CORRECTIONS TO HAWKING RADIATION FROM ASTEROID-MASS PRIMORDIAL BLACK HOLES: DESCRIPTION OF THE STOCHASTIC CHARGE EFFECT IN QUANTUM ELECTRODYNAMICS

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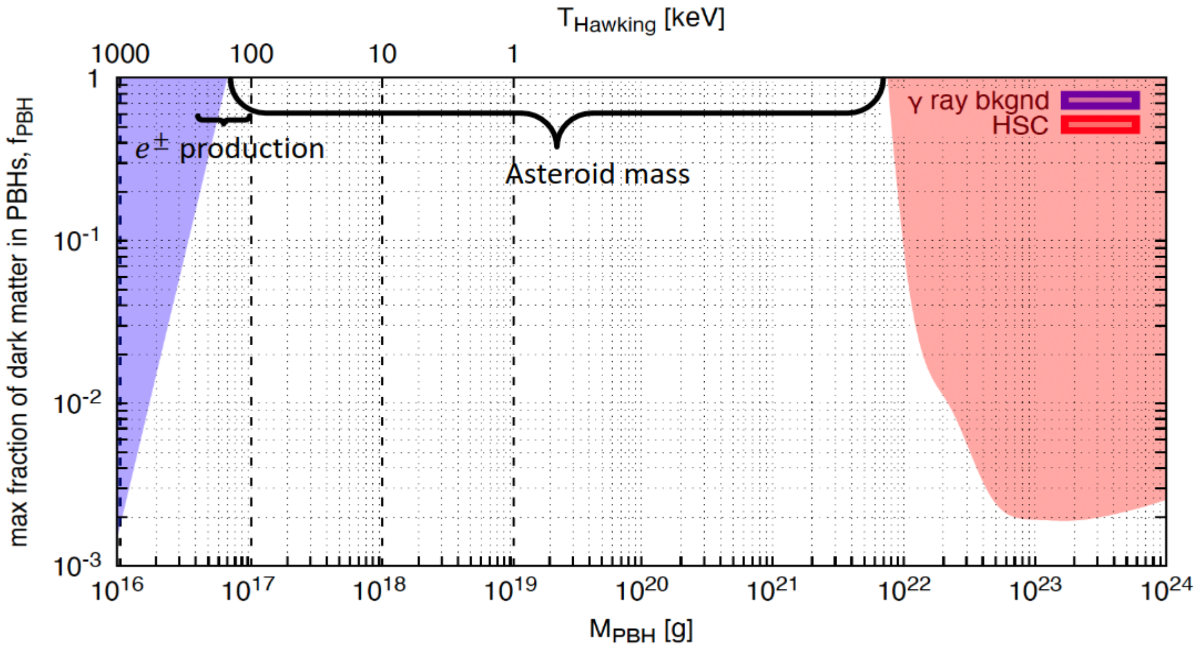
OHIO STATE

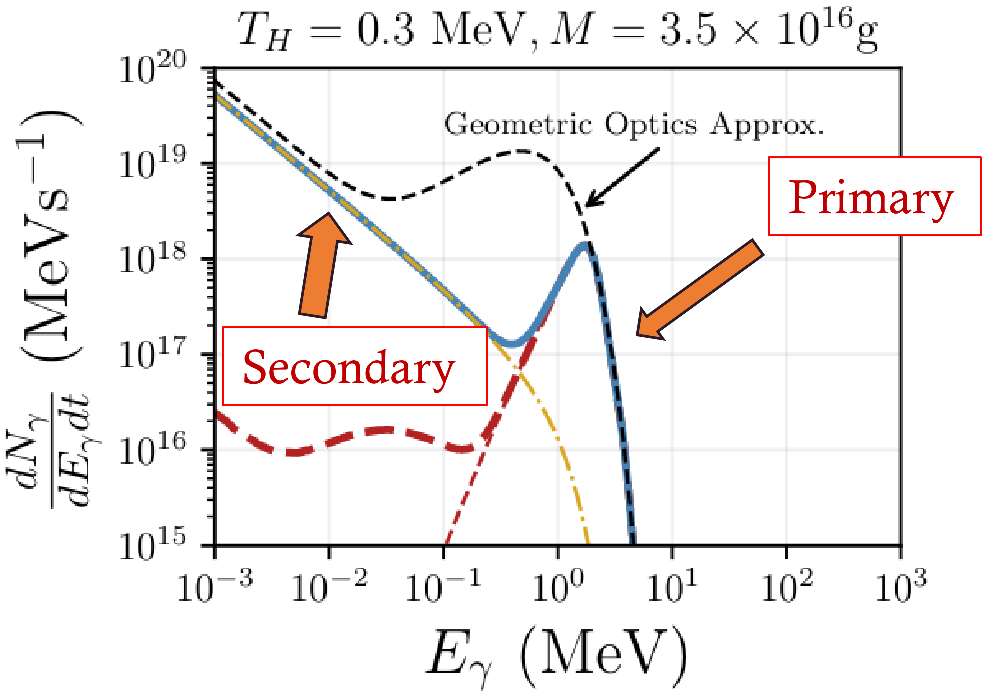
CONSTRAINTS IN THE ASTEROID-MASS REGIME



- Asteroid-Mass Regime: 10^{16}-10^{22} g .
- Hawking constraints are placed due to how the evaporation products interact with an astrophysical observable (e.g., diffuse gamma-ray background).
- $$T_H = \frac{1}{8\pi M}$$
- For a black hole of mass M~10^{16} g, the prominent evaporation products are photons, electrons, positrons, and neutrinos.

Credit: Fig. 9, Montero-Camacho et. al 2019

DIRECT DETECTION OF HAWKING RADIATION



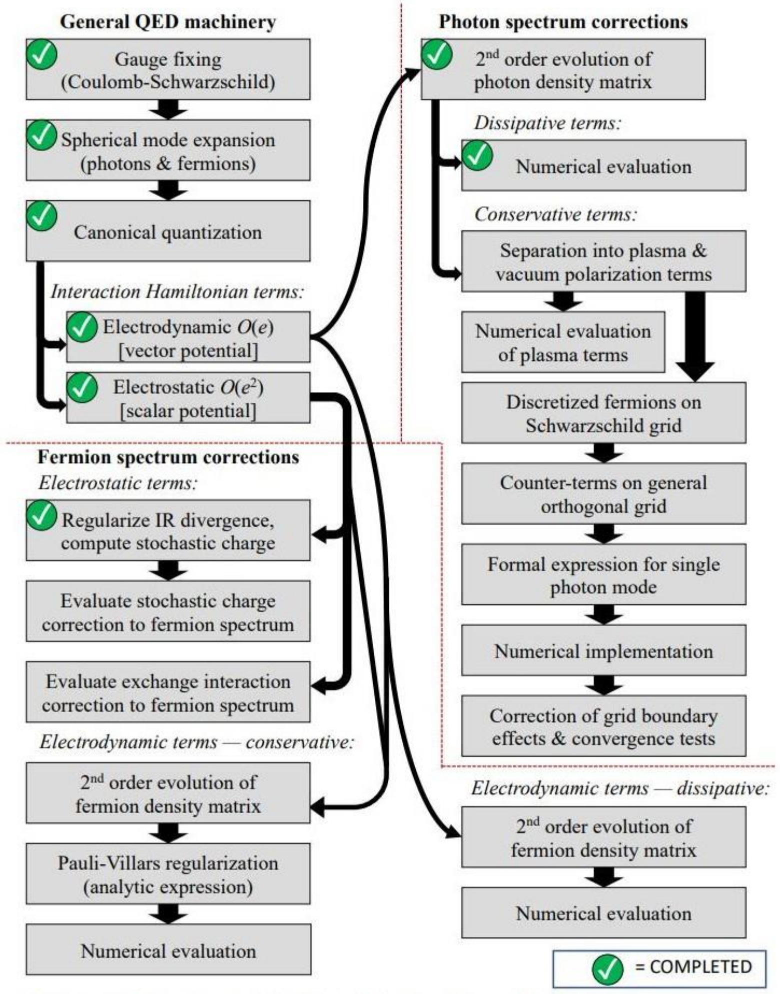
Credit: Fig. 2 from Coogan et. al 2021

- Coogan et. al 2021 states that current and future MeV telescopes may be able to detect Hawking radiation if PBHs make a considerable amount of dark matter at M~10^{16} g.

- **Previous Approach:** Used a code known as BlackHawk to calculate primary and secondary spectra. Secondary spectra (up to order α) is achieved by "stitching" flat spacetime interactions with black hole. See Coogan et. al 2019.

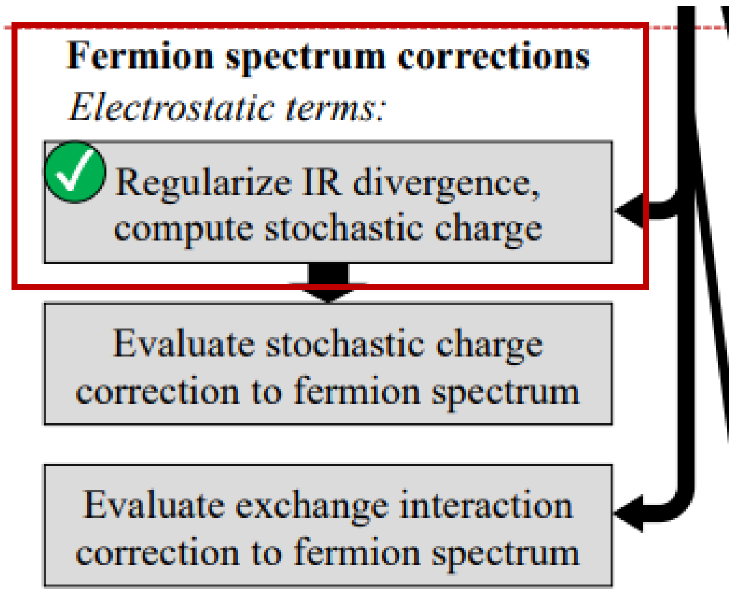
- **Our approach:** Calculate the O(α) change to the photon, electron, and positron spectra when performing the full QED calculation on a Schwarzschild spacetime (neutral and non-rotating).

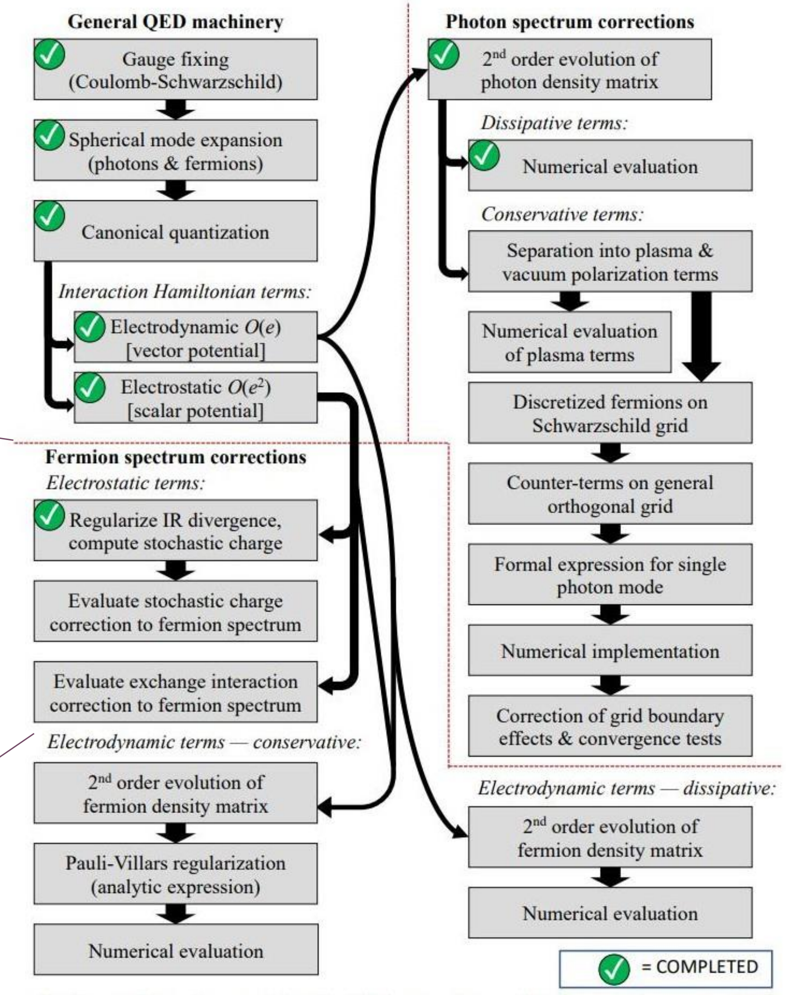
QUANTUM FIELD THEORY ON CURVED SPACETIME APPROACH



QUANTUM FIELD THEORY ON CURVED SPACETIME APPROACH

This talk!

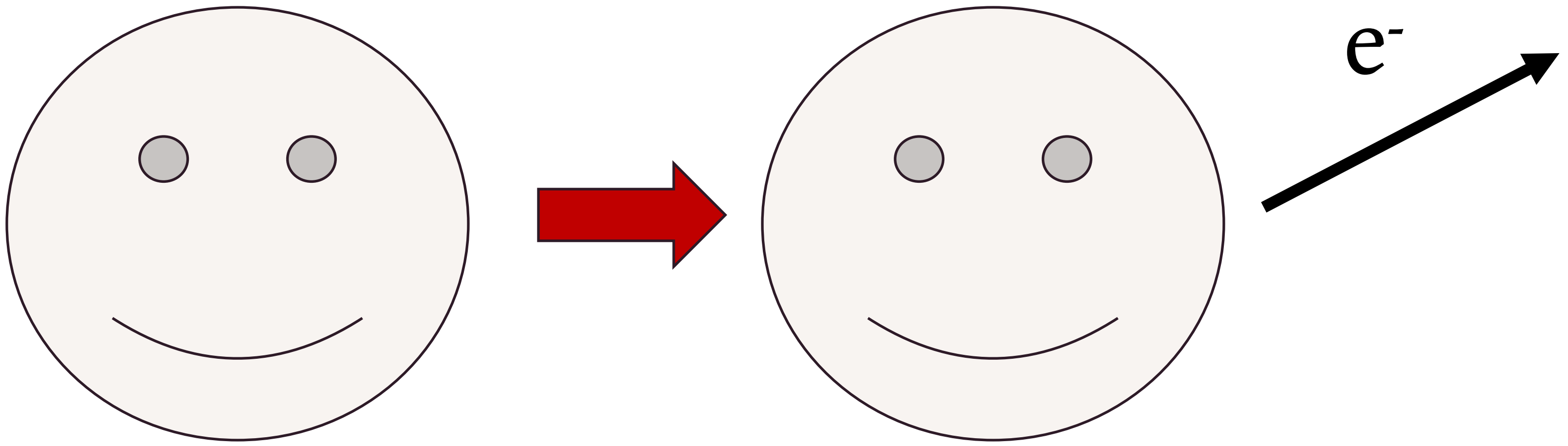




LET'S DO A SIMPLE THOUGHT EXPERIMENT

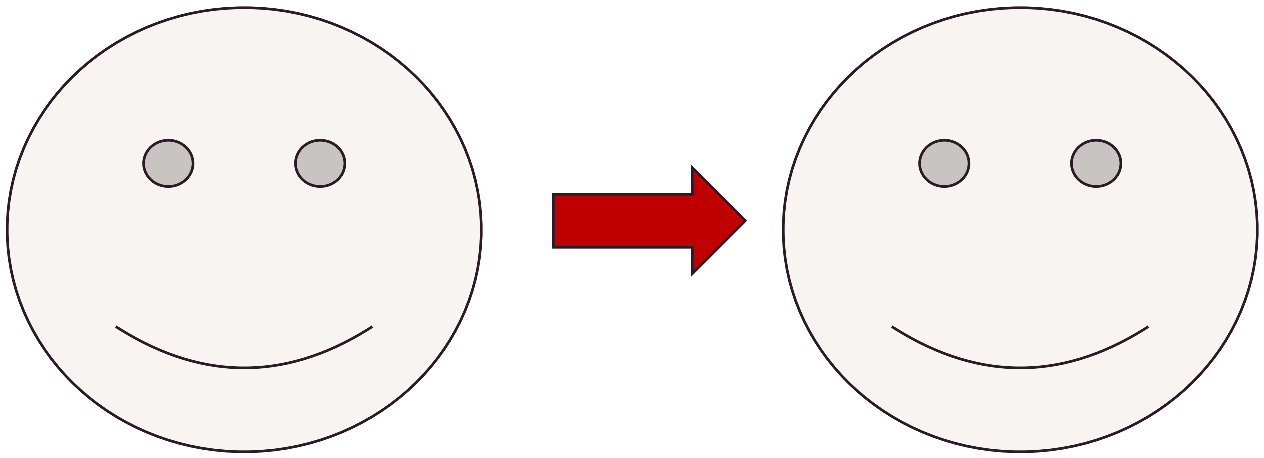
$Q_{BH} = 0$

LET'S DO A SIMPLE THOUGHT EXPERIMENT



$Q_{BH} = +|e|$

LET'S DO A SIMPLE THOUGHT EXPERIMENT

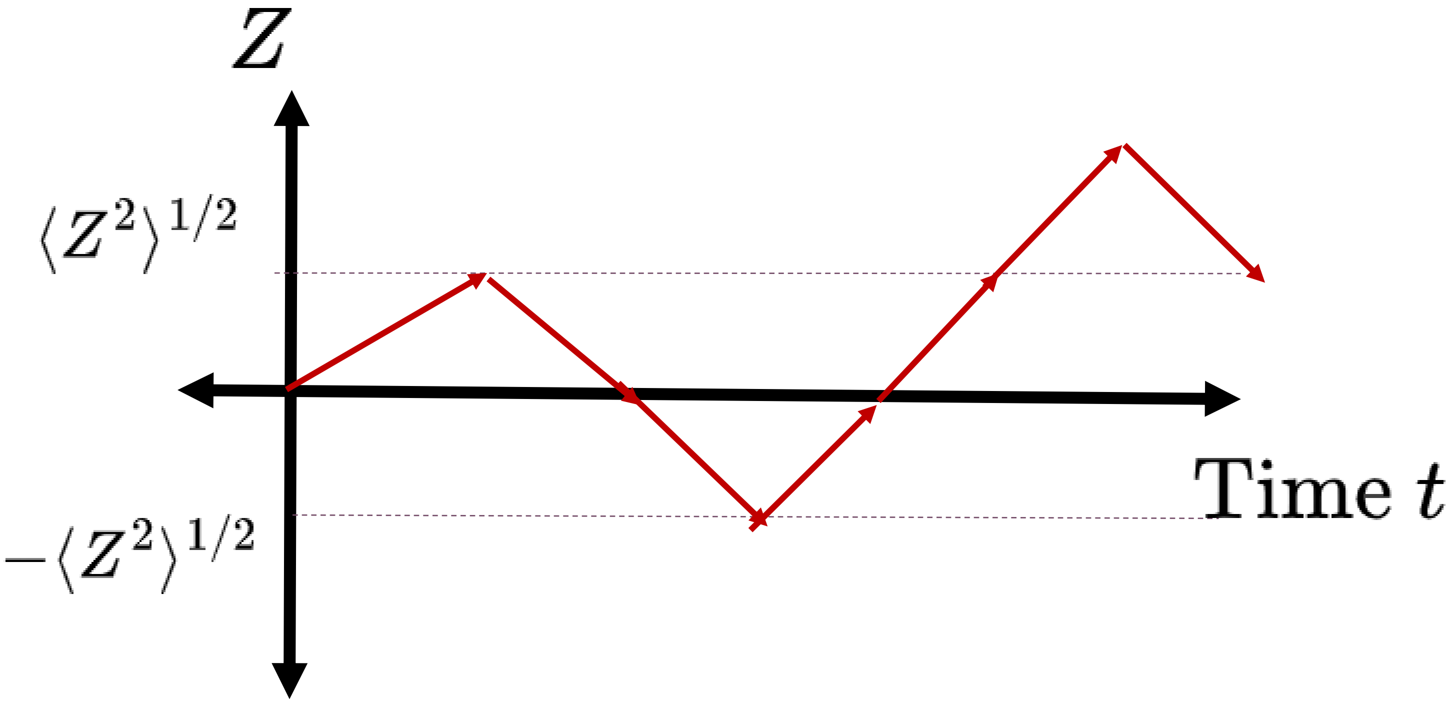


$Q_{BH} = +|e|$

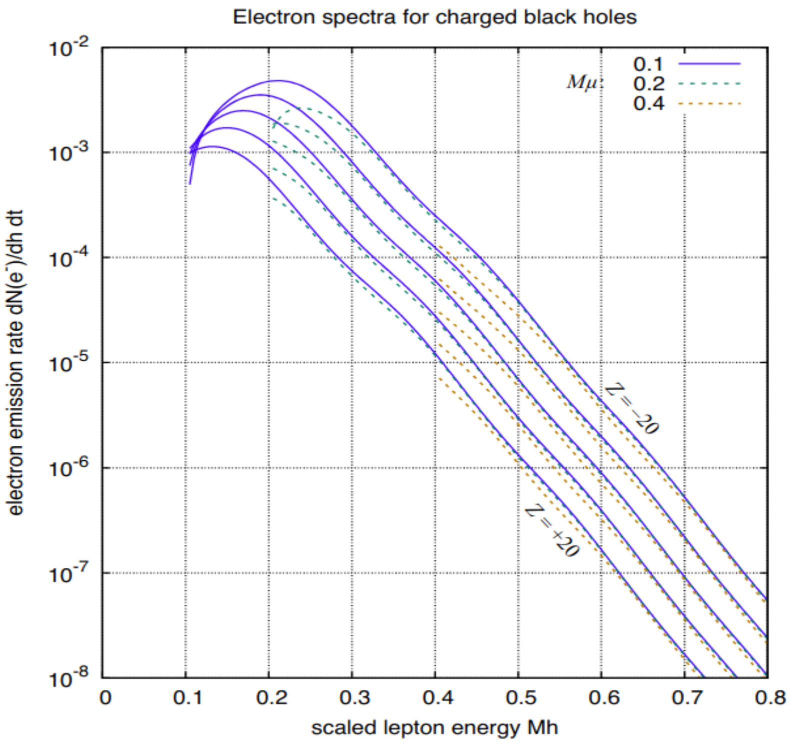
- Will the black hole be more likely to emit an electron or positron next?
- A positron is more likely!
- Known as the stochastic emission of charged particles (stochastic charge).
- Why? Emission rate depends not only on particle energy, but also on the product of the particle's and black hole's charge.

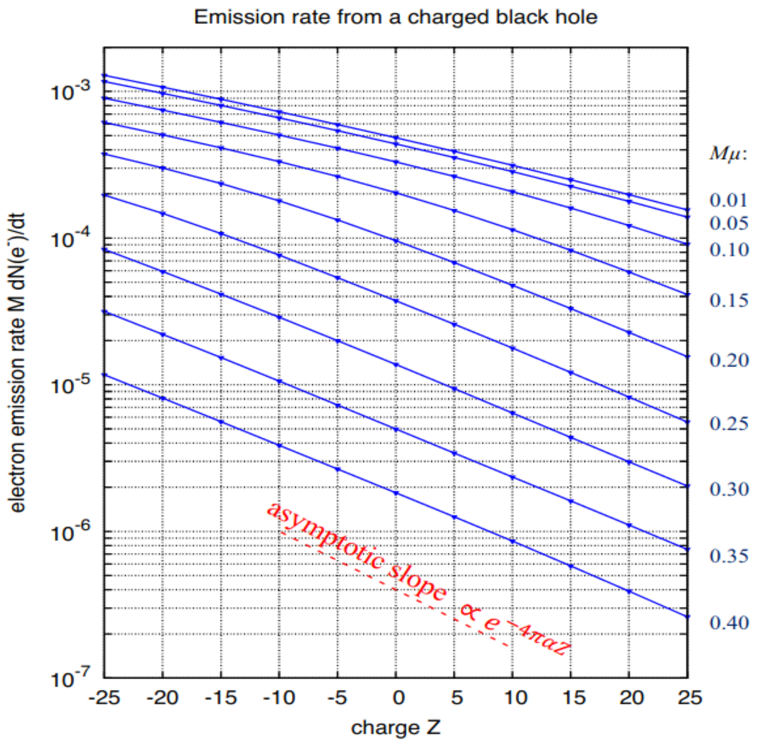
STOCHASTIC EMISSION OF CHARGED PARTICLES

- Probability of emitting a particle with a charge of the same sign as the black hole is more likely!
- Undergoes random walk driving black hole to neutrality.
- Can define a variance.
- Depends on the mass of the black hole.



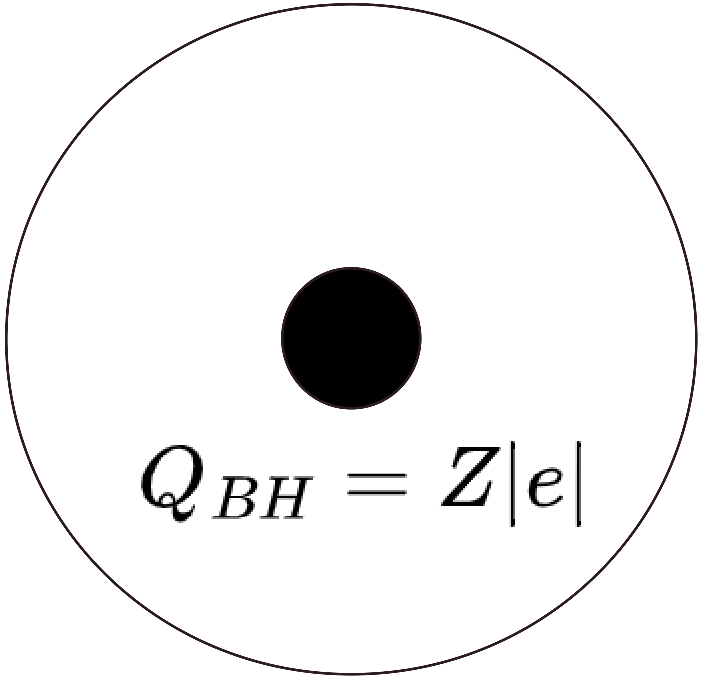
SEMI-CLASSICAL SPECTRA AND EMISSION RATE





Semi-classical Approach: Z

Charge of the black hole changes. Treated like an atomic number.



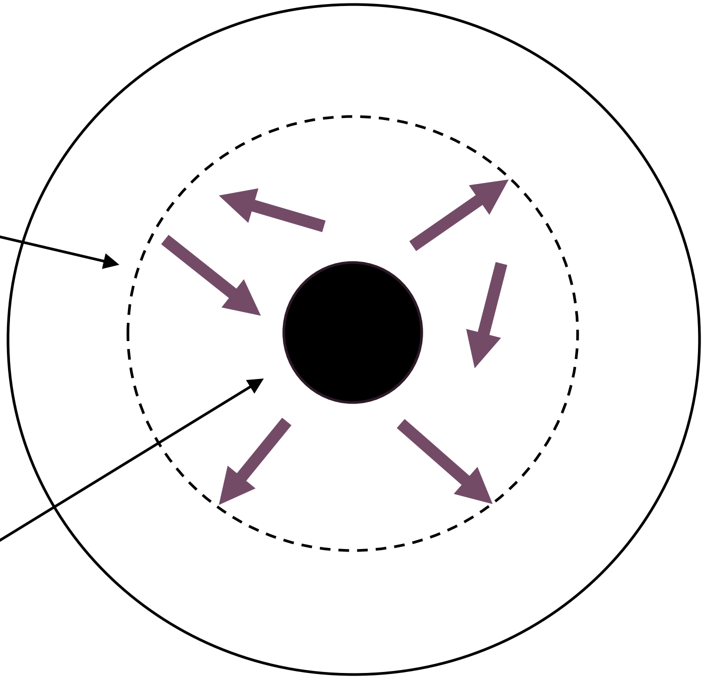
The e^-e^+ plasma screens the charge (weighted by 2M/r)

$\hat{b}_A^\dagger \hat{b}_B,\ \hat{d}_A^\dagger \hat{d}_B,\ \hat{b}_A \hat{d}_B,\ \hat{d}_A^\dagger \hat{b}_B^\dagger$

Charge of the black hole: $\mathcal{Q}_{BH}$

Our Approach: $\hat{\mathcal{Z}}$

Charge of black hole is fixed. Describes the charge distribution of the black hole and the surrounding plasma as an operator.



= Fermions

GOAL OF OUR FIRST PAPER:

$$\langle Z^2 \rangle = \langle \hat{\mathcal{Z}}^2 \rangle$$

GOAL OF OUR FIRST PAPER:

$$\boxed{\langle Z^2 \rangle} = \langle \hat{\mathcal{Z}}^2 \rangle$$

Semi-classical
Prediction

GOAL OF OUR FIRST PAPER:

$$\langle Z^2 \rangle = \boxed{\langle \hat{\mathcal{Z}}^2 \rangle}$$

QFT Formalism

GOAL OF OUR FIRST PAPER:

$$\langle Z^2 \rangle = \langle \hat{\mathcal{Z}}^2 \rangle$$

Our formalism should be able to explain the semi-classical result. That is our conjecture!

AFTER A LONG PERTURBATIVE CALCULATION...

- We derived an analytic expression for the variance.

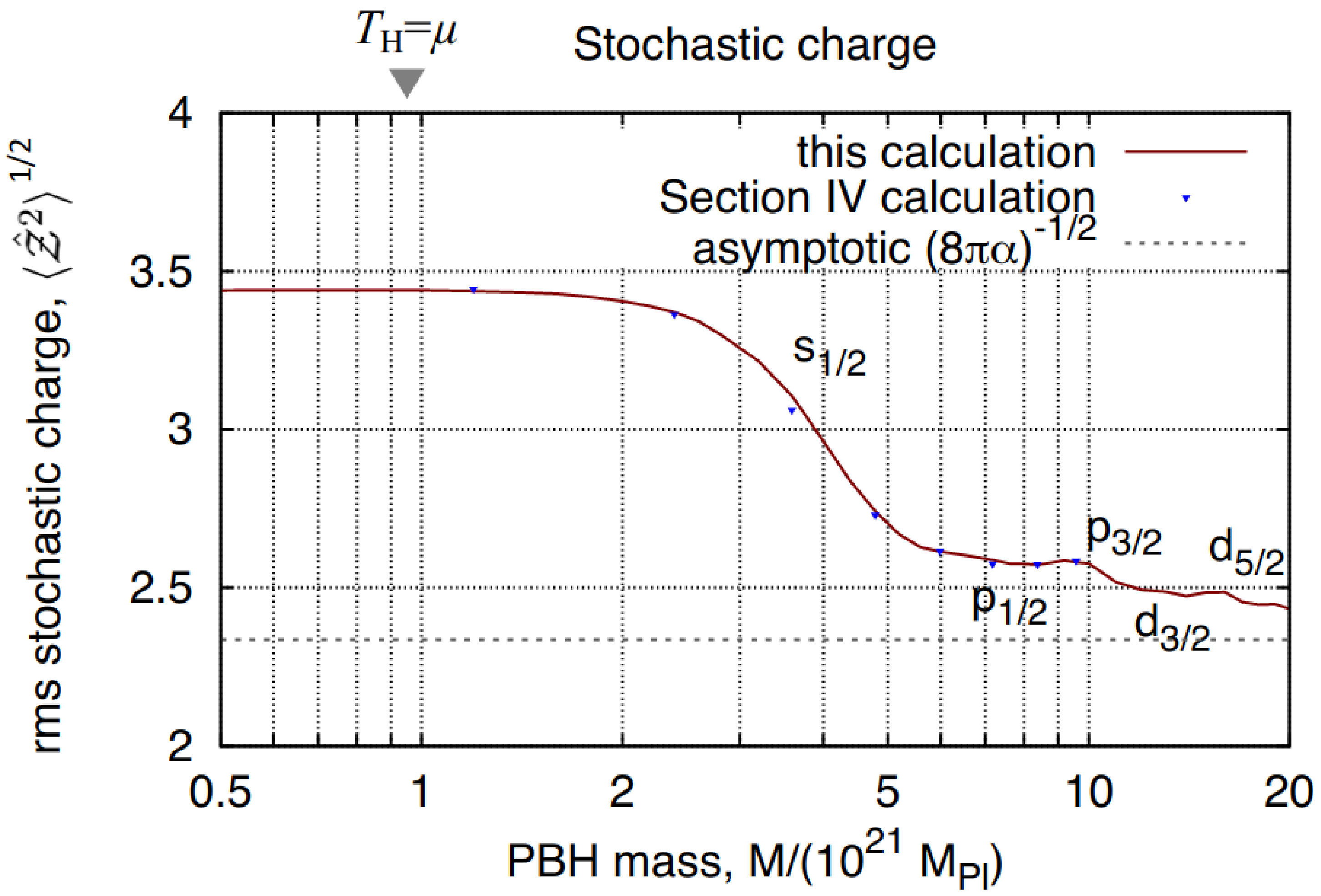
$$\langle \hat{\mathcal{Z}}^2 \rangle = \frac{1}{\Xi} \times 2 \int \frac{dh}{2\pi} \sum_k (2j+1) |T_{\frac{1}{2}kh}|^2 f^{e^-}_{\mathrm{up,up,k}}(h) \left[1 - |T_{\frac{1}{2}kh}|^2 f^{e^-}_{\mathrm{up,up,k}}(h) \right]$$

where

$$\Xi = 2 \times 2 \int \frac{dh}{2\pi} \sum_k (2j+1) \left\{ -\frac{e^2}{8\pi M} \mathbb{T}_{\frac{1}{2}kh} \left. \frac{df^{e^-}_{\mathrm{up,up},k}(h')}{dh'} \right|_{h=h'} - \frac{\partial \mathbb{T}_{\frac{1}{2}kh}}{\partial Z} f^{e^-}_{\mathrm{up,up},k}(h) \right\}$$

- Numerator may be thought of as shot noise.

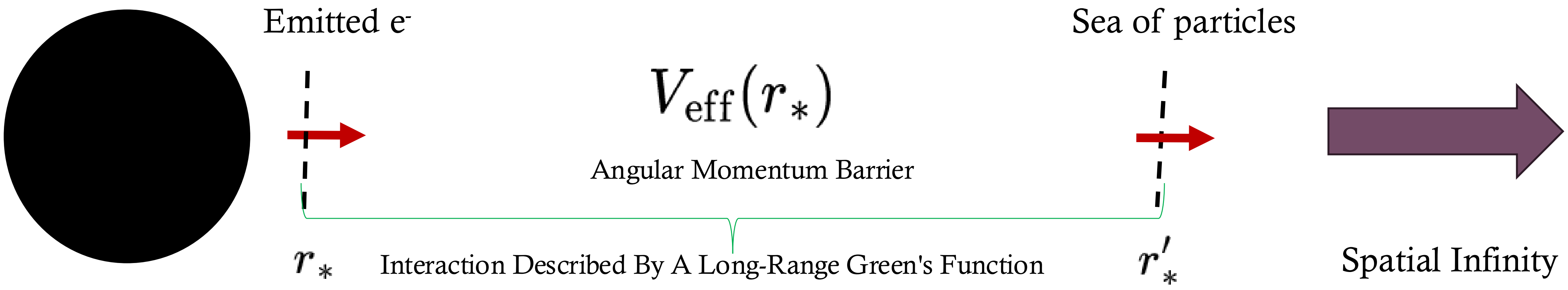
- Denominator may be thought of as all O(α) changes to the emission spectrum when a black hole acquires charge after emitting a particle.



$T_H=\mu$
Stochastic charge
this calculation
Section IV calculation
asymptotic $(8\pi\alpha)^{-1/2}$
$s_{1/2}$
$p_{1/2}$
$p_{3/2}$
$d_{3/2}$
$d_{5/2}$
rms stochastic charge, $\langle \hat{Z}^2 \rangle^{1/2}$
4
3.5
3
2.5
2
0.5
1
2
5
10
20
PBH mass, $M/(10^{21}\ M_{Pl})$

A DIFFERENCE OF PHILOSOPHY

- Our approach: the black hole's charge doesn't change.
- What is stochastic charge? The collective effect of an emitted particle interacting with the sea of other Hawking radiated particles from the BH horizon to spatial infinity.



CONCLUSIONS

- Our goal is to prove stochastic charge is a consequence of QED on a Schwarzschild spacetime.
- We proved the semi-classical variance $\langle Z^2 \rangle$ agrees with the variance when calculated in the full quantum formalism (Conjecture #1)
- Next step: calculating the emission spectra and proving it can replicate the semi-classical result (Conjecture #2).
- Fluctuations in the charge do not originate from the black hole (its charge remains fixed), but rather the collective effect of Hawking-emitted particles interacting with the plasma of electrons and positrons exterior to the black hole.

arxiv: 2407.09724

THANK YOU! QUESTIONS?

I want to also give a not-so-subtle shoutout that I am on the job market and looking for a post-doc. If you like my work, please contact me and I would love to chat! :)

Email: vasquez.119@osu.edu

APPENDIX

SEMI-CLASSICAL PREDICTION

- Emission rate: $$\frac{dN_{e^\pm}}{dh\,dt}(Z) = \frac{1}{2\pi}\sum_k (2j+1)\frac{\mathbb{T}_{\frac{1}{2}kh}(Z\mp 1;\pm)}{e^{8\pi M[h\mp\alpha(Z\mp1)/2M]}+1} \quad \text{for } \ h > \mu$$

- Assume charge conjugation invariance:

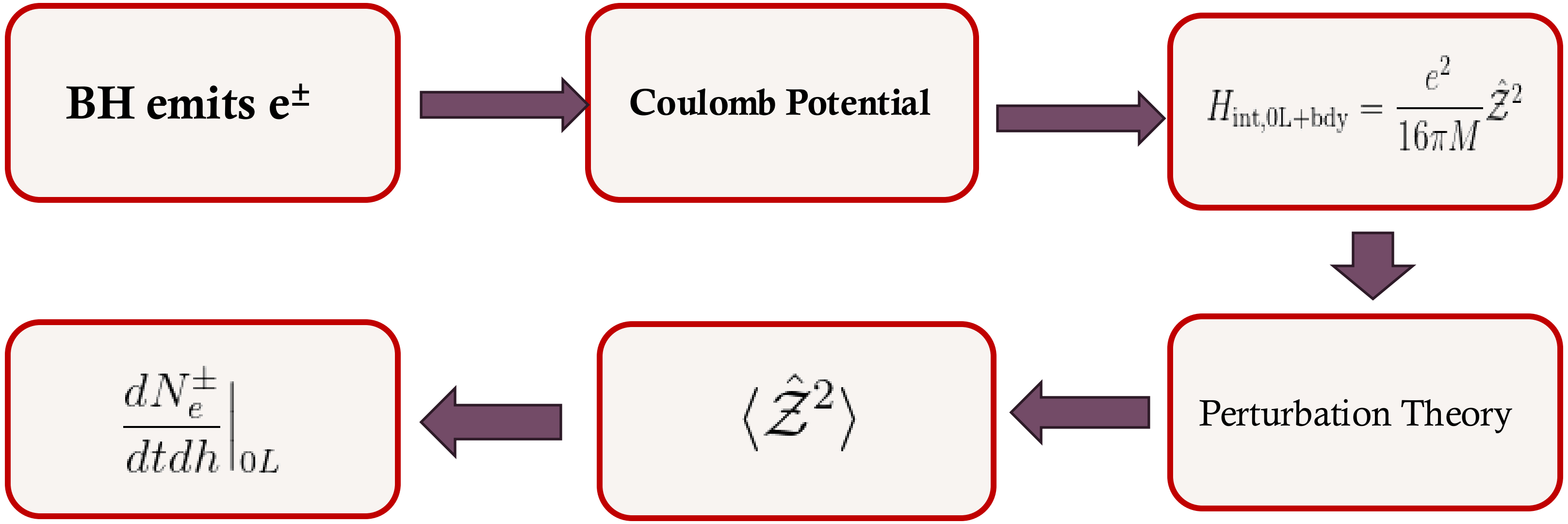
$$\frac{P(Z)}{P(Z-1)} = \frac{\Gamma(Z-1\to Z)}{\Gamma(Z\to Z-1)} \quad \text{or} \quad \ln P(Z) = \sum_{Z'=1}^{Z} \ln\frac{\Gamma(Z'-1\to Z')}{\Gamma(Z'\to Z'-1)} + \text{const}$$

- Expansion per powers of α: $\Gamma(Z\to Z+1) = \Gamma_{e,0} + \alpha(Z+1)\Gamma_{e,1} + [\alpha(Z+1)]^2\Gamma_{e,2}\ldots$

- To lowest order, the probability distribution is a Gaussian with variance: $$\langle Z^2\rangle \approx -\frac{\Gamma_{e,0}}{2\alpha\Gamma_{e,1}}$$

FLOWCHART OF CALCULATION

- I'm happy to discuss any of the intermediate steps, but we would be here for ages going through the whole thing.



LONG-RANGE INTERACTION

- Since the black hole acquires a charge, it will also acquire a Coulomb potential.

$$H_\Phi = \frac{1}{2}e^2 \sum_{\ell m} G_\ell(r_*, r'_*) D_{\ell m}(r_\star) D^\dagger_{\ell m}(r'_\star) dr_* dr'_*$$

- Green's function goes to zero at the BH horizon and at spatial infinity? Good! However, the pesky l=0 (monopole) Green's function doesn't.

- This long-range interaction introduces an IR divergence that we need to correct for.

- After some math, we derive this pesky long-range correction by calculating the spectra produced by the following Hamiltonian:

$$H_{\text{int,0L+bdy}} = \frac{e^2 \hat{\mathcal{Z}}^2}{16\pi M}$$

SCREENED CHARGE

- We define an operator, like an atomic number, that screens the charge of the black hole with the surrounding plasma.

$$\hat{\mathcal{Z}} = -\int \frac{dhdh'}{(2\pi)^2} \sum_{XX'} \sum_{km} \Big[\mathcal{I}^{(1)}_{XX'k}(h,h') \hat{b}^\dagger_{Xkmh} \hat{b}_{X'kmh'} + \mathcal{I}^{(2)}_{XX'k}(h,h') \hat{b}^\dagger_{Xkmh} \hat{d}^\dagger_{X'kmh'}$$

$$+ \mathcal{I}^{(2)*}_{X'Xk}(h',h) \hat{d}_{Xkmh} \hat{b}_{X'kmh'} - \mathcal{I}^{(1)*}_{X'X-k}(h',h) \hat{d}^\dagger_{Xkmh} \hat{d}_{X'kmh'} \Big] - \frac{\mathcal{Q}_-}{e}$$

- Time evolution: $\frac{d}{dt}\langle \hat{\mathcal{Z}}^2 \rangle = C_0 + C_1 \langle \hat{\mathcal{Z}}^2 \rangle \approx 0 \quad \Rightarrow \boxed{\langle \hat{\mathcal{Z}}^2 \rangle = -\frac{C_0}{C_1}}$

CONJECTURE #2: CURRENTLY BEING WORKED ON

$$
\begin{aligned}
\left.\frac{dN_-}{dtdh}\right|_{0\mathrm{L}} = -\frac{e^2}{16\pi^2 M}\Im\int\frac{dh'}{2\pi}\sum_{X'k}(2j+1)\Bigg\{ & |T_{\frac{1}{2}kh}|^2\Bigg[\mathcal{I}^{(1)}_{\mathrm{up,X'k}}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{\mathrm{up,X'}}(h,h') - \mathcal{I}^{(2)}_{\mathrm{up,X'k}}(h,h')\Gamma^{\hat{\mathcal{Z}}d^\dagger b^\dagger(k)}_{X',\mathrm{up}}(h',h) \\
& + \mathcal{I}^{(2)\star}_{\mathrm{up,X'k}}(h,h')\Gamma^{\hat{\mathcal{Z}}bd(k)}_{\mathrm{up,X'}}(h,h') - \mathcal{I}^{(1)\star}_{\mathrm{up},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{X',\mathrm{up}}(h',h)\Bigg] \\
& + |R_{\frac{1}{2}kh}|^2\Bigg[\mathcal{I}^{(1)}_{\mathrm{in,X'k}}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{\mathrm{in,X'}}(h,h') - \mathcal{I}^{(2)}_{\mathrm{in},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}d^\dagger b^\dagger(k)}_{X',\mathrm{in}}(h',h) \\
& + \mathcal{I}^{(2)\star}_{\mathrm{in},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}bd(k)}_{\mathrm{in,X'}}(h,h') - \mathcal{I}^{(1)\star}_{\mathrm{in},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{X',\mathrm{in}}(h',h)\Bigg] \\
& + T_{\frac{1}{2}kh}R^\star_{\frac{1}{2}kh}\Bigg[\mathcal{I}^{(1)}_{\mathrm{up,X'k}}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{\mathrm{in,X'}}(h,h') - \mathcal{I}^{(2)}_{\mathrm{up},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}d^\dagger b^\dagger(k)}_{X',\mathrm{in}}(h',h) \\
& + \mathcal{I}^{(2)\star}_{\mathrm{in},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}bd(k)}_{\mathrm{up,X'}}(h,h') - \mathcal{I}^{(1)\star}_{\mathrm{in},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{X',\mathrm{up}}(h',h)\Bigg] \\
& + T^\star_{\frac{1}{2}kh}R_{\frac{1}{2}kh}\Bigg[\mathcal{I}^{(1)}_{\mathrm{in,X'k}}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{\mathrm{up,X'}}(h,h') - \mathcal{I}^{(2)}_{\mathrm{in},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}d^\dagger b^\dagger(k)}_{X',\mathrm{up}}(h',h) \\
& + \mathcal{I}^{(2)\star}_{\mathrm{up},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}bd(k)}_{\mathrm{in,X'}}(h,h') - \mathcal{I}^{(1)\star}_{\mathrm{up},X'k}(h,h')\Gamma^{\hat{\mathcal{Z}}b^\dagger b(k)}_{X',\mathrm{in}}(h',h)\Bigg]\Bigg\},
\end{aligned}
$$