

Using Directional Flavour Compositions of High Energy Astrophysical Neutrinos to Constrain Lorentz Invariance Violation

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Outline

1. What are high energy astrophysical neutrinos?
2. How do we know their flavour ratios?
3. How can Lorentz Invariance Violation cause anisotropies?
4. How much can we constrain the effects with data now?



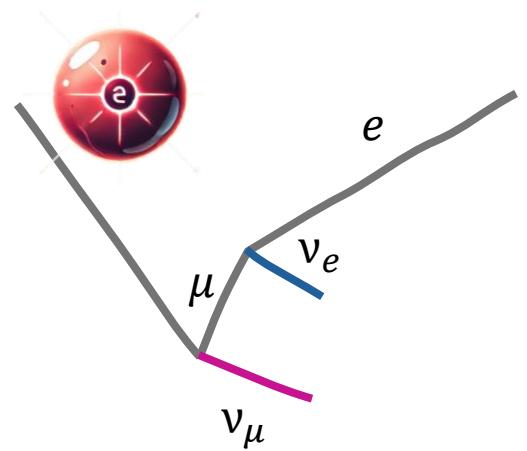
High Energy Astrophysical Neutrinos



A cosmic ray
produces pions



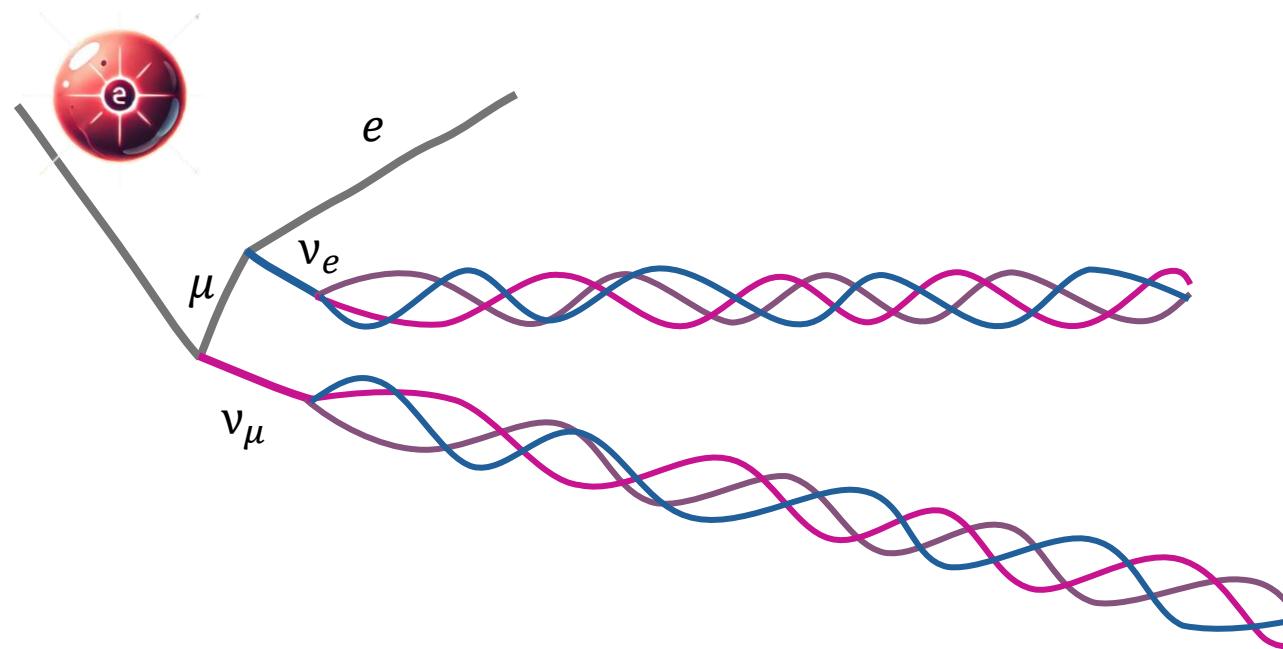
High Energy Astrophysical Neutrinos



Pions decay to leptons
and neutrinos

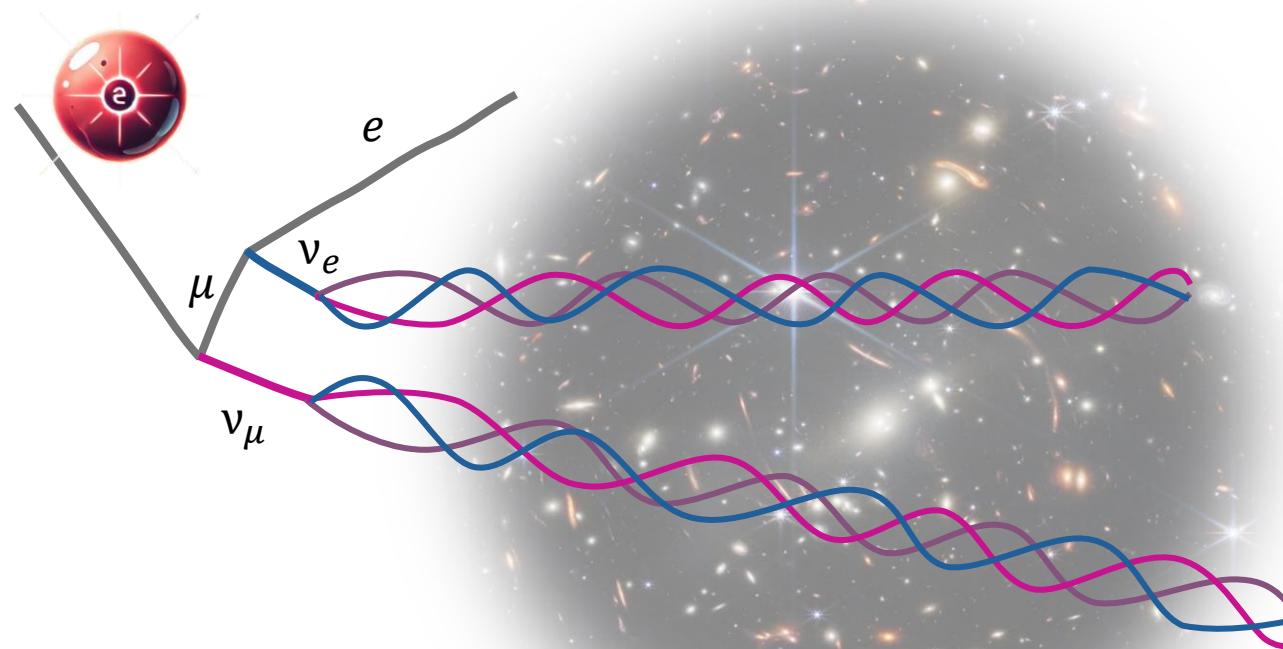


High Energy Astrophysical Neutrinos



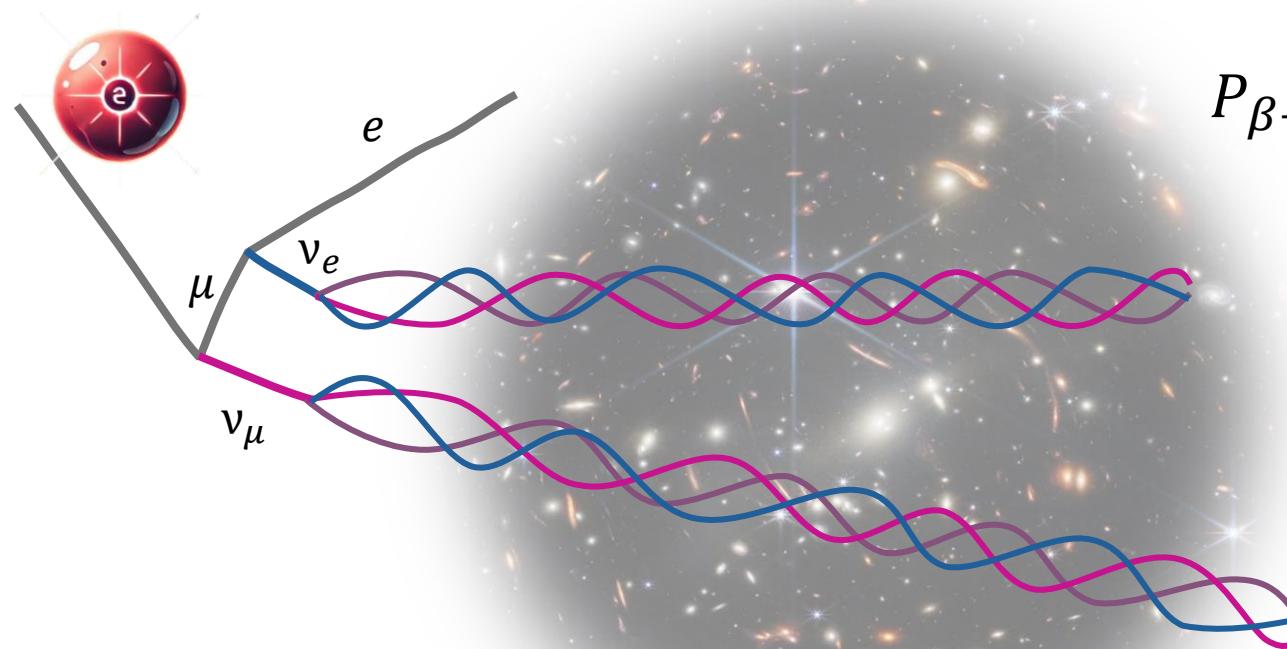
Neutrinos oscillate
over several Gpc

High Energy Astrophysical Neutrinos



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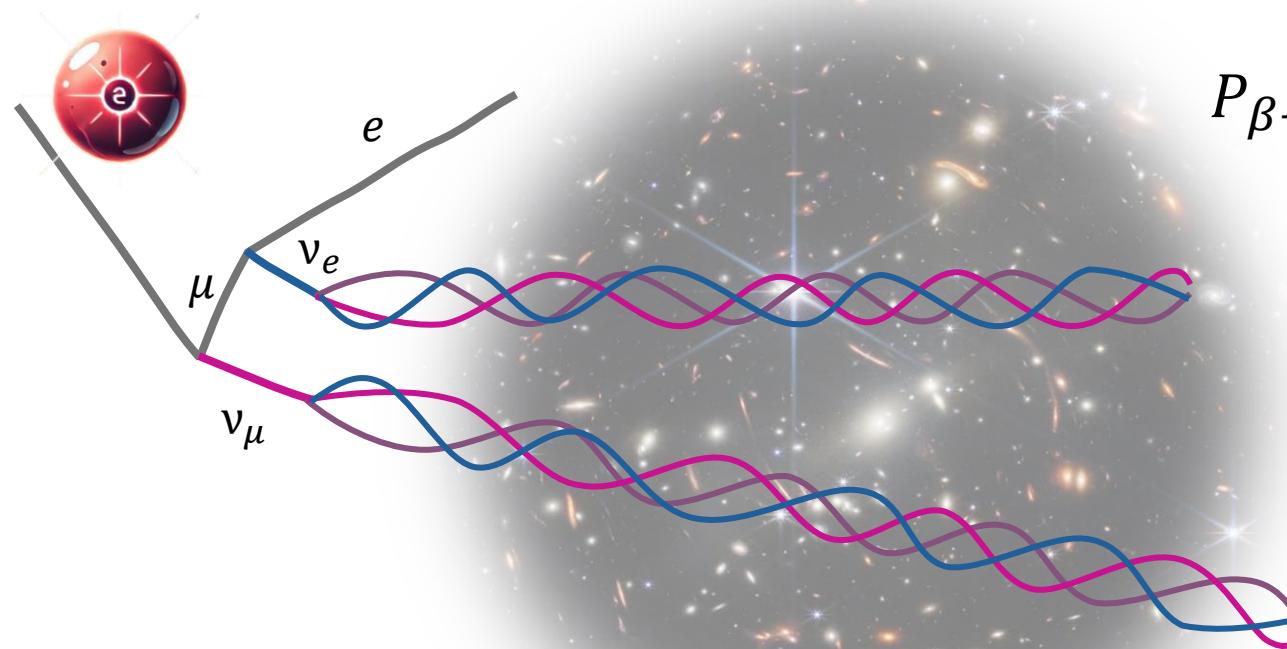
High Energy Astrophysical Neutrinos



$$P_{\beta \rightarrow \alpha} \sim \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Neutrinos oscillate
over several Gpc

High Energy Astrophysical Neutrinos



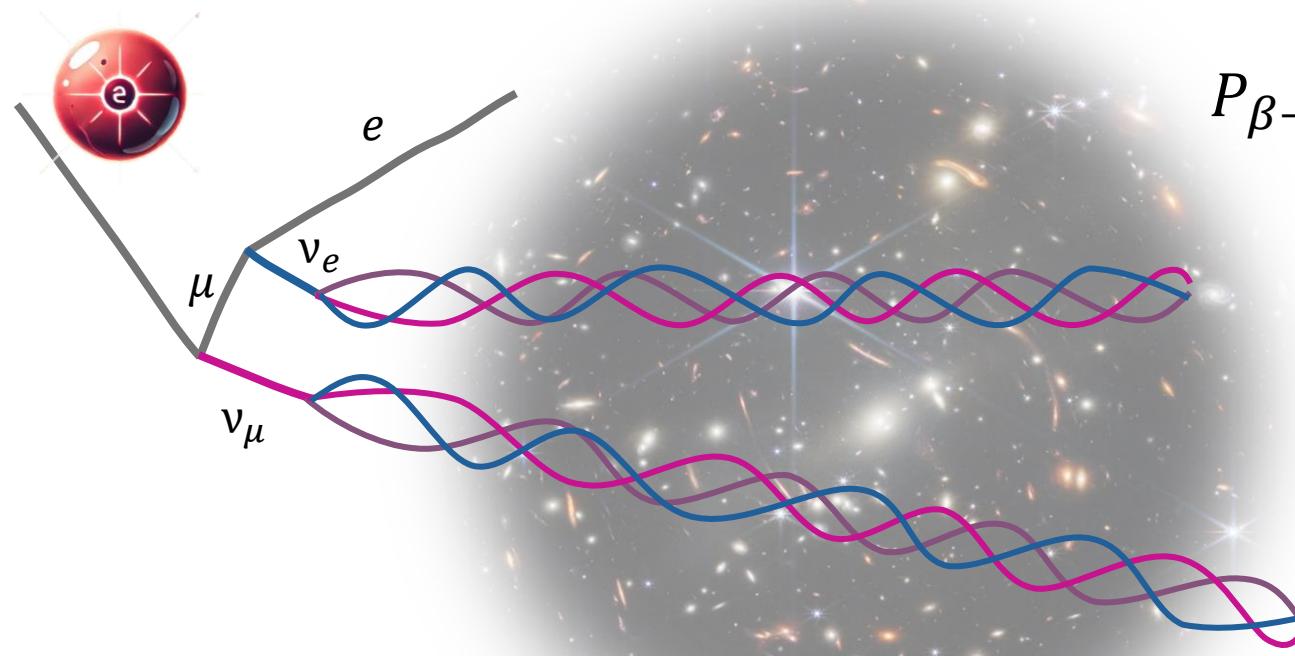
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↓

$$U = U_{PMNS}$$

High Energy Astrophysical Neutrinos



Neutrinos oscillate
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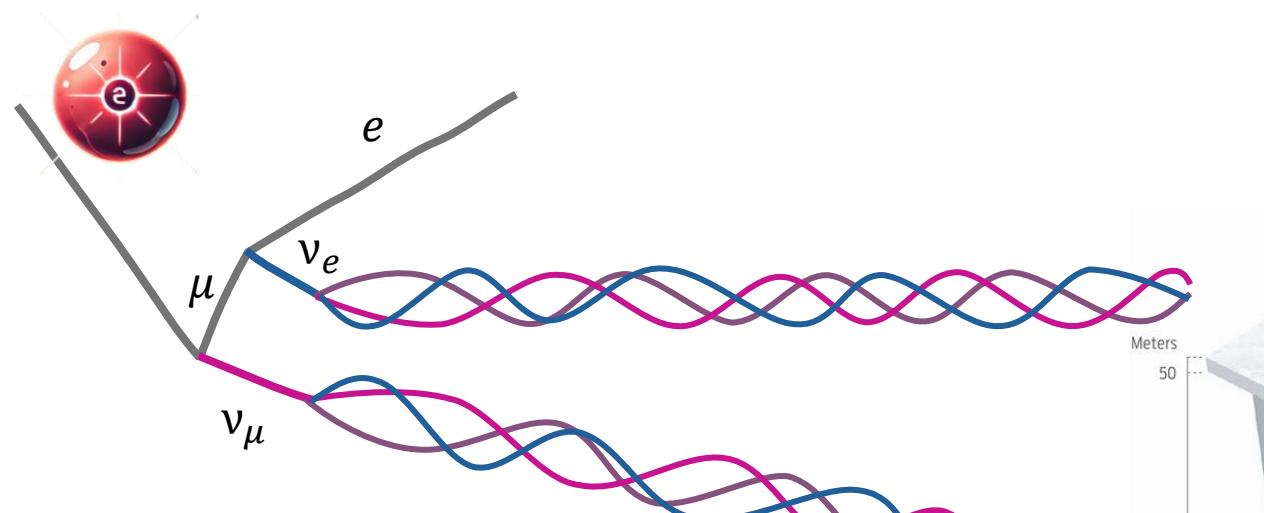
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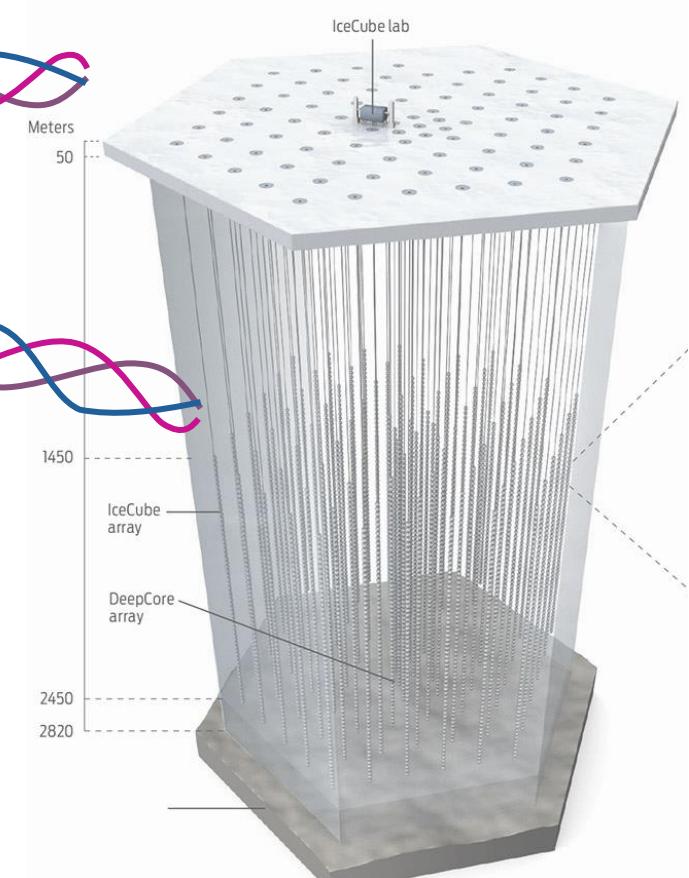
$$U = U_{PMNS}$$

New physics:
different U

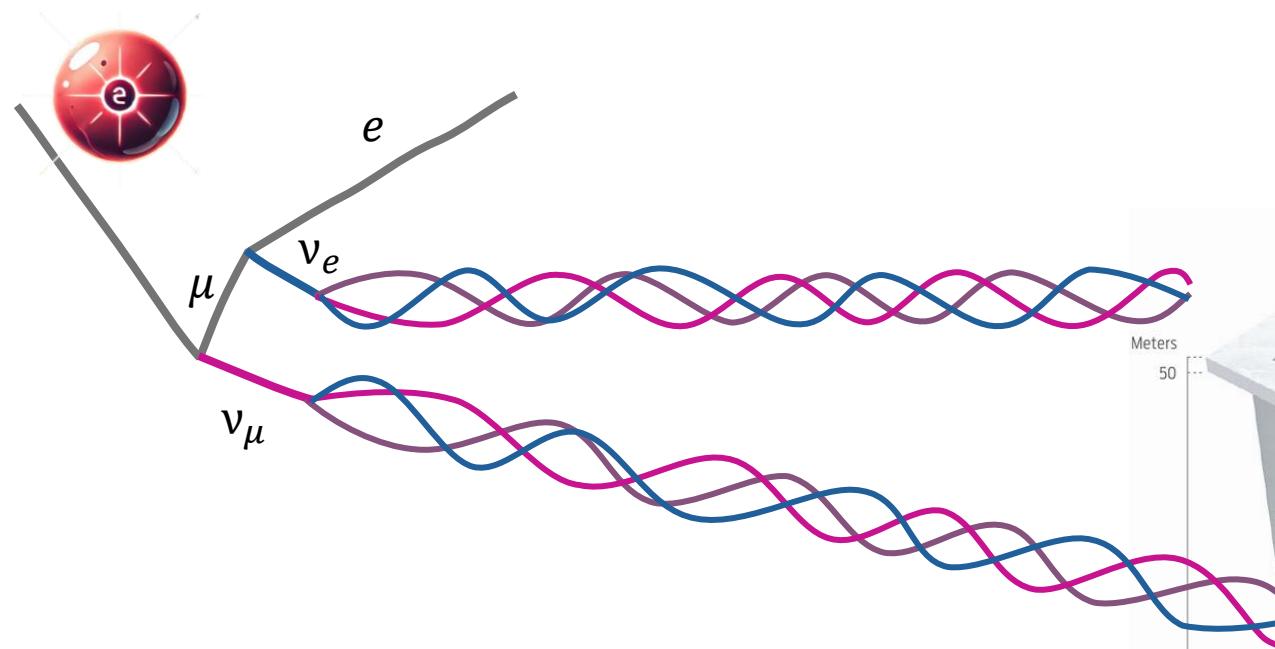
High Energy Astrophysical Neutrinos



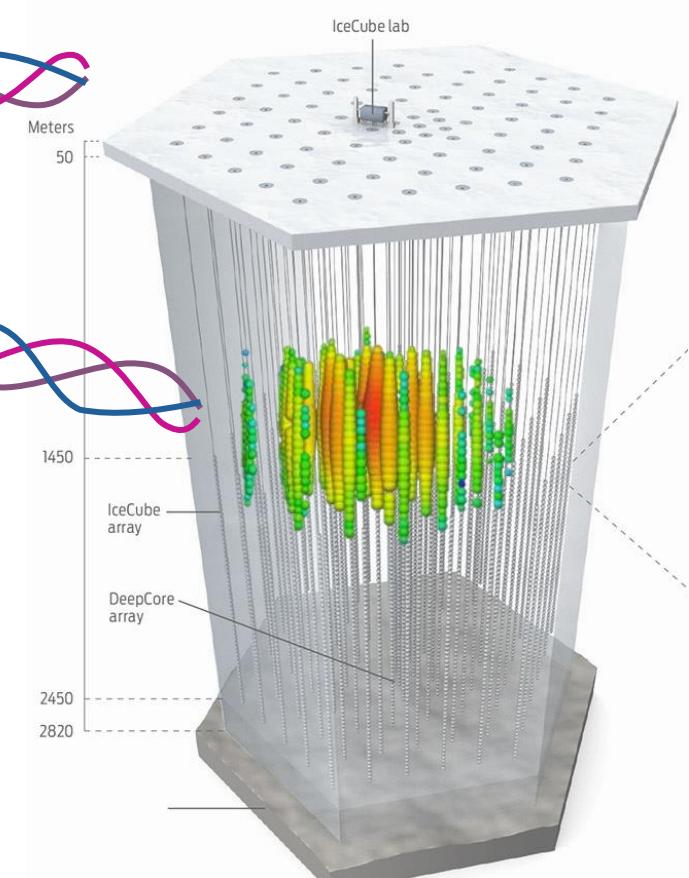
...then are detected in IceCube
as



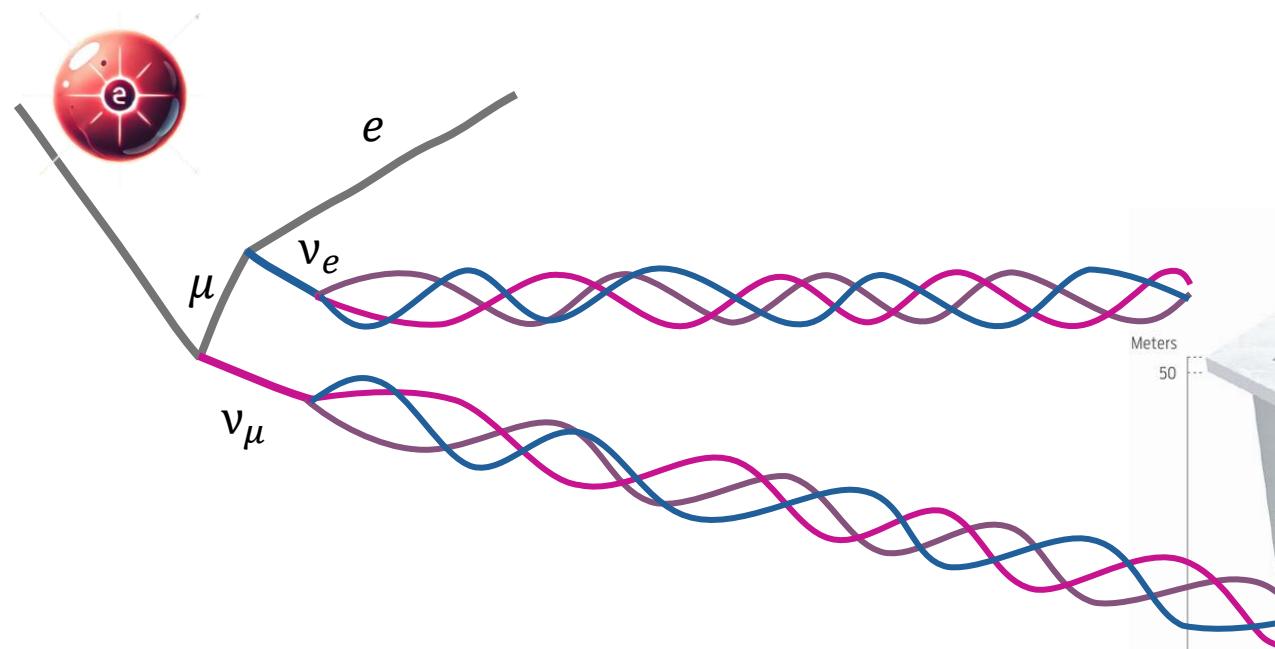
High Energy Astrophysical Neutrinos



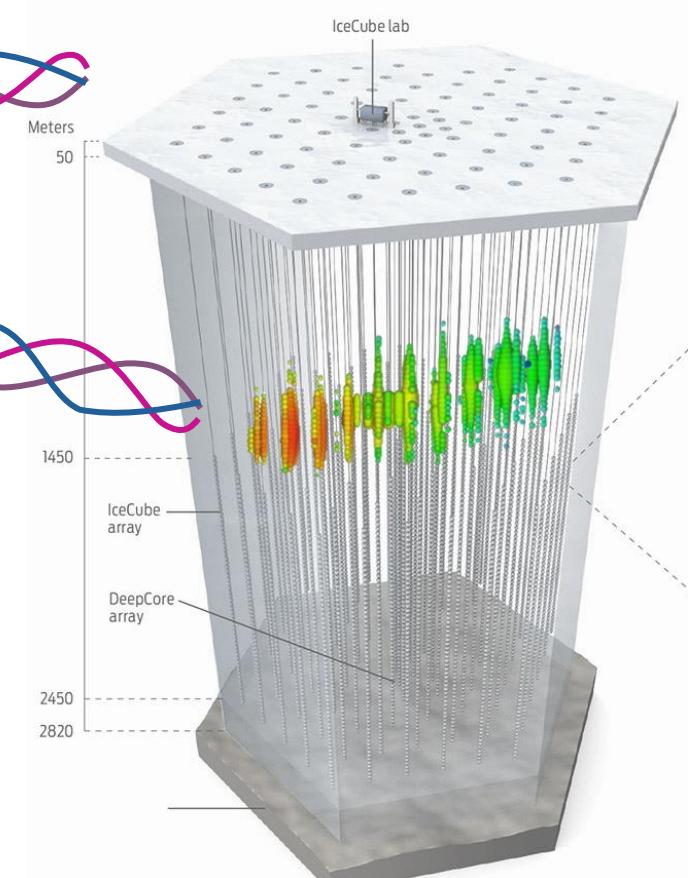
...then are detected in IceCube
as cascades



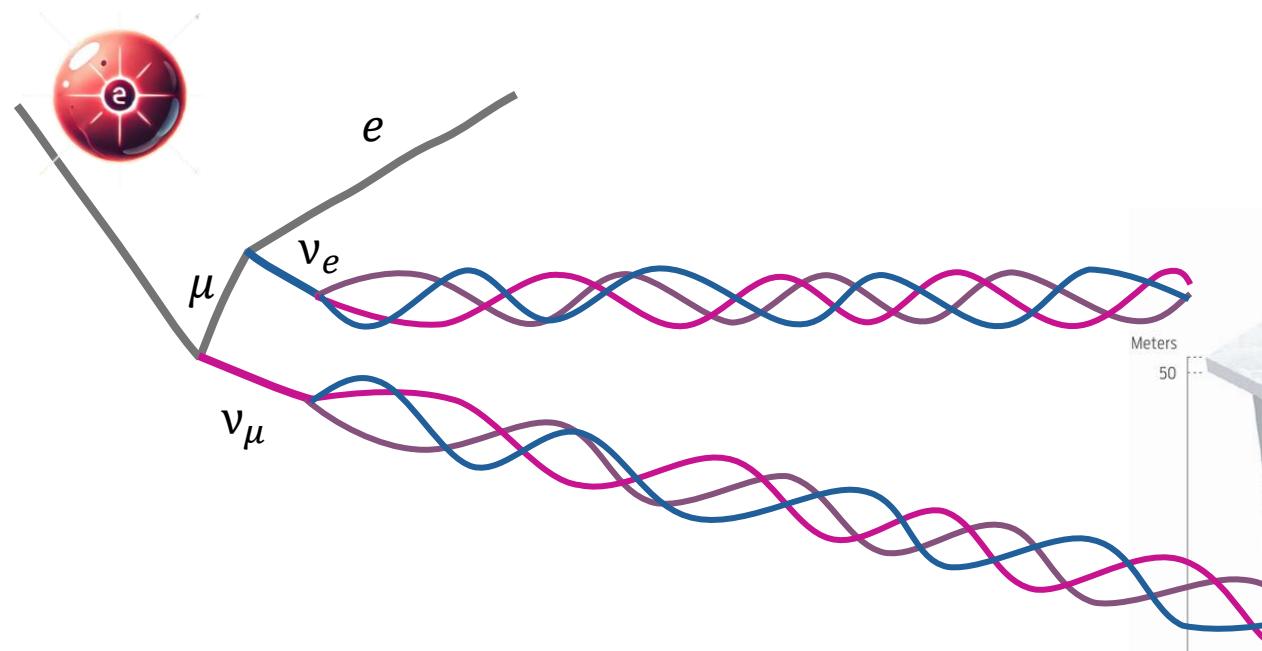
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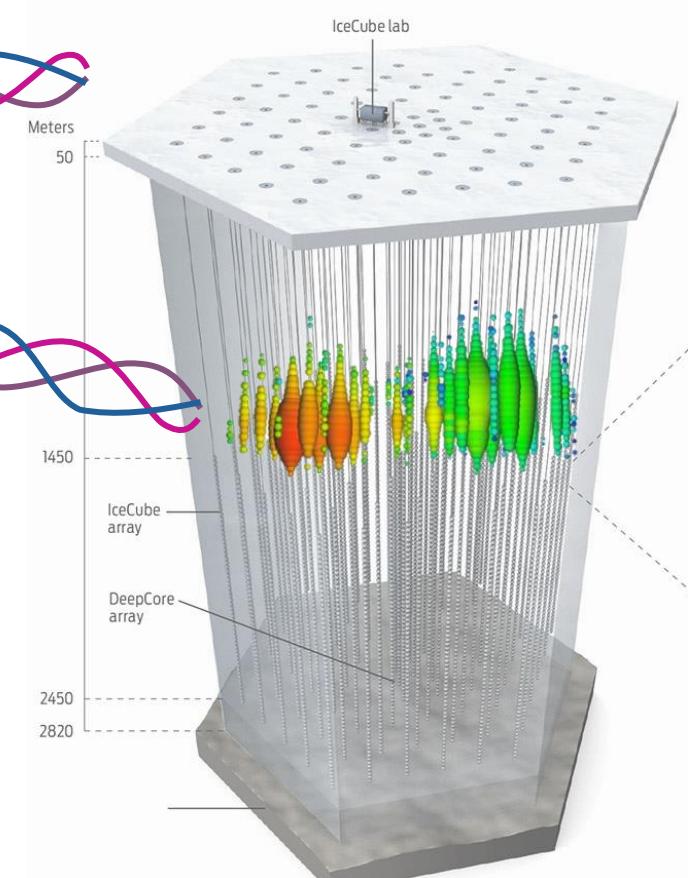
...then are detected in IceCube
as cascades, tracks



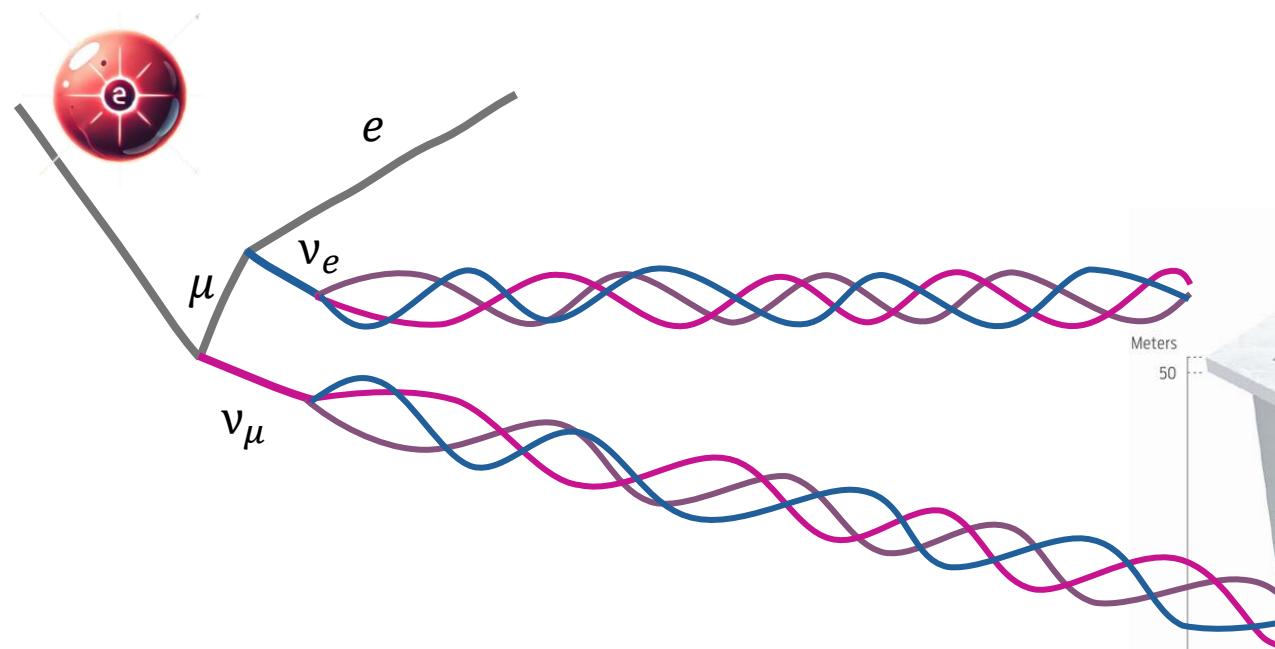
High Energy Astrophysical Neutrinos



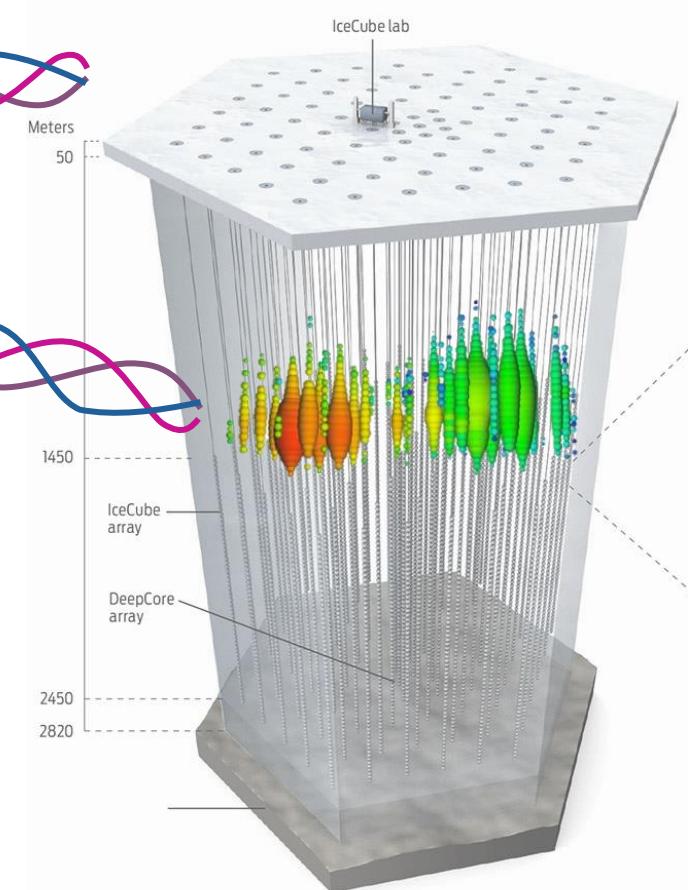
...then are detected in IceCube
as cascades, tracks or double
cascades



High Energy Astrophysical Neutrinos

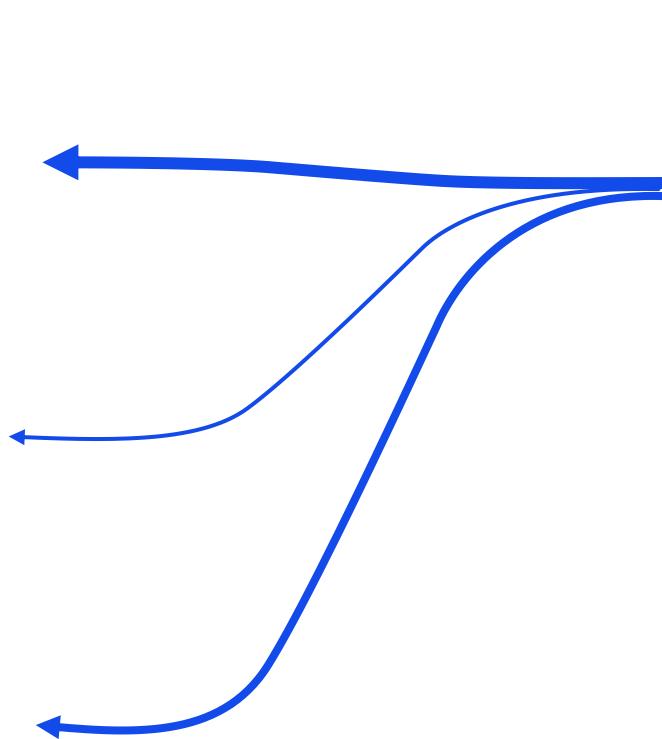
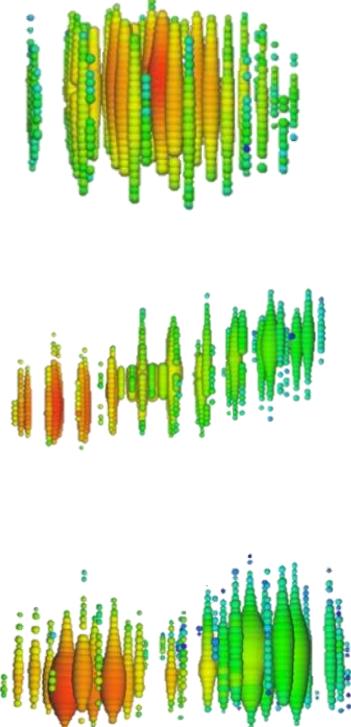


depending on the
neutrino flavour

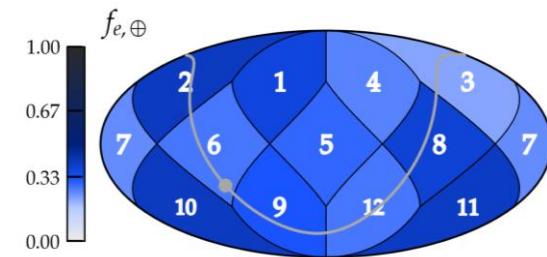


Directional Flavour Ratios in HESE data

We can reconstruct neutrino energy, direction and flavour



not to scale

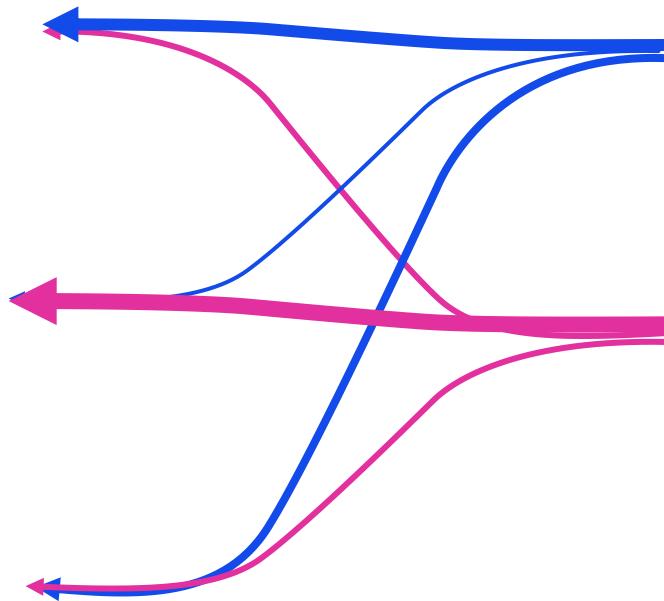
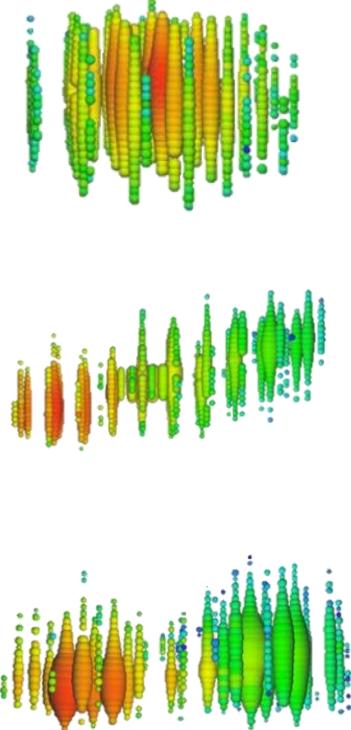


See: <https://arxiv.org/abs/2310.15224>

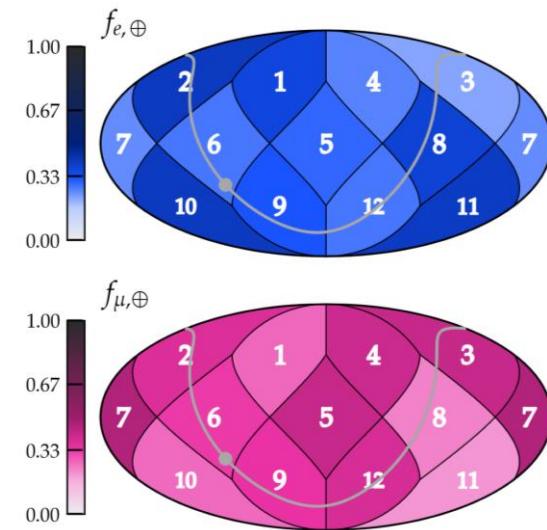
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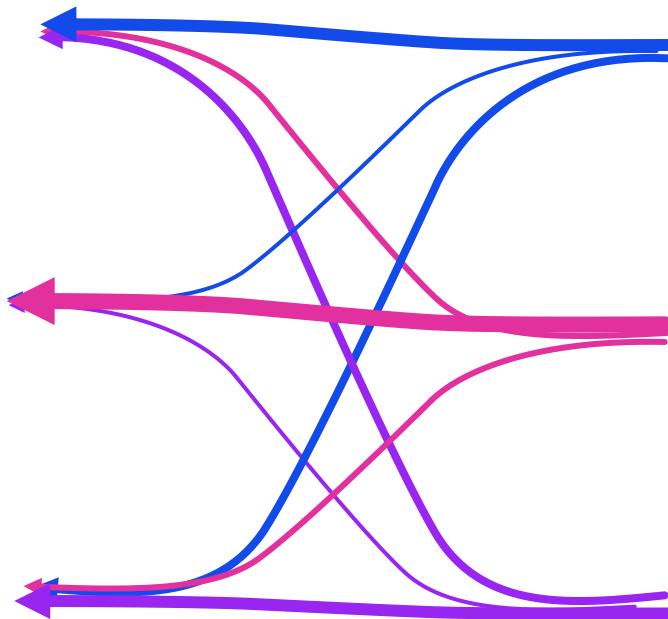
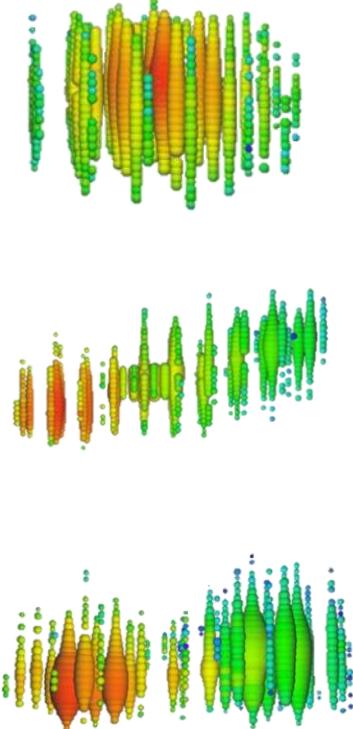


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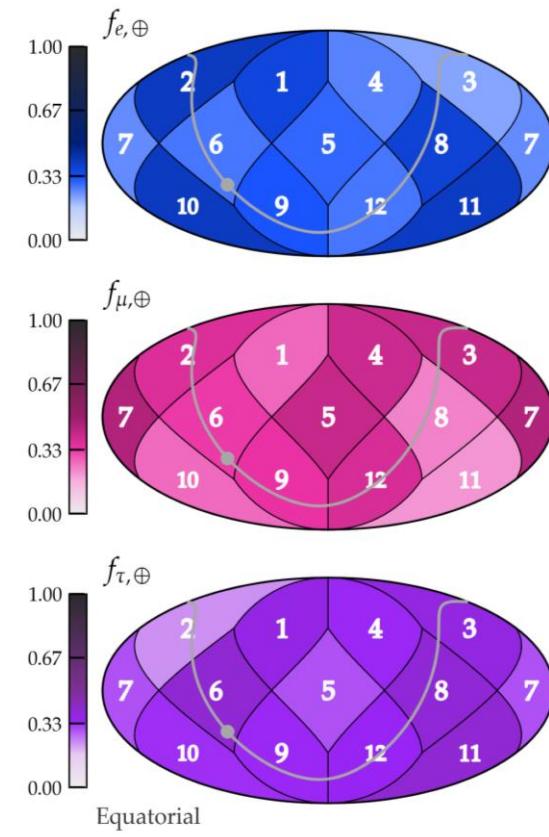
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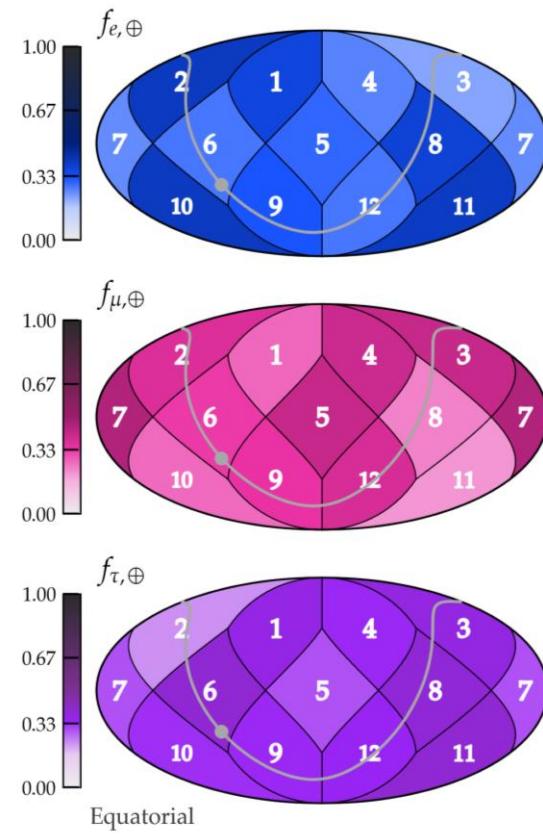
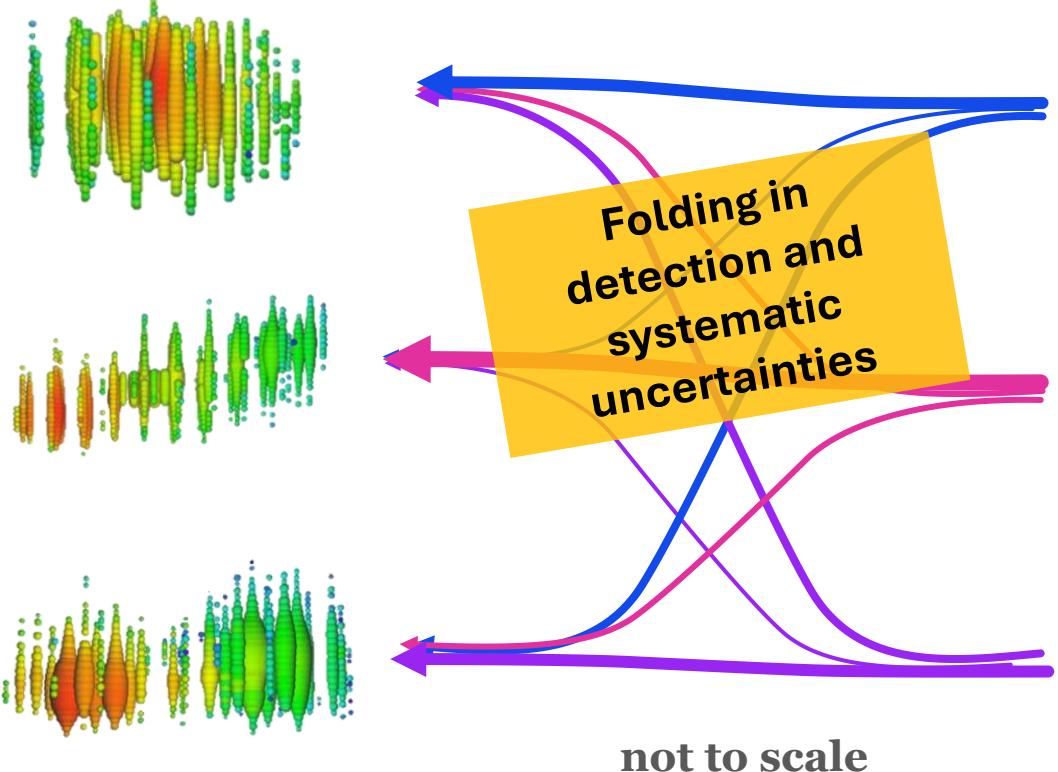
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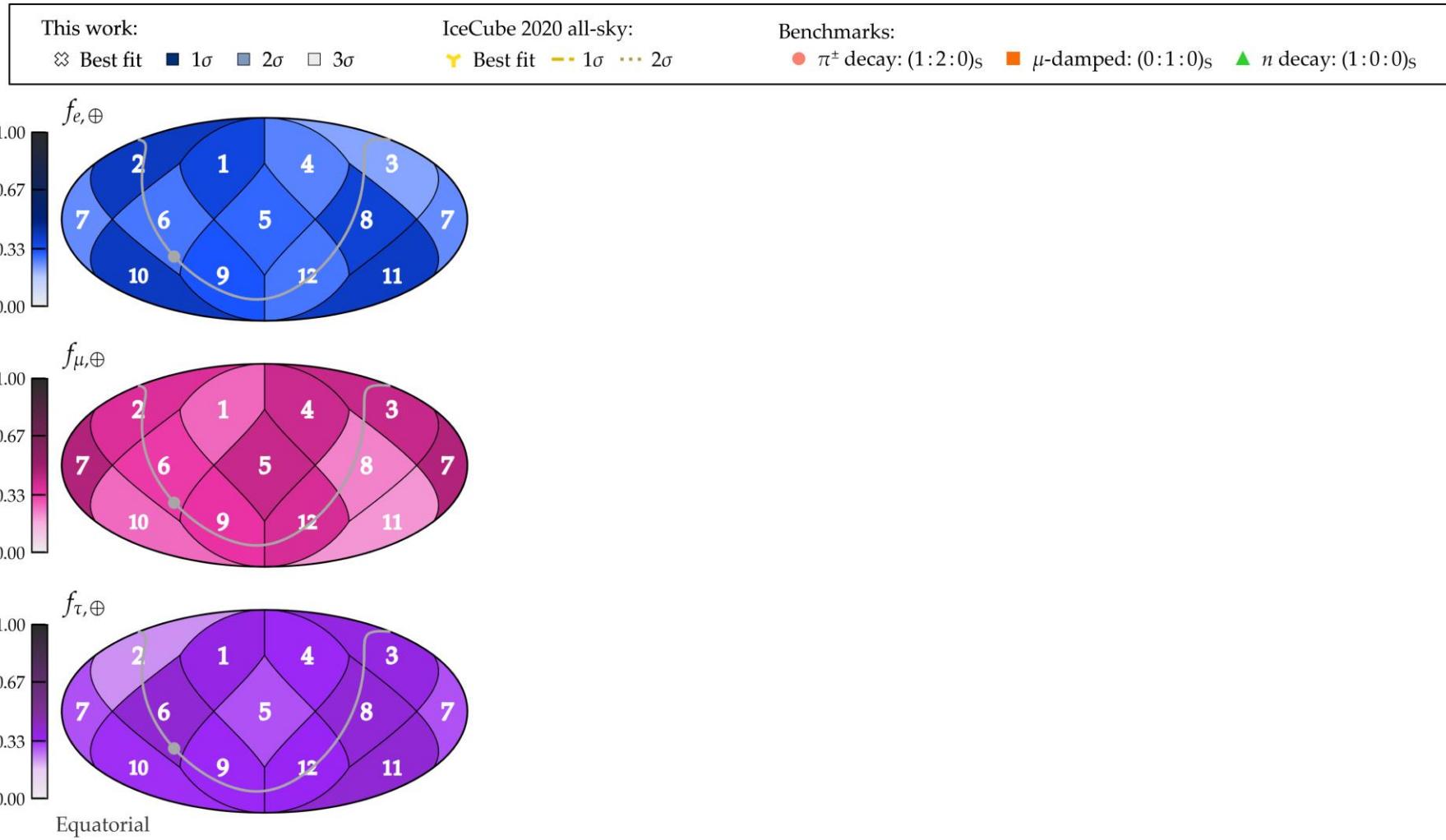


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Directional Flavour Ratios in HESE data

Directional high-energy astrophysical neutrino flavor composition: IceCube HESE (7.5 yr)

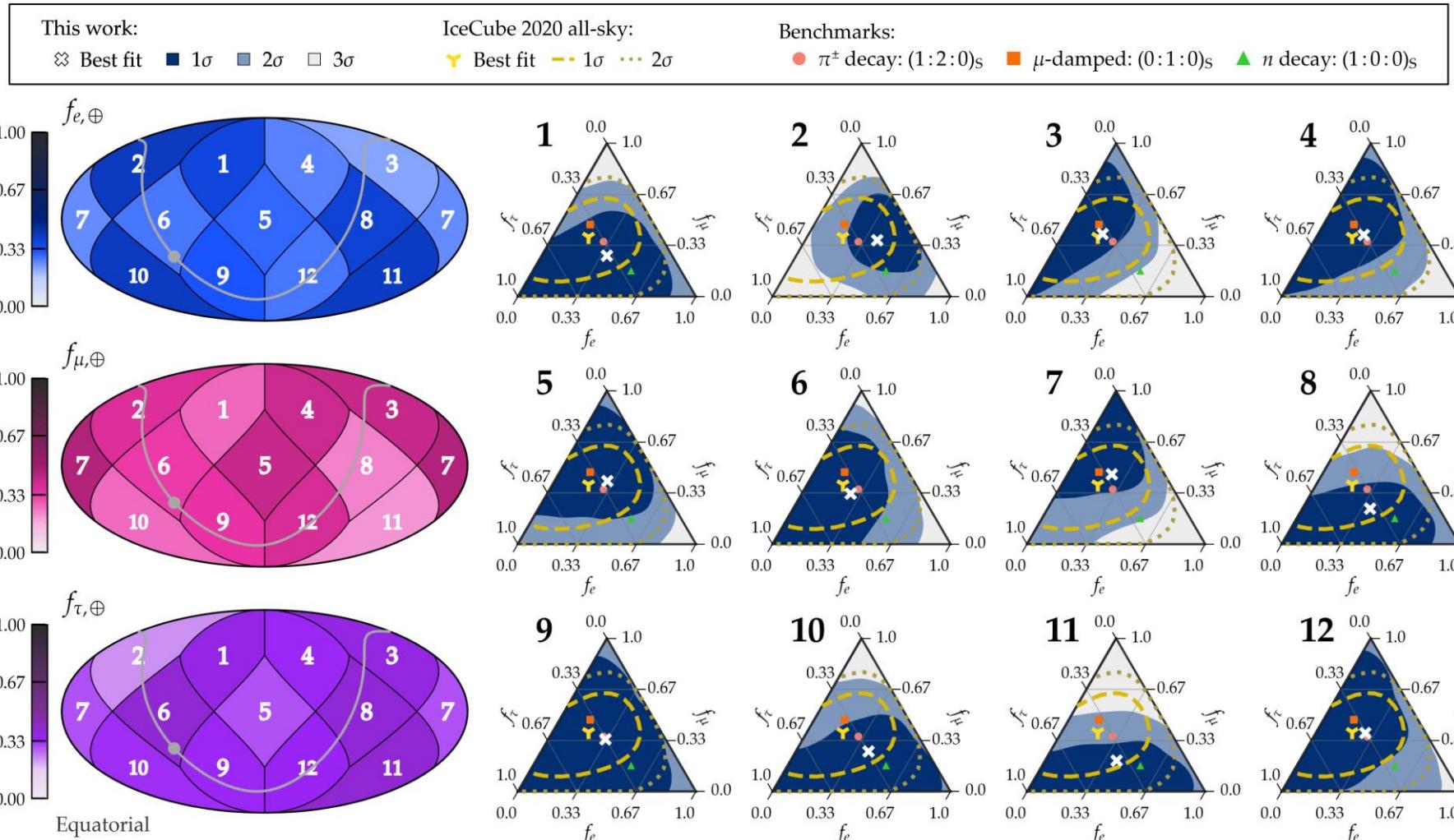


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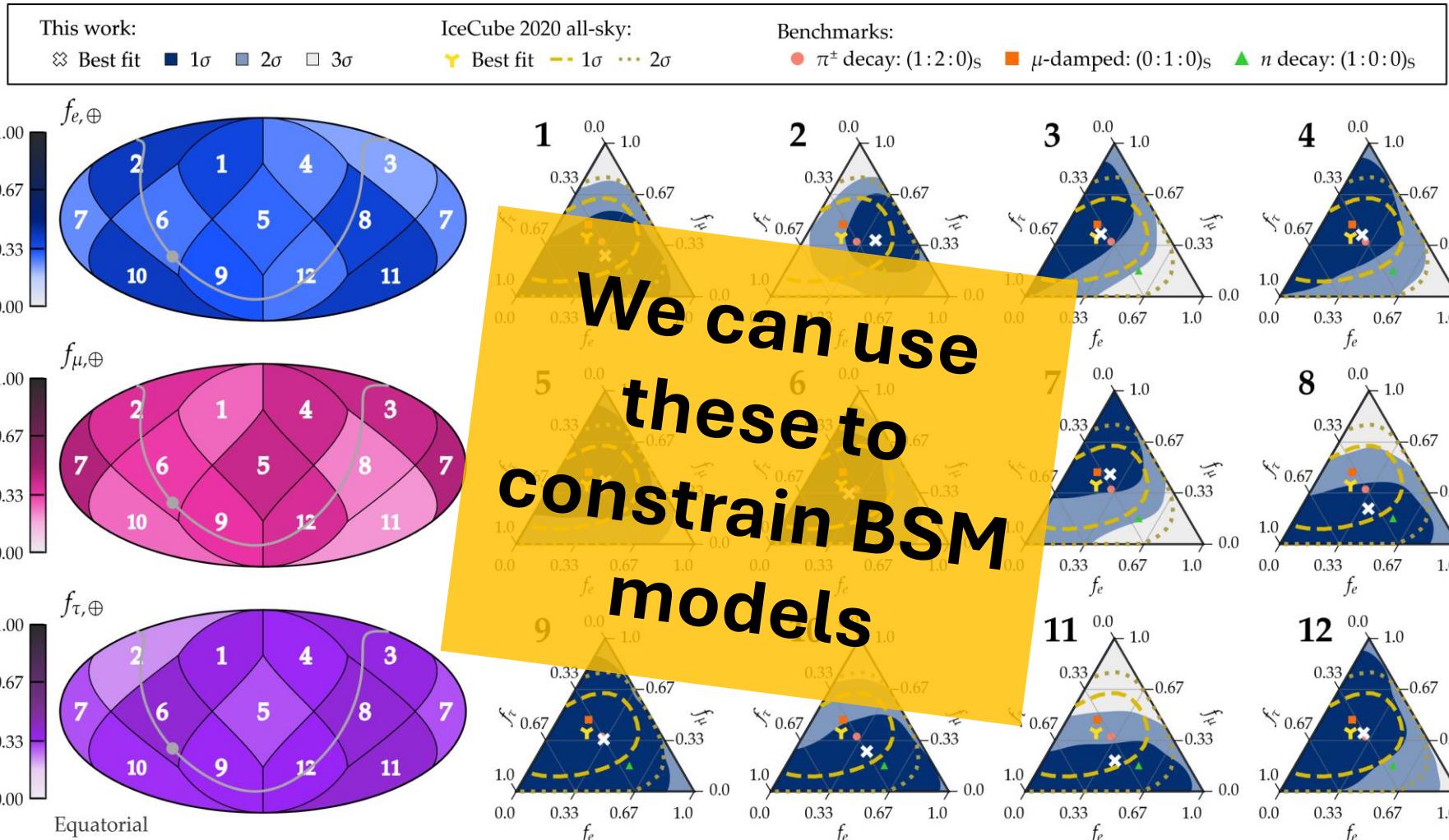
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Flavour ratios?

f_α - ratio of flavour $\alpha = e, \mu, \tau$ at Earth

$$f_\alpha = \frac{\Phi_\alpha}{\sum_\beta \Phi_\beta} \quad \text{with} \quad \sum_\alpha f_\alpha = 1$$

Φ_α - flux/amount of ν_α neutrinos

- energy integrated: TeV–PeV (HESE range)
- time integrated: 7.5 years at IceCube



How do we model the flux at Earth?

$$\begin{aligned}\frac{d\Phi_\alpha}{dE \ dz} &= \Phi_0 \rho_0 H_0^{-1} \\ &\times [E(z+1)]^{2-\gamma} \\ &\times \frac{\rho(z)}{h(z)(z+1)^2} \\ &\times \sum_{\beta} P_{\beta \rightarrow \alpha} f_{\beta,s}\end{aligned}$$



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constants



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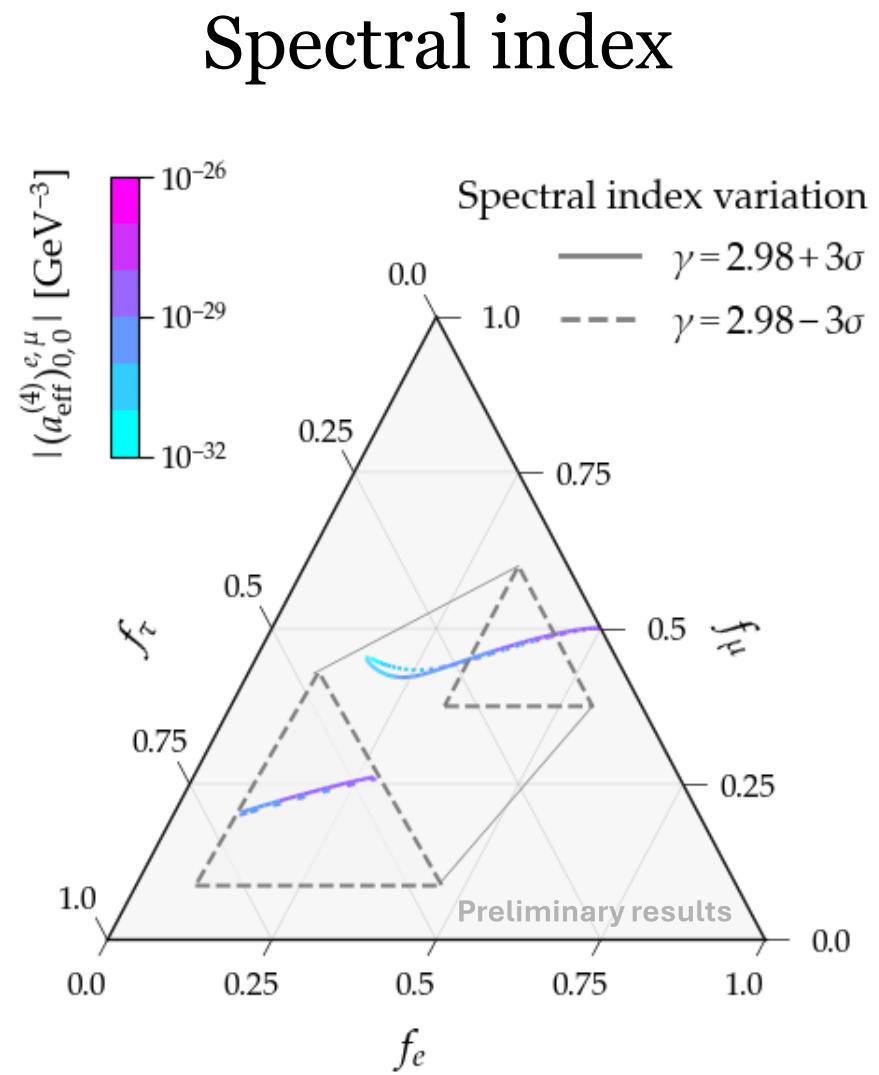
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Energy–redshift dependence
changes all-flavour flux shape



How do we model the flux at Earth?

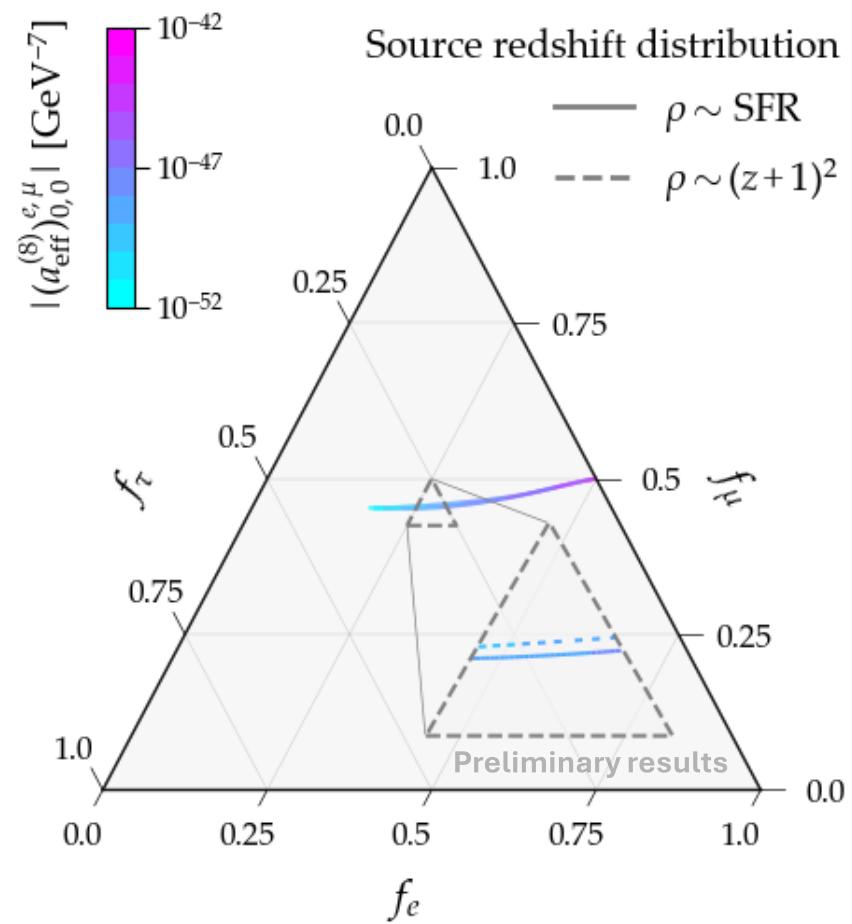
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Source distribution over redshift

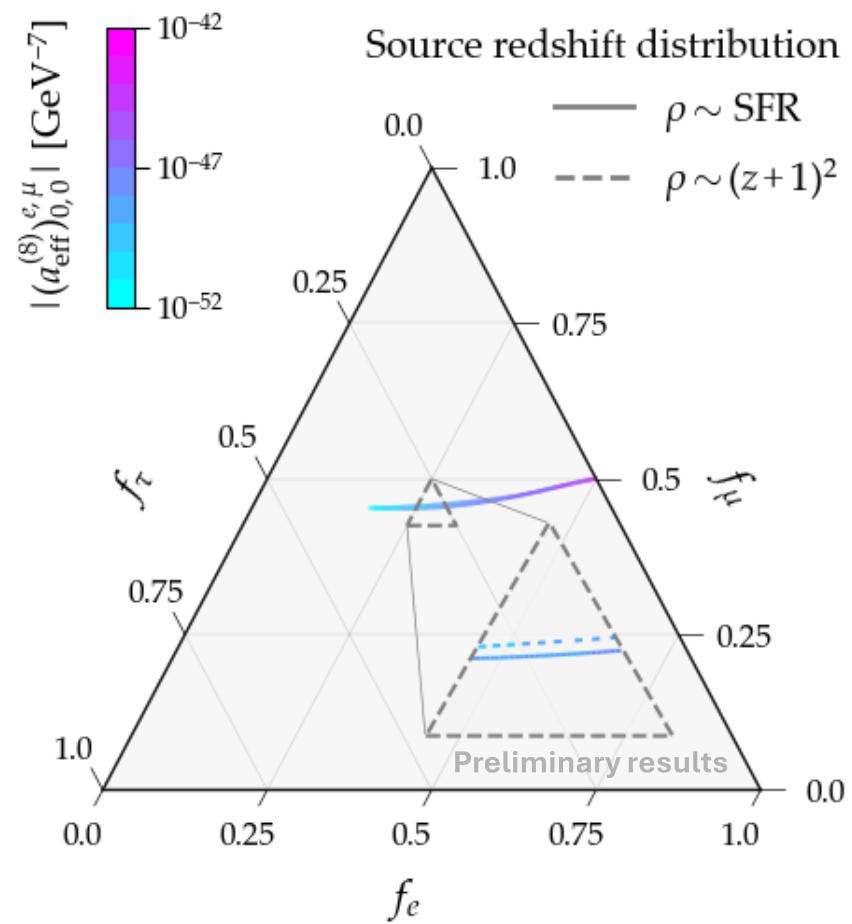


How do we model the flux at Earth?

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So far, no anisotropies are introduced!

Source distribution over redshift



How do we model the flux at Earth?

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Anisotropy in flavour oscillation



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Anisotropy in flavour oscillation

$$H_{tot} = \frac{1}{2E} U_{PMNS} M U_{PMNS}^\dagger + H_{LIV}$$



How do we model the flux at Earth?

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How do we model the flux at Earth?

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Averaged over long distances

$$P_{\beta \rightarrow \alpha} \sim \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$



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U diagonalises H_{tot}

Anisotropy in flavour oscillation

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Lorentz Invariance Violation

$$P_{\beta \rightarrow \alpha} \sim \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Lorentz Invariance – no preferred inertial reference frame

Violation – there is a preferred inertial reference frame



Lorentz Invariance Violation

$$P_{\beta \rightarrow \alpha} \sim \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

$$H_{LIV} = \sum_{d=2}^{\infty} E^{d-2} \sum_{\ell=0, m}^{d-1} \hat{a}_{\ell, m}^{(d)} Y_{\ell, m} + h.c.$$

Parametrises any
preferred reference
frame in the
Universe

See: Standard Model Extension <https://arxiv.org/abs/1112.6395>

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Lorentz Invariance Violation

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Energy dependence

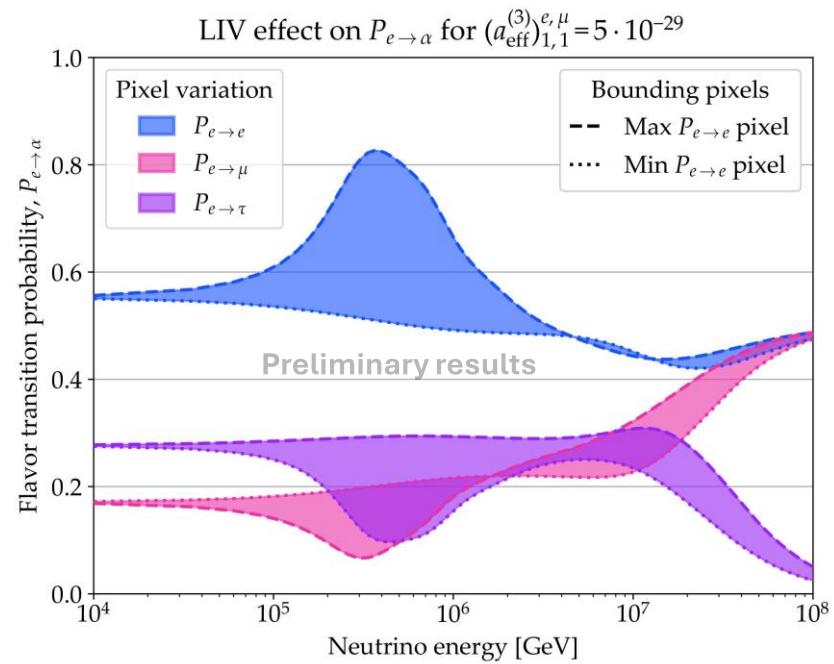


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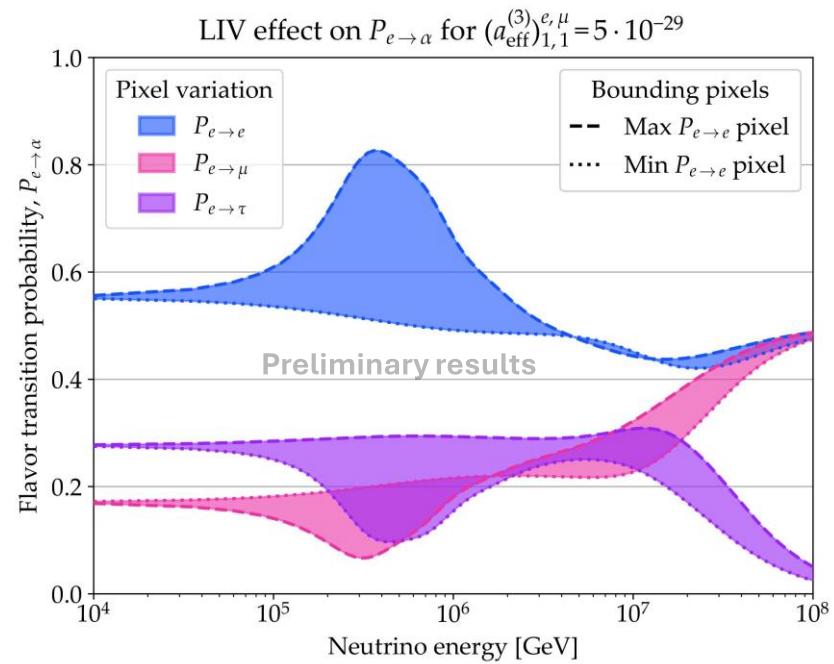
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Energy dependence

effect is strongest
at resonance



See: Standard Model Extension <https://arxiv.org/abs/1112.6395>



Lorentz Invariance Violation

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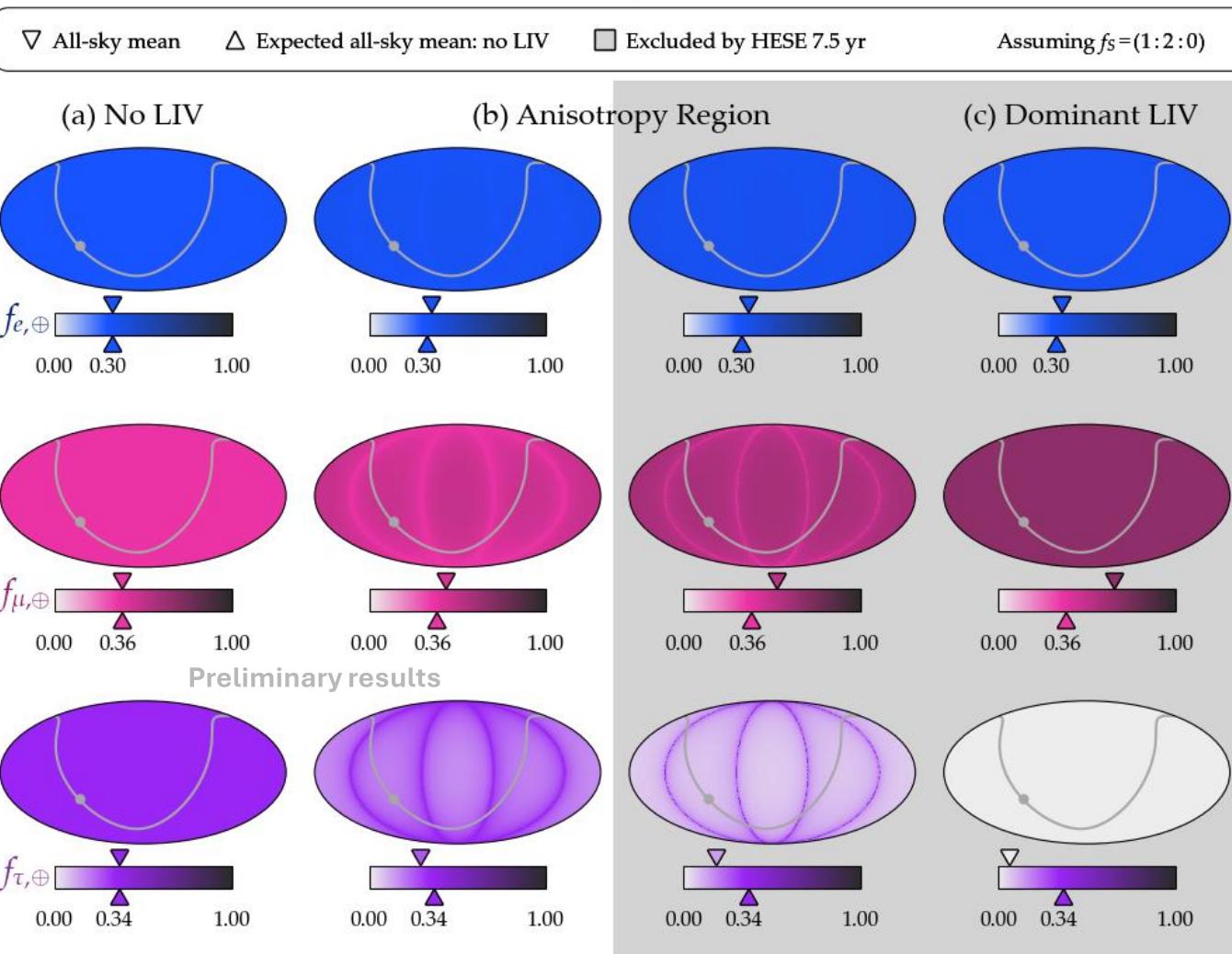
Angular dependence



Lorentz Invariance Violation

$$P_{\beta \rightarrow \alpha} \sim \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Single-parameter, $(a_{\text{eff}}^{(5)})_{2,2}^{\tau,\tau}$, model predictions



Lorentz Invariance Violation

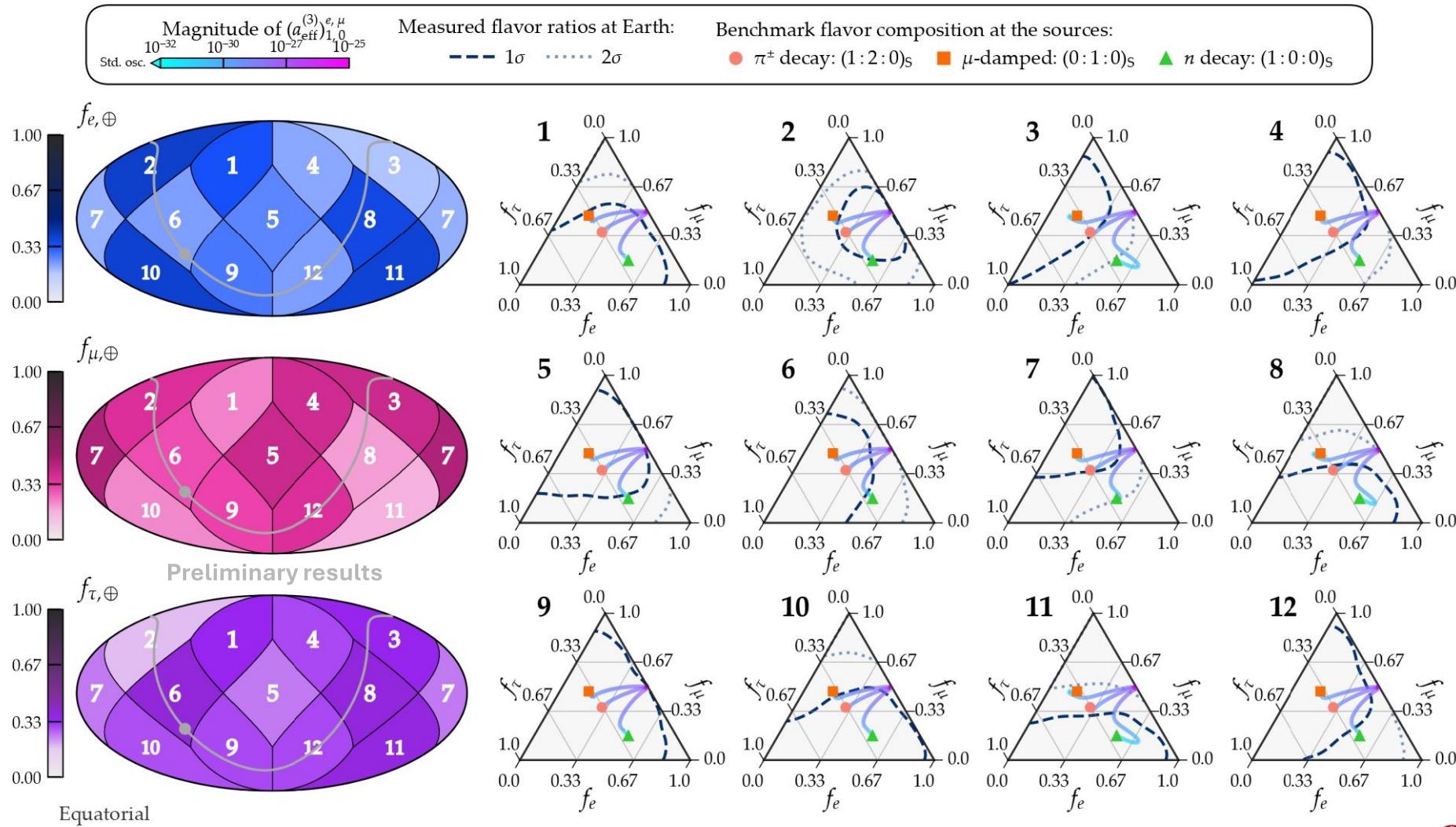
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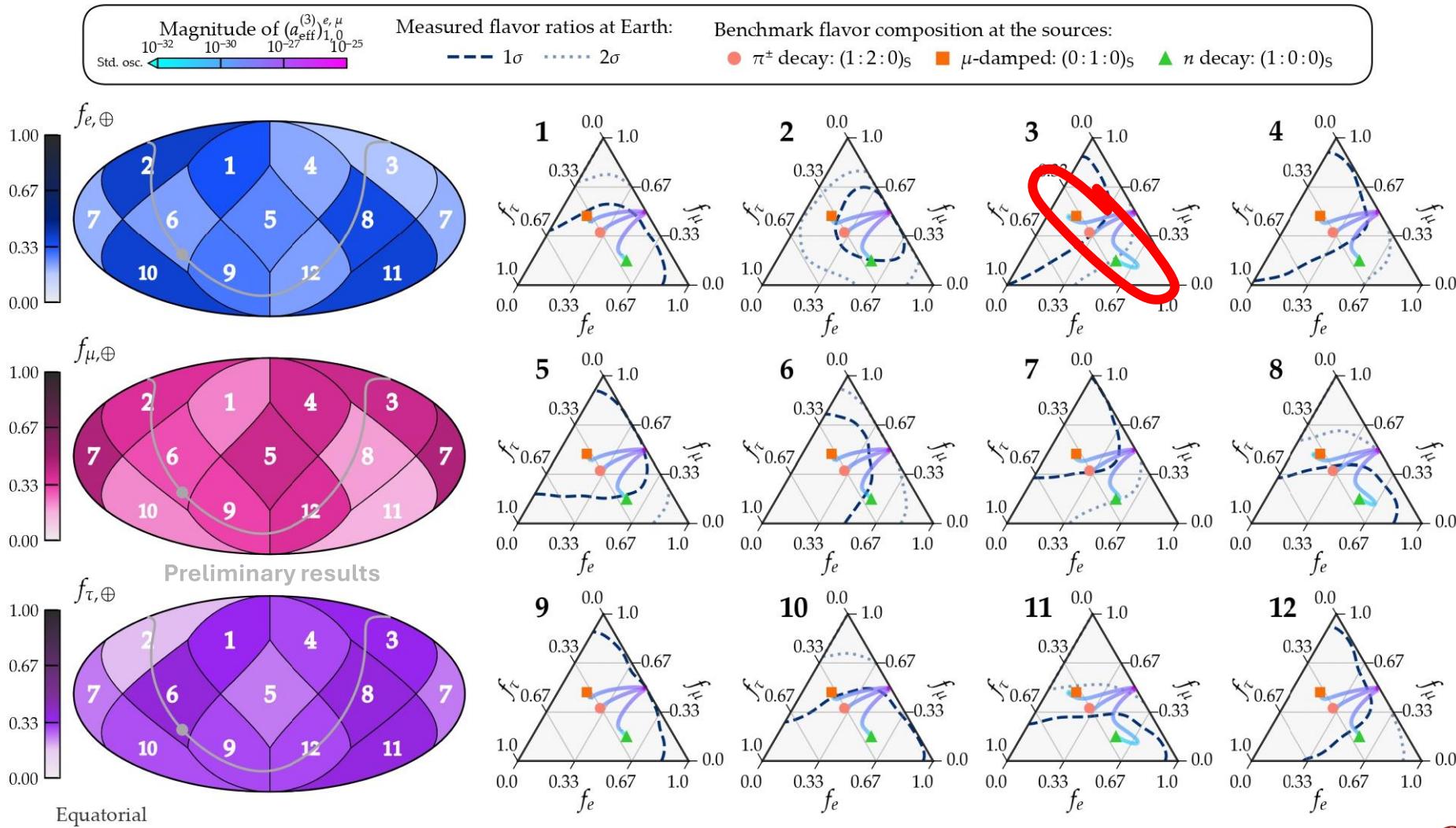
How do they manifest?

Lorentz-violating high-energy neutrino flavor anisotropy (IceCube HESE 7.5 years)



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Lorentz-violating high-energy neutrino flavor anisotropy (IceCube HESE 7.5 years)



How do we model the flux at Earth

$$\frac{d\Phi_\alpha}{dE \ dz} = \Phi_0 \rho_0 H_0^{-1} \times [E(z+1)]^{2-\gamma} \times \frac{\rho(z)}{h(z)(z+1)^2} \times \sum_{\beta} P_{\beta \rightarrow \alpha} f_{\beta,s}$$

What flavours are produced?



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What flavours are produced?

- assume negligible ν_τ production
- otherwise, stay agnostic



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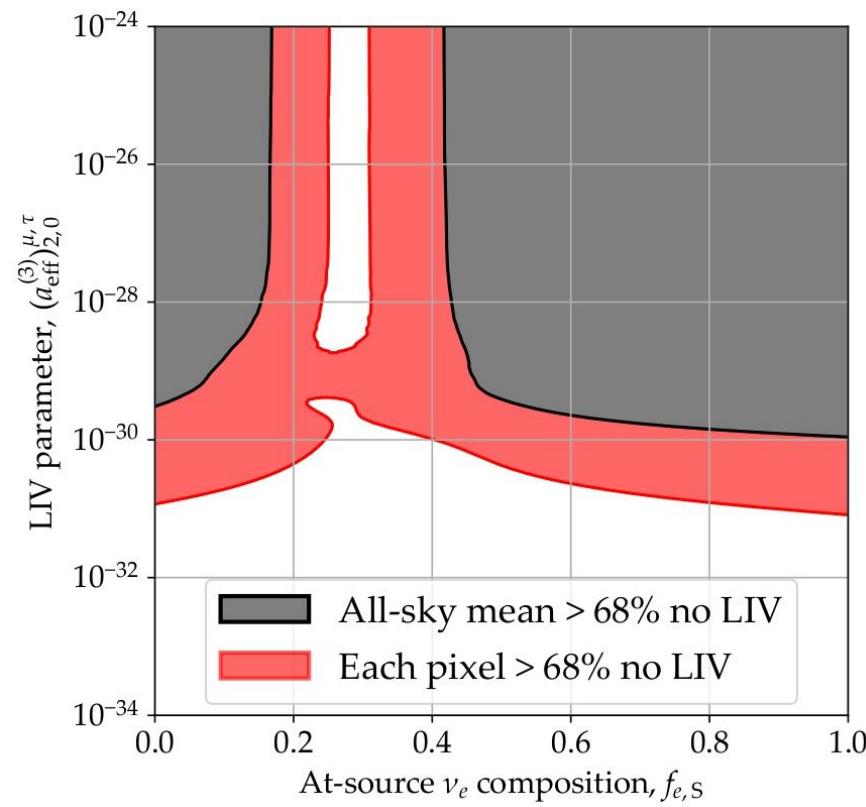


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Bayesian Procedure

$$\frac{d\Phi_\alpha}{dE \ dz}$$

Bayesian Procedure

$$\frac{d\Phi_\alpha}{dE \ dz} \xrightarrow{\int dEdz} \Phi_\alpha$$

Bayesian Procedure

$$\frac{d\Phi_\alpha}{dE \ dz} \xrightarrow{\int dEdz} \Phi_\alpha \longrightarrow f_\alpha$$

Bayesian Procedure

$$\frac{d\Phi_\alpha}{dE \ dz} \xrightarrow{\int dEdz} \Phi_\alpha \longrightarrow f_\alpha \longrightarrow \mathcal{L} = \prod_{\text{pixels}} \int \omega p(f_{\vec{\alpha}}(\omega)) \pi(\omega)$$

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p - measurement in each pixel



Bayesian Procedure

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ω - all model parameters

Bayesian Procedure

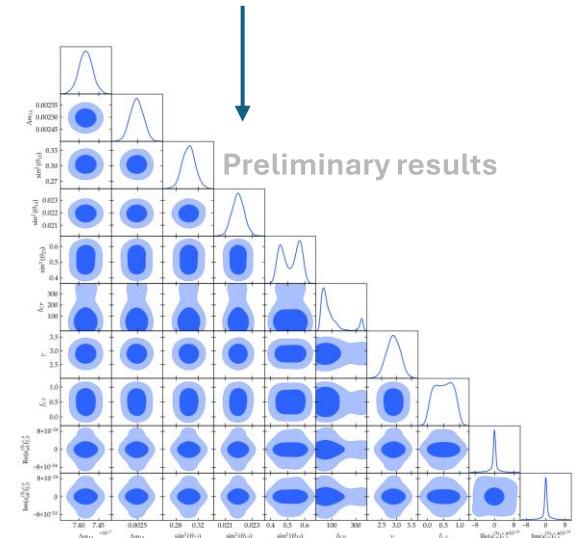
$$\frac{d\Phi_\alpha}{dE \ dz} \xrightarrow{\int dEdz} \Phi_\alpha \longrightarrow f_\alpha \longrightarrow \mathcal{L} = \prod_{\text{pixels}} \int p(f_{\vec{\alpha}}(\omega)) \pi(\omega)$$

p - measurement pdf in each pixel

$f_{\vec{\alpha}}$ - predicted flavour ratio in that pixel

π - priors on all parameters

ω - all model parameters



Bayesian Procedure

We fit each H_{LIV} parameter one-at-a-time

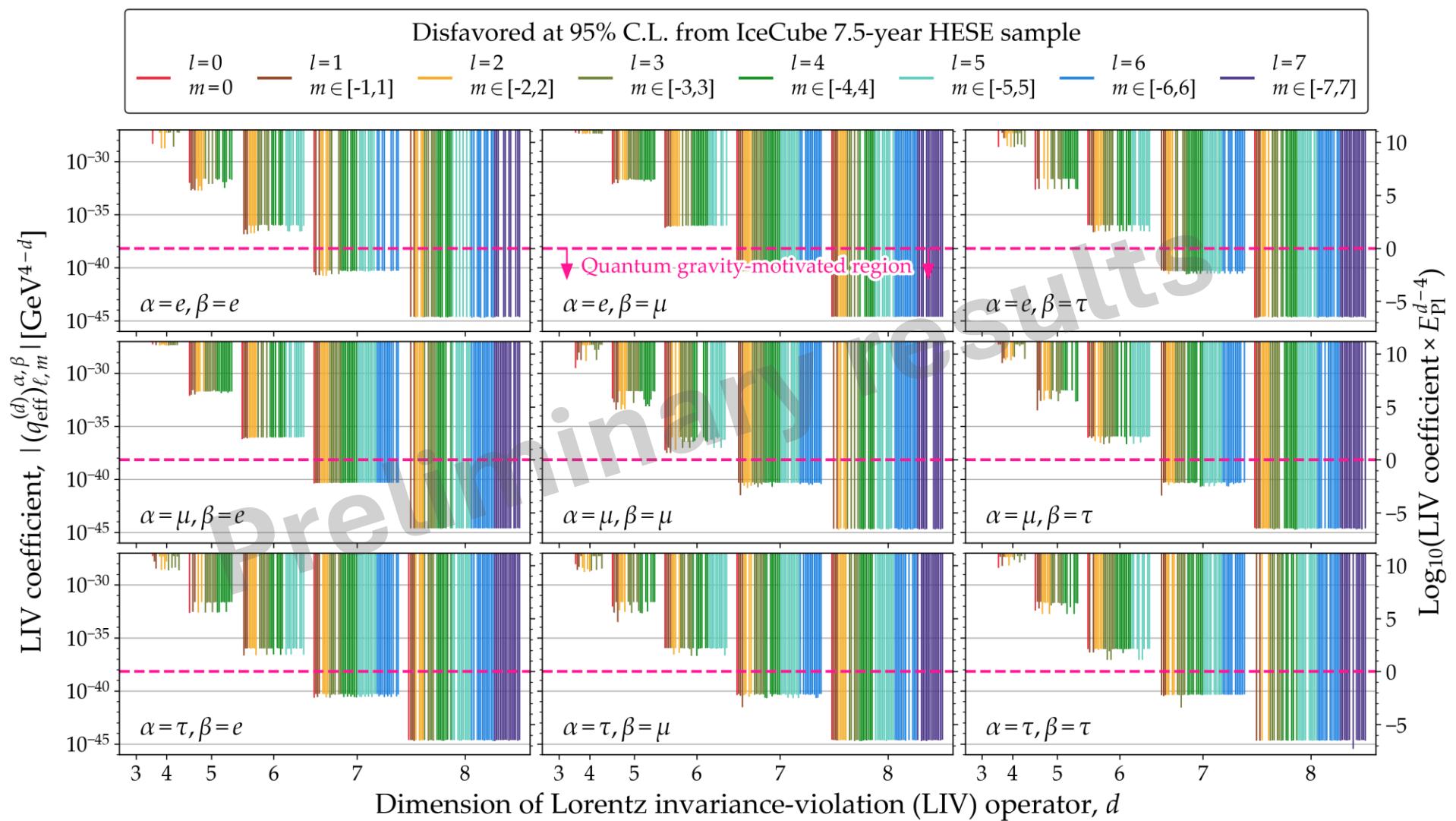
$$d = 2, \dots, 8$$

for each $d > 3$, there are $9d^2$ parameters

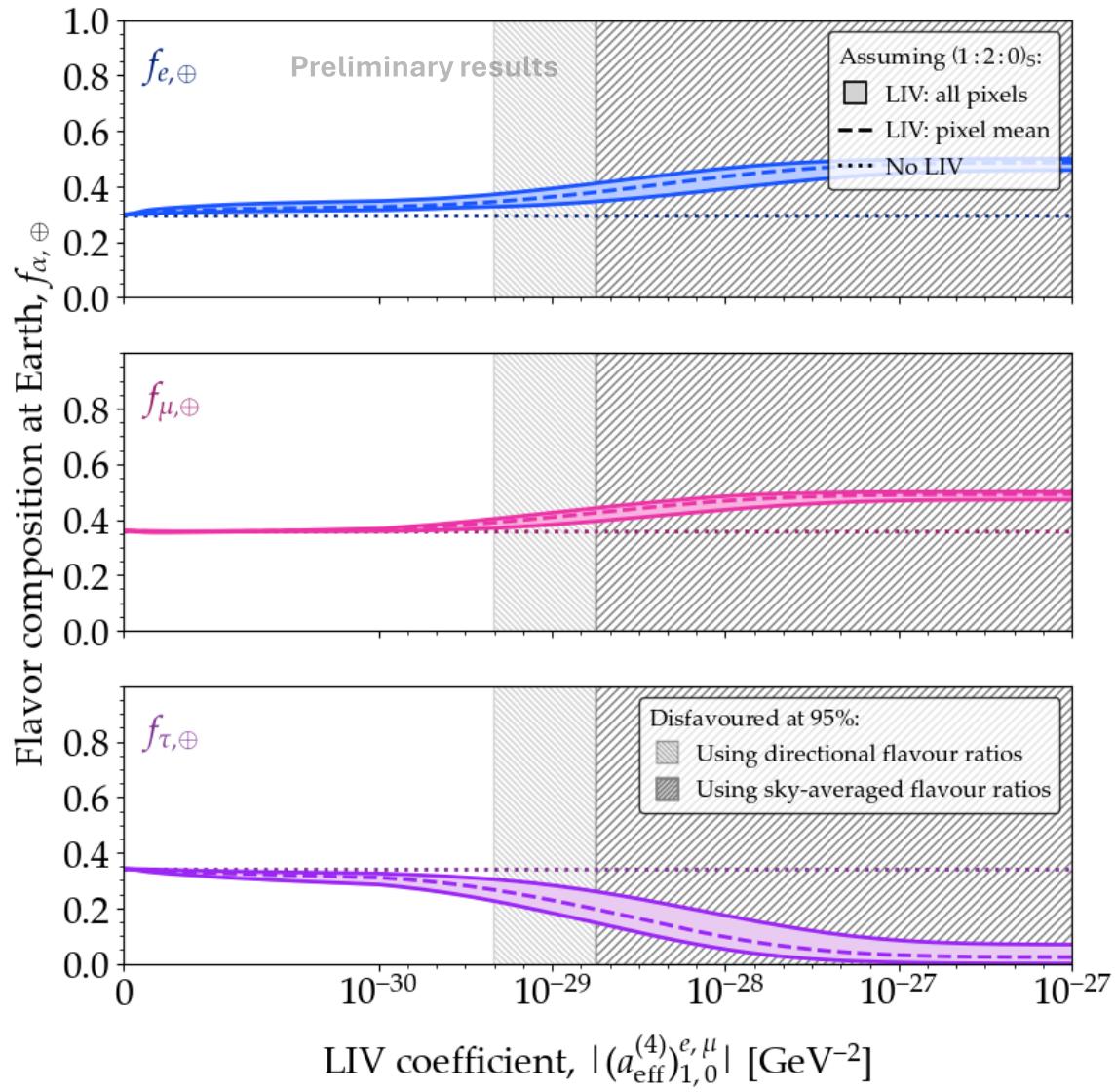
Results



Results



Directional info helps!



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