rm BW}, q_{\rm SW}, q_{\rm turb}) = (2, 1, 1)$$

- Potential suppression due to wall velocity, spectral fns and peak frequencies :

$$\Delta_{\rm BW} = \frac{0.11 v_{\rm w}^3}{(0.42 + v_{\rm w}^2)}, \quad f_{\rm p,BW} = \frac{0.62\beta}{1.8 - 0.1 v_{\rm w} + v_{\rm w}^2}, \quad s_{\rm BW}(x) = \frac{3.8\, x^{2.8}}{1 + 2.8\, x^{3.8}},$$

$$\Delta_{\rm SW} = v_{\rm w} \min(1, H_* \tau_{\rm sh}), \quad f_{\rm p,SW} = \frac{2\beta}{\sqrt{3} v_{\rm w}}, \quad s_{\rm SW}(x) = x^3 \left(\frac{7}{4 + 3\, x^2}\right)^{7/2},$$

$$\Delta_{\rm turb} = v_{\rm w}, \quad f_{\rm p,turb} = \frac{3.5\beta}{2 v_{\rm w}}, \quad s_{\rm turb}(x) = \frac{x^3}{(1+x)^{11/3}(1 + 8\pi\, x \frac{f_{\rm p,turb}}{H})}.$$

- Additional velocity suppression for sound waves given by:

$$H_* \tau_{\rm sh} \approx \frac{3.38 \max[v_{\rm w}, c_{\rm s}]}{\beta/H} \sqrt{\frac{1+\alpha}{\kappa_{\rm SW}\alpha}}$$

GW signal efficiency factors

- Efficiency factor for bubble walls give a measure of the fraction of energy stored in the bubble wall w.r.to the total energy gained in the transition.
- For sound waves, it is the fraction of energy density imparted to the plasma w.r.to the total energy gained in the transition.
- For runaway scenarios, it can be calculated from the bubble walls.
- For non-runaway scenarios, bubble wall contribution is generally neglected.
- Turbulence efficiency factor can be estimated from sound wave as,

$$\kappa_{\rm Turb} = \epsilon \kappa_{\rm SW} \,,$$

$$\epsilon = \left(1 - \min\left(H_* \tau_{\rm sh}, 1\right)\right)^{\frac{2}{3}}$$

GW signal efficiency factors

- Turbulence efficiency factor can be estimated from sound wave as,

$$\kappa_{\text{Turb}} = \epsilon \kappa_{\text{SW}} \, ,$$

$$\epsilon = \left(1 - \min\left(H_* \tau_{\text{sh}}, 1\right)\right)^{\frac{2}{3}}$$

- Sound wave efficiency factor differs in run-away vs non-runaway scenarios.
- For runaway scenarios, we have,

$$\kappa_{\text{SW}} = (1 - \kappa_{BW}) \frac{\alpha_{eff}}{0.73 + 0.083\sqrt{\alpha_{eff}} + \alpha_{eff}}, \quad \alpha_{eff} = \alpha_h (1 - \kappa_{\text{BW}})$$

- Efficiency factor for runaway in Phase 2 is based on numerical analysis in the works of Blasi et al on inverse transitions.

GW signal efficiency factors

- Sounds wave efficiency factor for non-runaway transitions is:

$$\kappa_{\rm SW} = \begin{cases} \frac{c_s^{11/5}\kappa_A\kappa_B}{\left(c_s^{11/5}-v_w^{11/5}\right)\kappa_B+v_w c_s^{6/5}\kappa_A}, & \text{if } v_w < c_s \\ \kappa_B + (v_w - c_s)\,\delta\kappa + \frac{(v_w-c_s)^3}{(v_J-c_s)^3}\left[\kappa_C - \kappa_B - (v_J - c_s)\,\delta\kappa\right], & \text{if } c_s < v_w < v_J \\ \frac{(v_J-1)^3 v_J^{5/2} v_w^{-5/2}\kappa_C\kappa_D}{\left[(v_J-1)^3-(v_w-1)^3\right]v_J^{5/2}\kappa_C+(v_w-1)^3\kappa_D}, & \text{if } v_J < v_w \end{cases}$$

with

$$\kappa_A \simeq v_w^{6/5}\frac{6.9\,\alpha_h}{1.36-0.037\sqrt{\alpha_h}+\alpha_h}, \qquad \kappa_B \simeq \frac{\alpha_h^{2/5}}{0.017+(0.997+\alpha_h)^{2/5}},$$

$$\kappa_C \simeq \frac{\sqrt{\alpha_h}}{0.135+\sqrt{0.98+\alpha_h}}, \qquad \kappa_D \simeq \frac{\alpha_h}{0.73+0.083\sqrt{\alpha_h}+\alpha_h},$$

$$\delta\kappa \simeq -0.9\log\frac{\sqrt{\alpha_h}}{1+\sqrt{\alpha_h}}.$$

Energy injection : Moduli decay

- Consider a scenario where moduli field is coupled to the hidden sector in early universe and undergoes decay as it becomes comparable to Hubble,

$$\Gamma_\chi \approx \frac{m_\chi^3}{M_{pl}^2} \approx H = \frac{\sqrt{\frac{1}{3}\rho_R(T)}}{M_{pl}}$$

- Mass of the moduli field can be correlated with the the scale of hidden sector from the above equation as,

$$m_\chi \approx 2.4 \times 10^8\,\mathrm{GeV} \times \left(\frac{\mathrm{T}}{100\,\mathrm{GeV}} \times \frac{0.1}{\xi}\right)^{2/3}$$

$\xi = 1$

- Relevant parameters for hidden sector having similar temperature as SM.
- Note the temperature being similar as all three are nearly concurrent.
- Phase 2 has different rate and strength due to its different nature of transition.

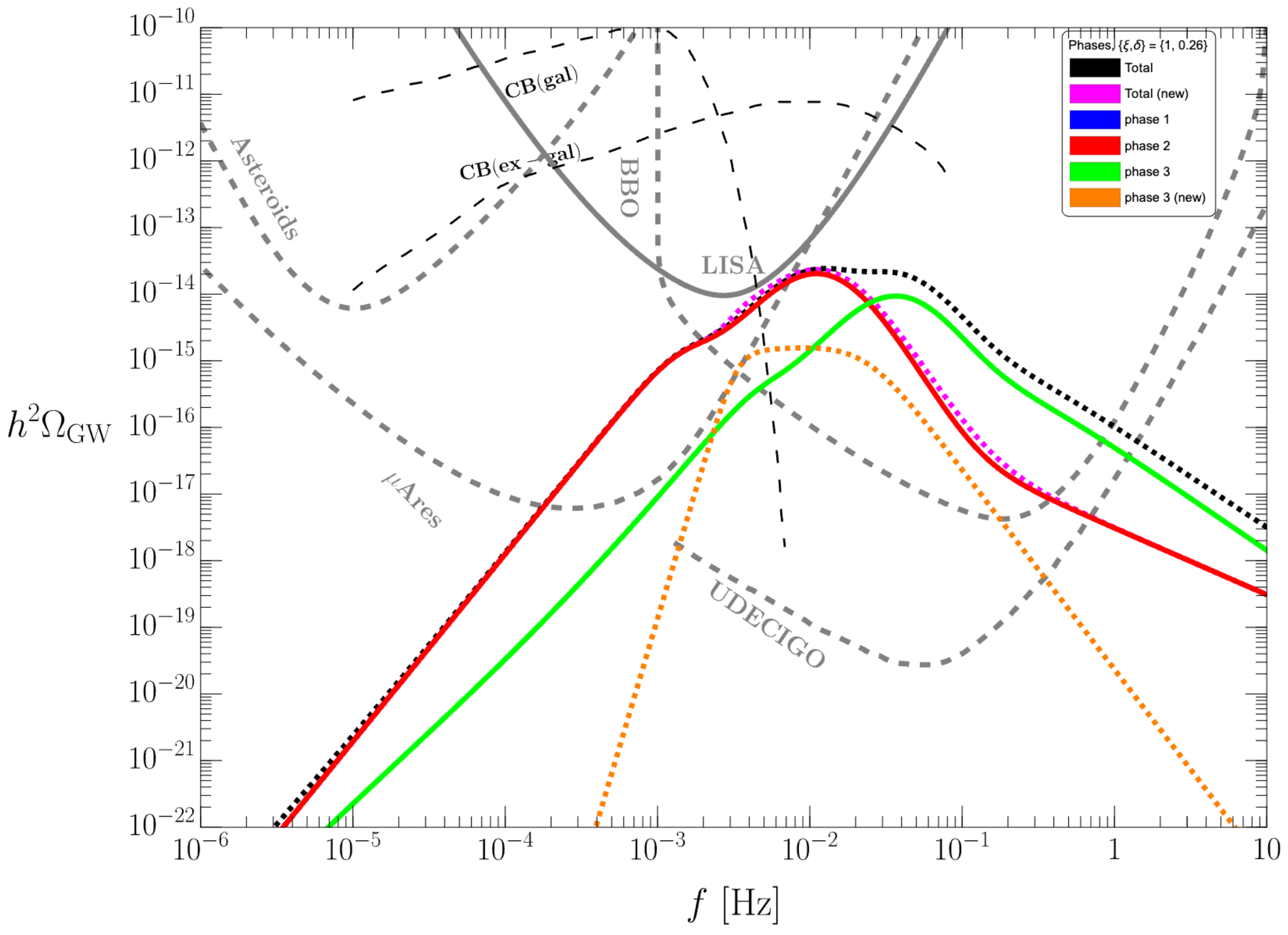
$\xi = 1$	α	β/H	α_h	T_N^{SM} (GeV)	f_P (Hz)	$\Omega_P h^2$
I	0.011	729.399	0.2167	232.939	0.036	9.37×10^{-15}
II	0.00837	201.494	0.162	244.01	0.011	1.34×10^{-14}
III	0.011	729.399	0.2167	232.94	0.036	9.37×10^{-15}

Results : $\xi = 0.2$

- Increasing the injection(δ) leads to separation between Phase 1 and Phase 3.
- As expected, the strength is quite suppressed owing to large hierarchy in hidden sector and SM energies

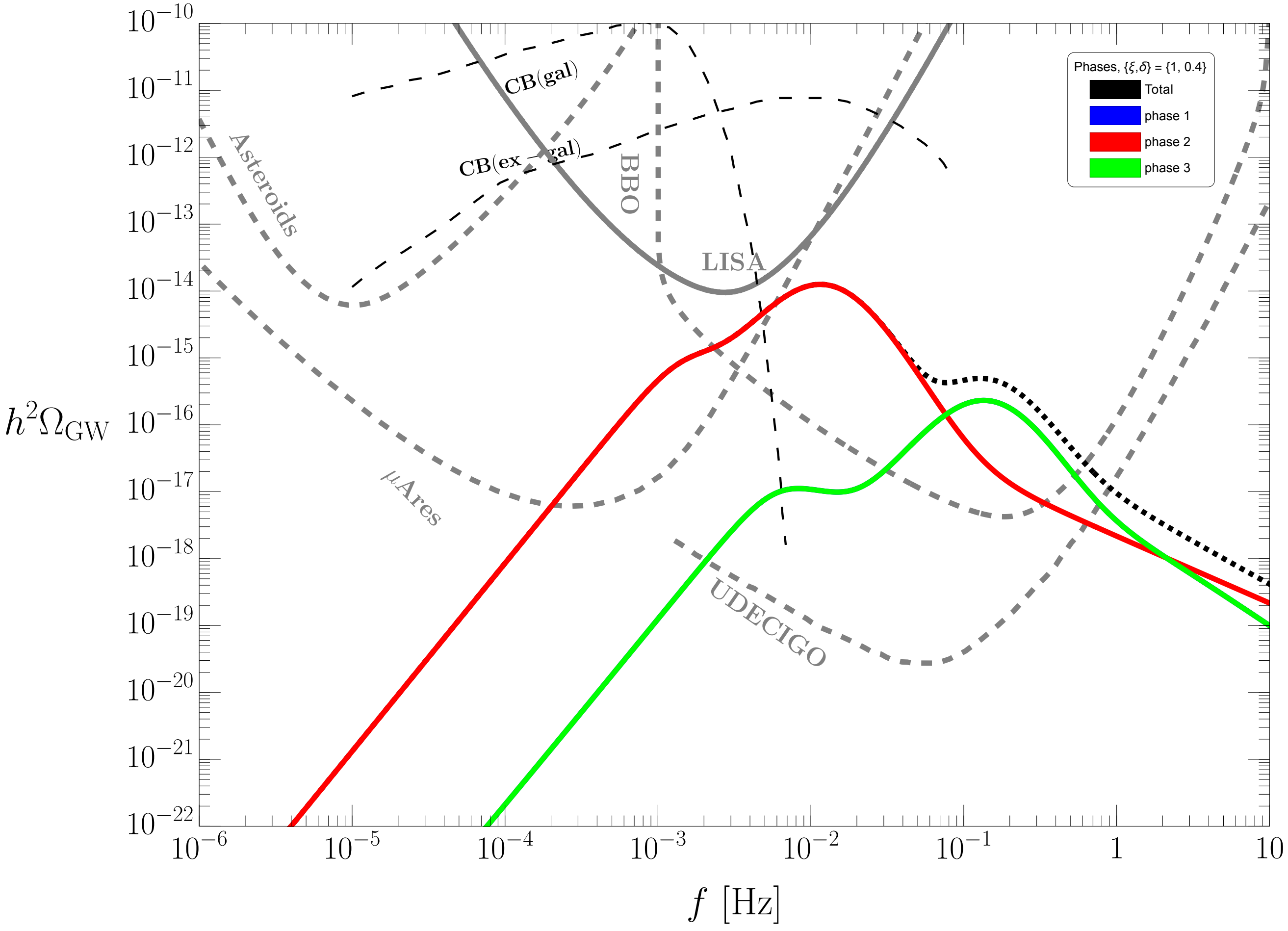
$\xi = 0.2$	α	β/H	α_h	$T_N^{\rm SM}$ (GeV)	f_P (Hz)	Ω_P
I	1.9×10^{-5}	691.154	0.221	1159.3	0.169	6.49×10^{-19}
II ($\delta = 0.83$)	6.5×10^{-5}	216.316	0.206	847.951	0.04	2.89×10^{-18}
II ($\delta = 0.07$)	2.16×10^{-5}	214.756	0.201	1117.78	0.052	4.08×10^{-19}
III($\delta = 0.83$)	6.99×10^{-5}	698.892	0.22	837.446	0.124	5.07×10^{-18}
III($\delta = 0.07$)	2.3×10^{-5}	692.445	0.22	1097.95	0.16	9.85×10^{-19}

Including new fit

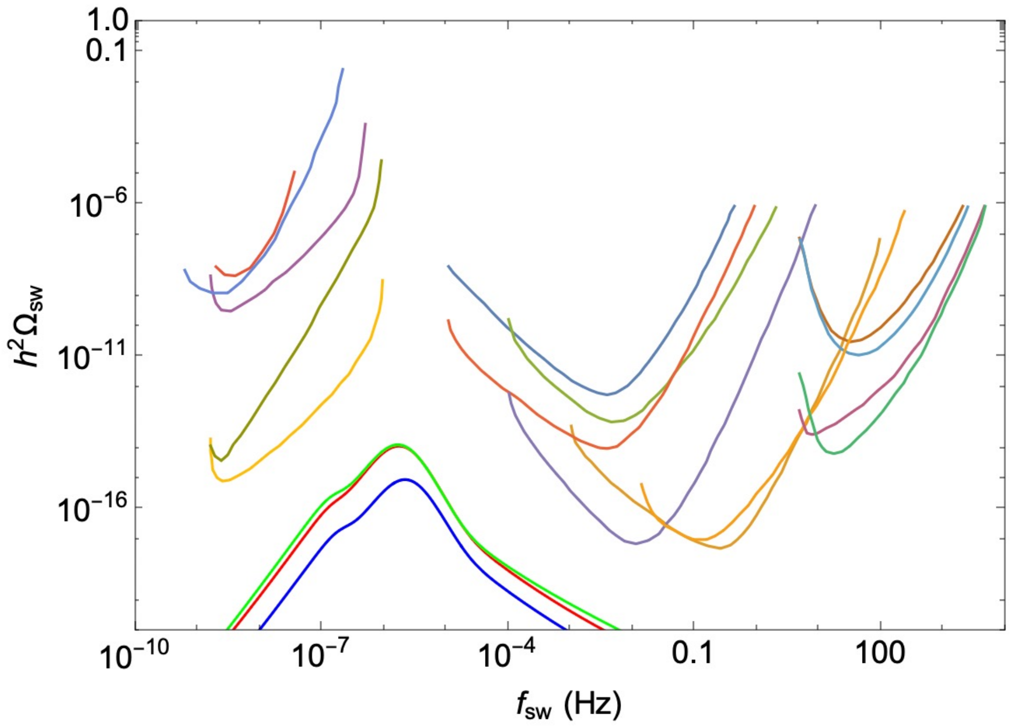


Effect of wall velocity

- Here we let wall velocity for Phase 1 and 3 be a free parameter and set $v_w = 0.2$, which leads to reduction in amplitude and enhancement of f_p



GW spectrum $\mu = 10\ MeV$

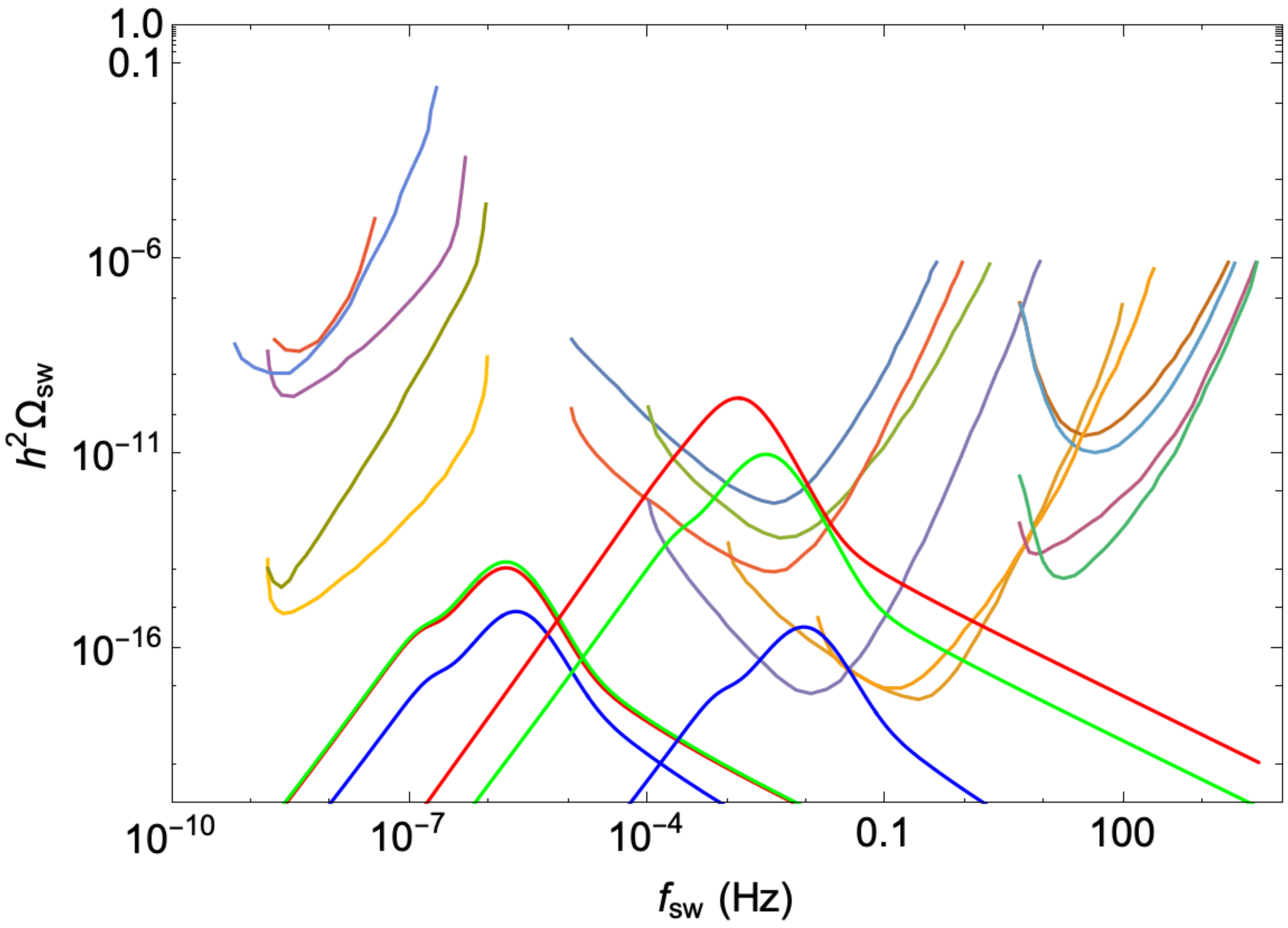


$\{\alpha_{kick}, \beta_{kick}/H, T_{SM,kick}, \delta\theta\}$ = 0.000521944, 91.9744, 0.108399, 0.397682

$\{\alpha_{late}, \beta_{late}/H, T_{SM,late}\}$ = 0.000772954, 92.2871, 0.102398

$\{\alpha, \beta/H, T_{SM,first}\}$ = 0.000203883, 92.2871,0.14312

GW spectrum : comparing scales



Slow reheating