Modeling the refractive index profile of polar ice with the Askaryan Radio Array (ARA)

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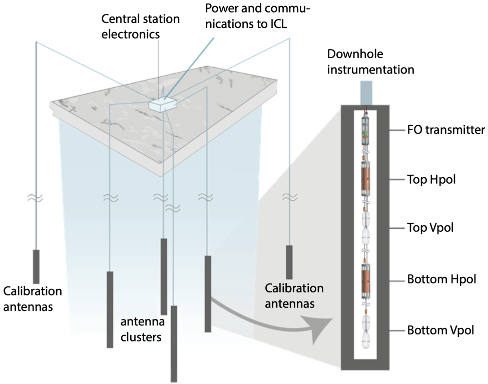
Overview

Ultra-high energy neutrino (UHEN) experiments rely on a depth dependent refractive index ($n(z)$) for calibration and simulations.

- $n(z)$ affects propagation paths through the ice, determining effective volume via a shadow zone
- ARA includes 5 stations that receive signal from a single deep pulser source
- We compare results of two $n(z)$ models to ARA data
  - single exponential
  - 3 stage exponential

ARA Detector

- ARA consists of 5 stations (typical station shown right)
- 4 strings each contain 4 antennas ranging 170-200m in depth
    - 2 Hpol and 2 Vpol antennas per string
- Calibration antennas used to calibrate antenna positions and test source reconstruction



Snow densification

Sorge's Law [1]

- Given constant snow accumulation and temperature, snow density $\rho(z,t)$ is constant for a given depth.
- $\rho(z)$ and therefore $n(z)$ are modeled as functions of depth

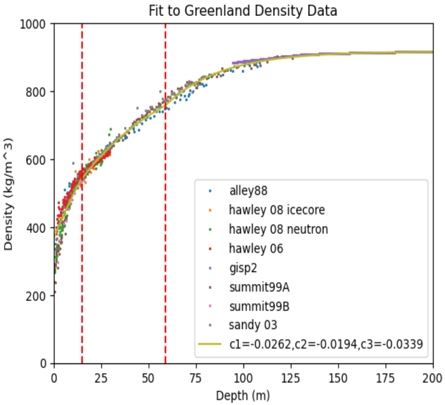
Density as a function of depth is modeled similar to an exponential scale height model.

$$\rho(z) = \rho_f - b_0 e^{-Cz}$$

where ρ_f is the density of pure ice, $\rho_f - b_0$ is the density of snow at the surface, and C is a proportionality constant describing the densification rate.

3 stage exponential model

- Glaciology studies ice densification [3], [4] occurs in 3 stages
  - snow
  - firn
  - bubbly ice
- Density boundary conditions
  - 550 kg/m^3 from previous empirical fits to polar density data [3]
  - 758 kg/m^3 fit to compiled greenland density data (shown right).



Conversion from ice density to n(z)

- Study of specific gravity vs dielectric constant of ice in the McMurdo ice shelf suggests a linear relationship between density and index of refraction [2]

The following equation describes the relation

$$n(z) = 1 + A\rho$$

where A is a proportionality constant and ρ is the specific gravity.

- A = 0.845 from the McMurdo study
- Data drawn from a specific gravity range of 0.2-0.8 (snow/firn region)

Applying conversion to ice density data

- Single exponential and 3 stage models
  - C_1 and C_2 fit to SPICE density data, C_3 fit to station A2 deep pulser data (see slide 12).
  - Single exponential fit to A2 data
- Follows density boundary conditions
  - 20.5m: snow to firn
  - 96.6m: firn to bubbly ice

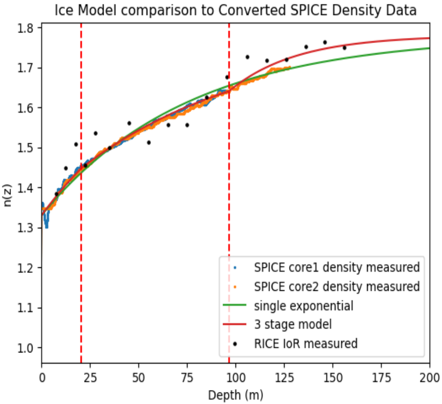


Figure: Allison *et al* (2020), Kravchenko I, Besson D, Meyers J. (2004)

A2 n(z) models

Single exponential model:

$$n(z) = 1.78 - 0.45e^{-0.0135z}$$

3 stage parameterization:

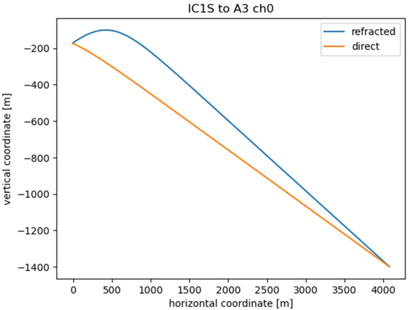
$$z < 20.5m : n(z) = 1.78 - 0.45e^{-0.0148z}$$

$$20.5m \leq z < 96.6m : n(z) = 1.78 - 0.33e^{-0.0114(z-20.5)}$$

$$z \geq 96.6m : n(z) = 1.78 - 0.14e^{-0.0290(z-96.6)}$$

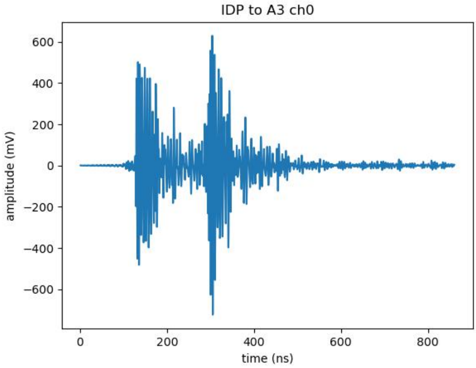
Simulated dt(D,R) timing differences

- Maxwell's equations admit two ray propagation solutions from source to receiver
- The two D and R ray paths are simulated using numerical ray tracer RadioPropa for a given n(z) model
- After calculating the travel time for each ray path, the simulated dt(D,R) is compared to measured values
- Example simulated ray paths from a 1400m deep pulser shown right



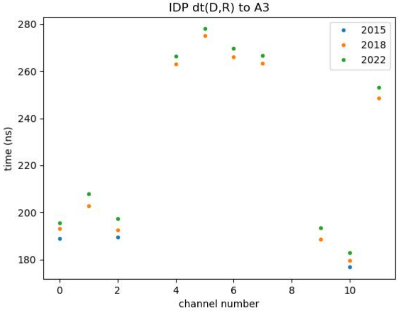
Direct and Refracted timing difference data (dt(D,R))

- Observed waveforms from deep pulser sources show two signals
- Signals correspond to the direct (D) and refracted (R) ray propagation paths
- Measured timing differences taken using the leading edges of the D and R signals
- dt(D,R) taken channel by channel minimizes additional sources of uncertainty



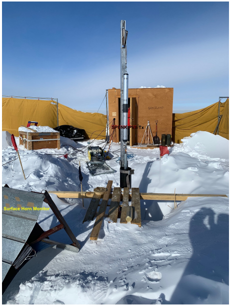
Snow accumulation

- Comparison of 2015, 2018, and 2022 DP signals to A3 show how snow accumulation affects dt(D,R)
- Accounts for years between deployment and SPICE pulsing runs
- $\pm$ 0.85 ns statistical uncertainty among channels



SPICE dt(D,R) data

- Transmitter lowered into SPICE bolehole during 2018 season.
- Emitted signal at 1 pps as it was lowered and raised
- Data over depth range of 700-1300m



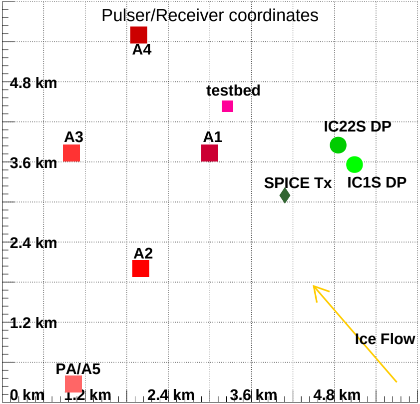
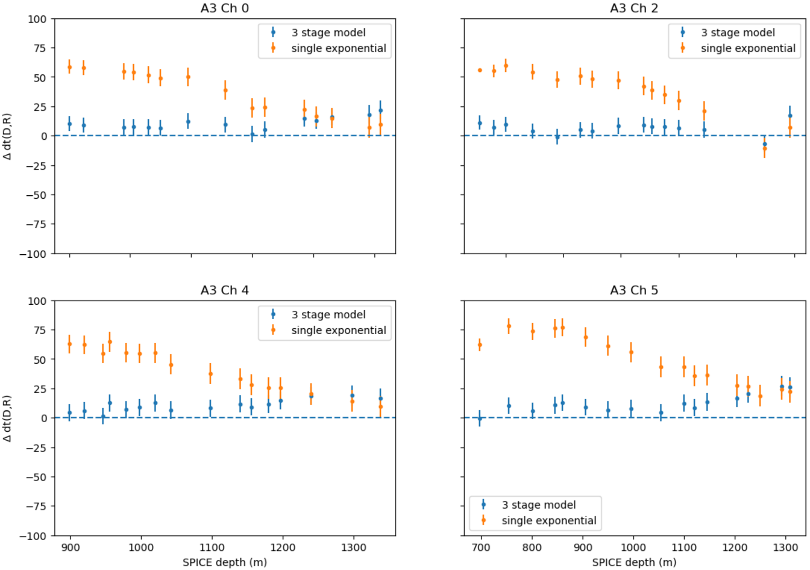
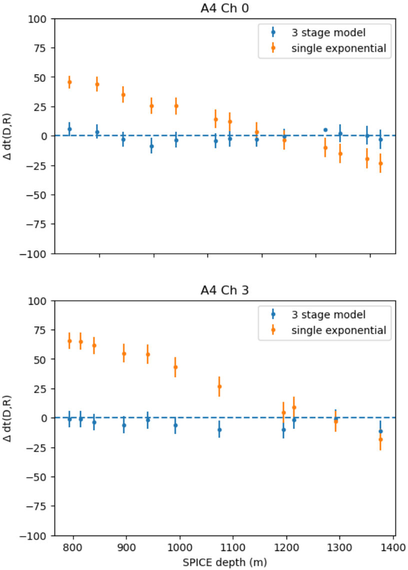


Figure: Allison *et al* (2020)

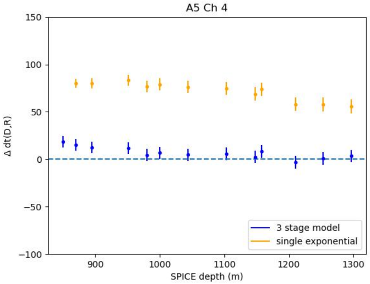
n(z) Model Results - Station A3

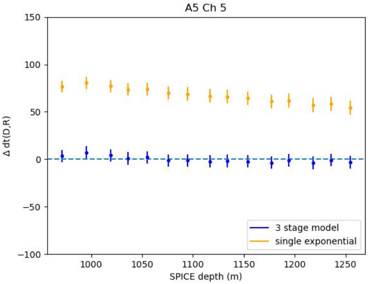


n(z) Model Results - Station A4



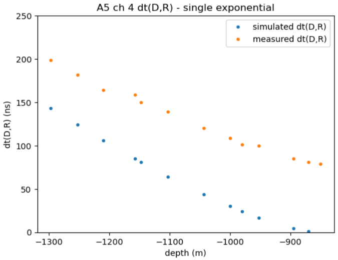
n(z) Model Results - A5

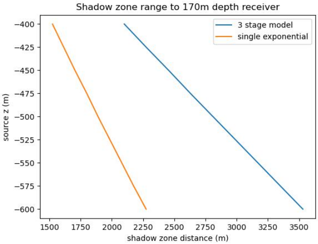




Shadow Zone Range

- dt(D,R) decreases the further distance between source and receiver
- As dt(D,R) approaches zero, bending of possible paths results in signal not reaching receiver
- Region with lack of signal known as the shadow zone
- Shadow zone differs for different $n(z)$ models, suggesting the simple exponential model underestimates effective volume





Conclusions

Measured dt(D,R) from the pulser lowered into SPICE borehole support a 3 stage exponential over single stage exponential $n(z)$ models

- Single exponential $n(z)$ model deviates from measured data as either source depth or source distance changes relative to initial fit
- Comparison to the 3 stage model suggests that a simple exponential overestimates $n(z)$ in the firn region (20.6-96.6m) and underestimates $n(z)$ for depths greater than 100m

Improved $n(z)$ model helps provide a more accurate effective volume and aid in calibration efforts for UHEN experiments in both Greenland and South Pole

References

- Henri Bader. Sorge's law of densification of snow on high polar glaciers. Journal of Glaciology, 2(15):319–323, 1954.
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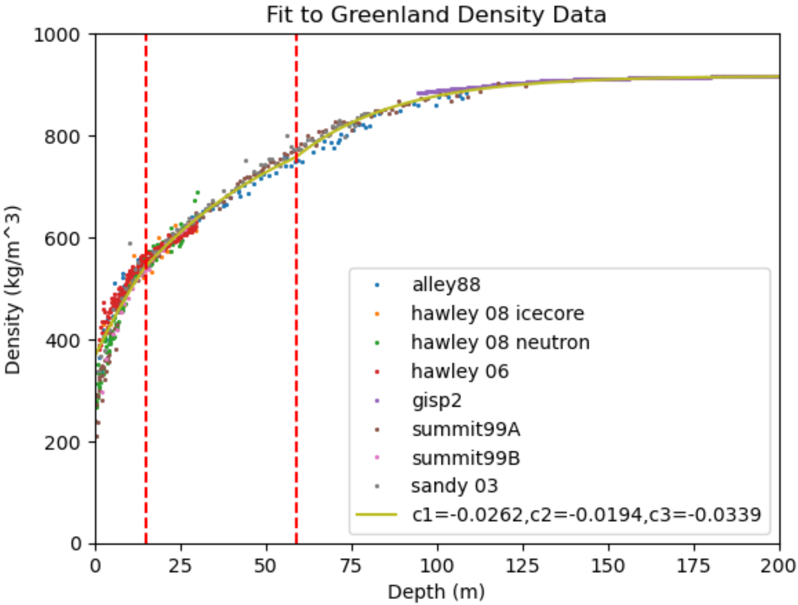
- Kravchenko I, Besson D, Meyers J. In situ index-of-refraction measurements of the South Polar firn with the RICE detector. Journal of Glaciology. 2004;50(171):522-532.
- P Allison, S Archambault, JJ Beatty, DZ Besson, CC Chen, CH Chen, P Chen, A Christenson, BA Clark, W Clay, et al. Long-baseline horizontal radio-frequency transmission through polar ice. Journal of Cosmology and Astroparticle Physics, 2020(12):009, 2020.

Backup Slides

3 stage density fit to greenland density data



2 stage density fit