Overconfidence in Non-Poissonian Template Fitting

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The NSF Institute for Artificial Intelligence and Fundamental Interactions



A long time ago in the galactic center far, far away...





Fermi telescope image <-- 40° -> data: 2009 - now

morphology: spherical-like, extended up to 15°





data: 2009 - now

spherical-like, extended up to 15°

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Small-scale structures

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(g)NFW

- - -



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~ stellar bulge

unresolved point sources



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. . .

Diffuse i.e. Poissonian data: $D \sim \text{Pois}\left(\sum S_i \Phi_i(x)\right)$

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 π^0 + bremsstrahlung



inverse Compton scattering



Diffuse i.e. Poissonian data:

+ unresolved point sources:

 $D \sim \operatorname{Pois}\left(\sum_{i} S_{i} \Phi_{i}(x)\right)$

- $D \sim \text{Pois}\left(\sum_{i} S_{i} \Phi_{i}(x) + \Phi_{\text{PS}}(x)\right)$ with $\Delta \Phi_{\text{PS}}(x) \sim \text{Pois}\left(S_{j} T_{j}(x)\right)$

Diffuse i.e. Poissonian data:

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To understanding the difference in likelihood:

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a double/compound poisson process.

Unresolved point sources:



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Non-Poissonian Template Fitting is a likelihood (-based fitting method) that include unresolved point sources. It achieves this by (implicitly) accounting for all the ways in which an observed count in a pixel is made up.

6

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$$3 = 1 + 1 + 1 = 1 + 2$$

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Diffuse i.e. Poissonian data:

+ unresolved point sources:

$$D \sim \operatorname{Pois}\left(\sum S_{i}\right)$$
$$D \sim \operatorname{Pois}\left(\sum S_{i}\right)$$

Likelihood from different pixels are still independent.

 $S_i \Phi_i(x)$

7

 $_{i}\Phi_{i}(x) + \Phi_{\rm PS}(x)$

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with point spread function (PSF)

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- $S_i \Phi_i(x)$
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Diffuse i.e. Poissonian data:

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$$D \sim \operatorname{Pois}\left(\sum S_i \Phi_i(x)\right)$$
$$D \sim \operatorname{Pois}\left(\sum S_i \Phi_i(x) + \frac{1}{2}\right)$$

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7

with point spread function (PSF)

$$D \sim \operatorname{Pois}\left(\sum S_i \tilde{\Phi}_i(x)\right)$$

 $\vdash \Phi_{\rm PS}(x)$

Diffuse i.e. Poissonian data:

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with point spread function (PSF)

$$\Phi_i(x)$$
 $D \sim \operatorname{Pois}\left(\sum S_i \tilde{\Phi}_i(x)\right)$

 $_{i}\Phi_{i}(x) + \Phi_{\text{PS}}(x)$ $D \sim \text{Pois}\left(\sum S_{i}\tilde{\Phi}_{i}(x) + \text{PSF}[\Phi_{\text{PS}}(x)]\right)$



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NPTF approximately accounts for the PSF effect by correctly^{*} computing the 1-pixel (marginal) likelihood...

...but still treating the total likelihood as a product of that for each pixel.

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$$3 = 1 + 1 + 1 \dots$$

Can be 10% of a 10-co

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ount source

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*A related approximation: point source templates are slow-varying compared to the PSF.

with point spread function (PSF)

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Fermi's point spread function (PSF)



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How reliable are the posteriors produced from this likelihood?

with point spread function (PSF)

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Example: fits to many simulations



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Example: fits to many simulations



For this talk, I will focus on one of sources of NPTF's overconfidence: un-modeled inter-pixel correlations.

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Pointed out in Collin et al 2018



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We fit for overall normalization, against simulated data, to test coverage.

PSF $\sigma = 0.8^{\circ}$ simulation

PSF of one source

With a single template, fitted normalization \sim total count. Total count likelihood given by NPTF is overconfident!

coverage

We fit for overall normalization, against simulated data, to test coverage.

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In this toy example, 2-pixel correlation can be recaptured with an Gaussian approximation to the image likelihood, yielding a fairly well-calibrated fit.

We fit for overall normalization, against simulated data, to test coverage.

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Modeling multi-pixel correlation in the likelihood is inherently hard.

In this toy example, 2-pixel correlation can be recaptured with an Gaussian approximation to the image likelihood, yielding a fairly well-calibrated fit.

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$$p(\theta|)$$

In this toy example, 2-pixel correlation can be recaptured with an Gaussian approximation to the image likelihood, yielding a fairly well-calibrated fit.

We fit for overall normalization, against simulated data, to test coverage.

Aside: share the same total count likelihood profile but NPTF with different PSF gives different results (gives correct result for no PSF case)

Modeling multi-pixel correlation in the likelihood is inherently hard.

In this toy example, 2-pixel correlation can be recaptured with an Gaussian approximation to the image likelihood, yielding a fairly well-calibrated fit.

Nominal coverage

0.4

Overconfident

0.8

0.6

1.0

coverage

NPTF

0.8

Actual coverage 9.0

0.2

0.0

0.0

Gaussian

0.2

Neural Posterior Estimator (as an example for SBI)

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Neural Posterior Estimator (as an example for SBI)

With energy binning: even more complicated correlations.

Summary

- Non-Poissonian Template Fitting may be overconfident in fits of the Galactic Center.
- Un-modeled positive correlations between pixels causes the likelihood for point source template normalization to be overconfident.
- Preliminarily, this effect accounts for a significant portion of the observed overconfidence in our tests. More careful study upcoming.
- Simulation-Based Inference may be a solution to this issue.

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- Preliminarily, this effect accounts for a significant portion of the observed overconfidence in our tests. More careful study upcoming.
- Simulation-Based Inference may be a solution to this issue.

Thank you!

Backup slides

NPTF fitting de-correlated data in toy example

