Probing Dark Matter Microphysics Using Stellar Streams

Aditya Parikh
YITP, Stony Brook University

In collaboration with P. Agrawal & M. Reece and
D. Adams, R. Essig, M. Kaplinghat, A. Price-Whelan & O. Slone
Based on 2003.00021, 2012.11606 + work to appear

Starting to Probe

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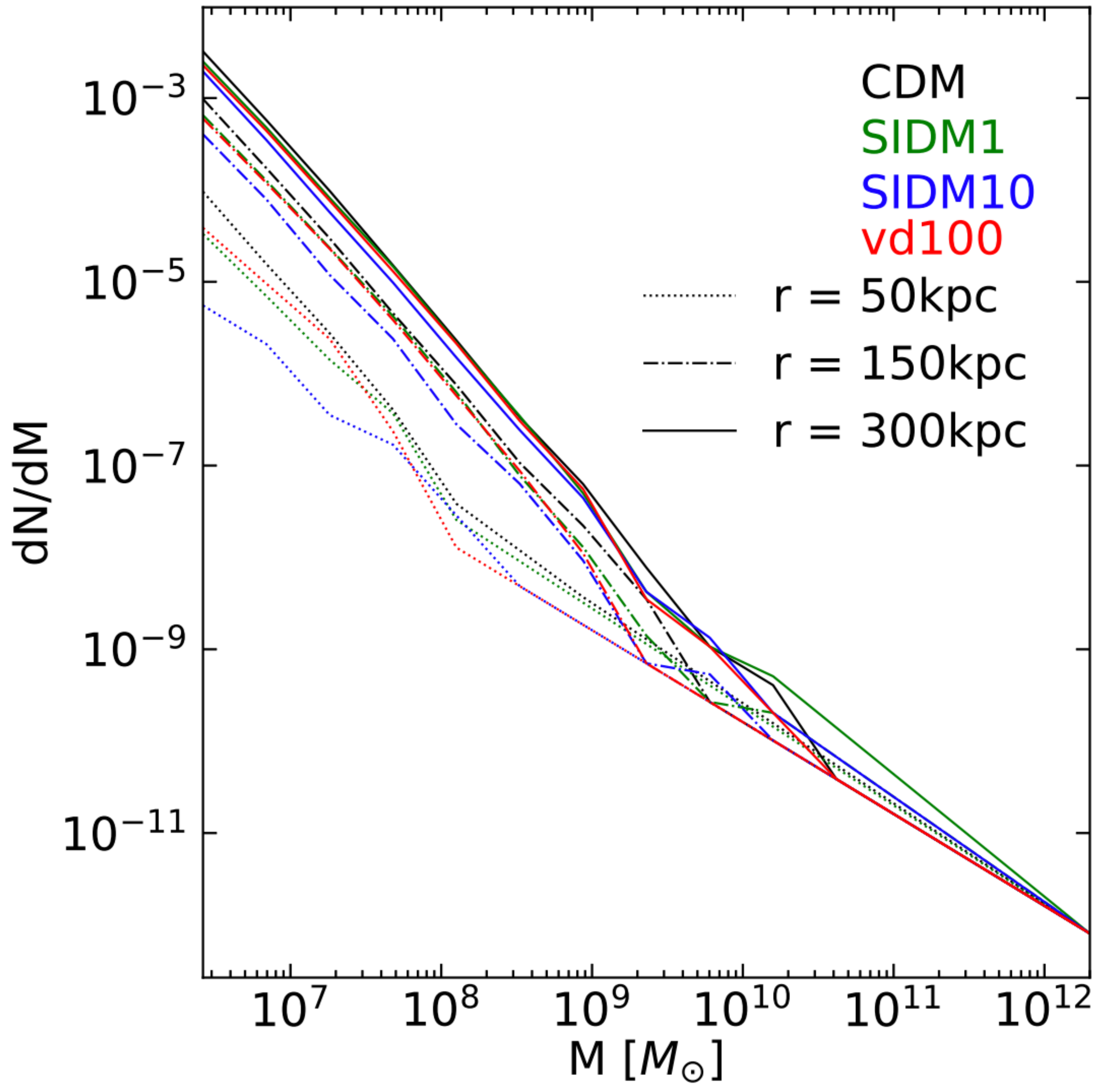
HARVARD

Why Consider Dark Matter Microphysics?

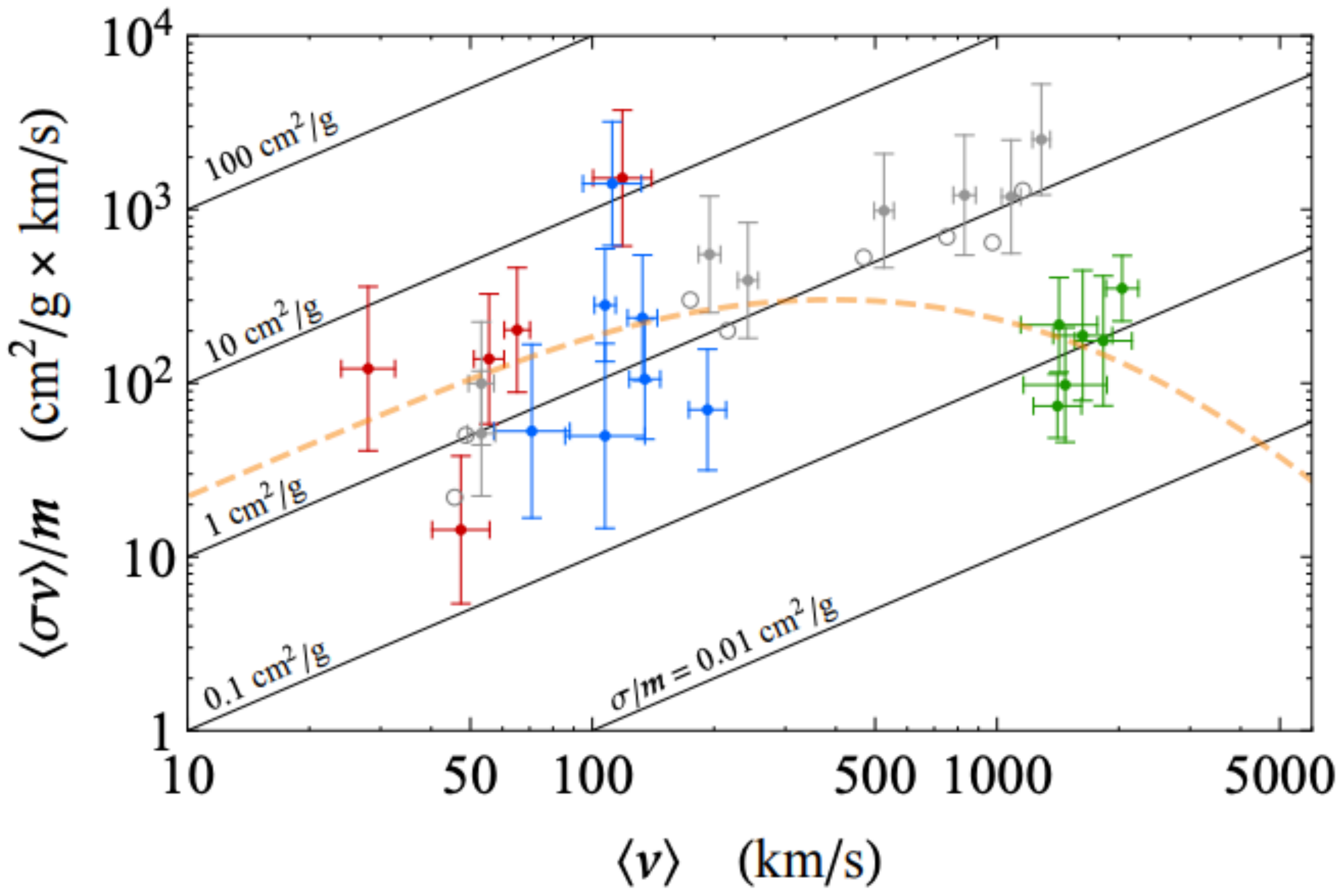
- CDM does extremely well at the largest scales we've tested it at.
- Deviations at smaller scales have been detected hinting at possible dark matter microphysics. It is also well-motivated theoretically.
- The imprints of dark matter microphysics can occur at a variety of scales, but are more pronounced for smaller masses.



[2010.02924]

Astrophysical Imprints?

- On smaller scales, introducing self-interactions can a change the dark matter density profile.
- Inner regions of subhalos are thermalized due to high interaction rates. Outer regions on the other hand are closer to an NFW profile. The characteristic radius is where a particle scatters once per the object's lifetime.
- Strong/weak lensing measurements, stellar kinematics, and rotation curves allow us to pin down the dark matter density profile.
- Different particle physics models produce different behaviors and Sommerfeld enhancement can significantly alter the naive scaling at low velocities. [See 2003.00021, 2012.11606 for a systematic EFT approach to this problem.]
- Larger cross sections carry more pronounced effects.



Long range Yukawa potential fit to results inferred from Dwarfs, LSBs, and Clusters. [1508.03339]

Gravothermal Collapse

- Halo evolution sequence for an initially NFW profile. The same sequence holds for truncated NFW profiles as well as tidally stripped profiles. [1901.00499]

    - Self-interactions begin thermalizing the inner region, increasing the temperature, and forming a gradually expanding core.

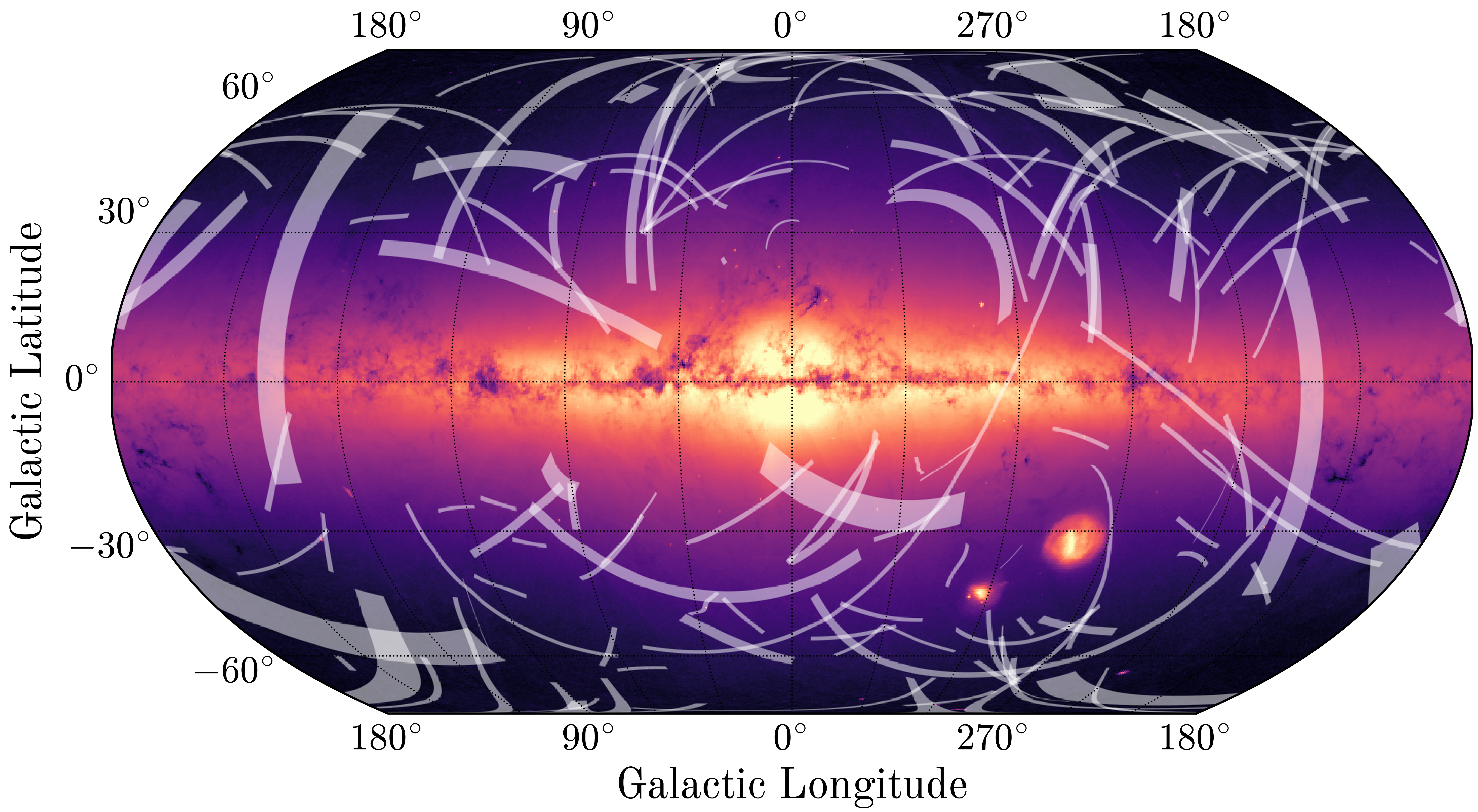
    - The expansion halts as an isothermal core is formed. The velocity is roughly constant.

    - Core collapse is initiated. The core becomes denser and hotter as it shrinks in size, but remains isothermal. The evolution is self-similar.

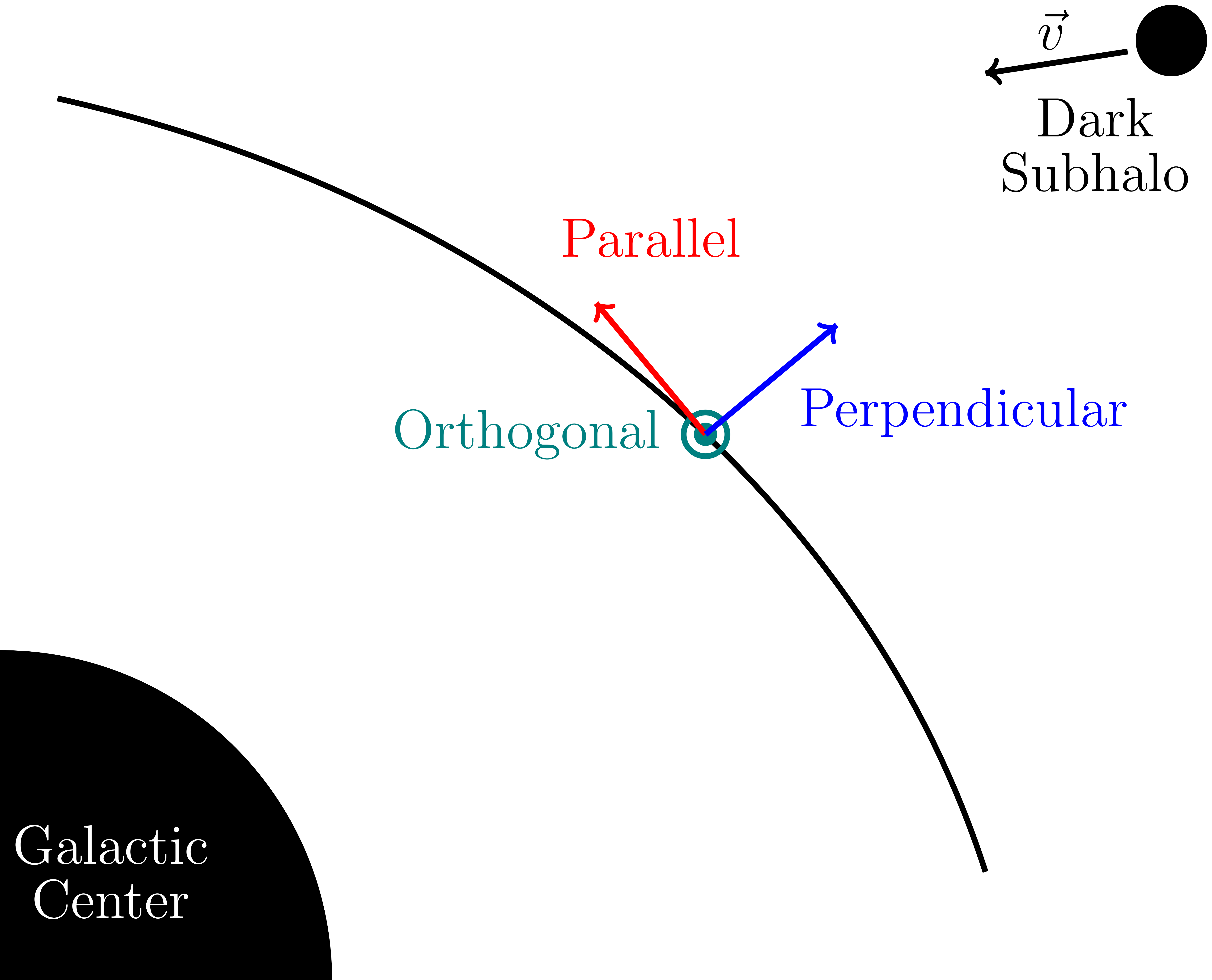
    - Finally, we enter the short mean free path regime and the core becomes optically thick to self-interactions. The core attains its maximum mass, and the density and temperature increase rapidly in this phase.

- How can we go after deviations in the subhalo mass function, especially at lower mass scales where the deviations are large and subhalos are dark?
- Can we probe gravothermally collapsed cores or other deviations to the dark matter density profile?

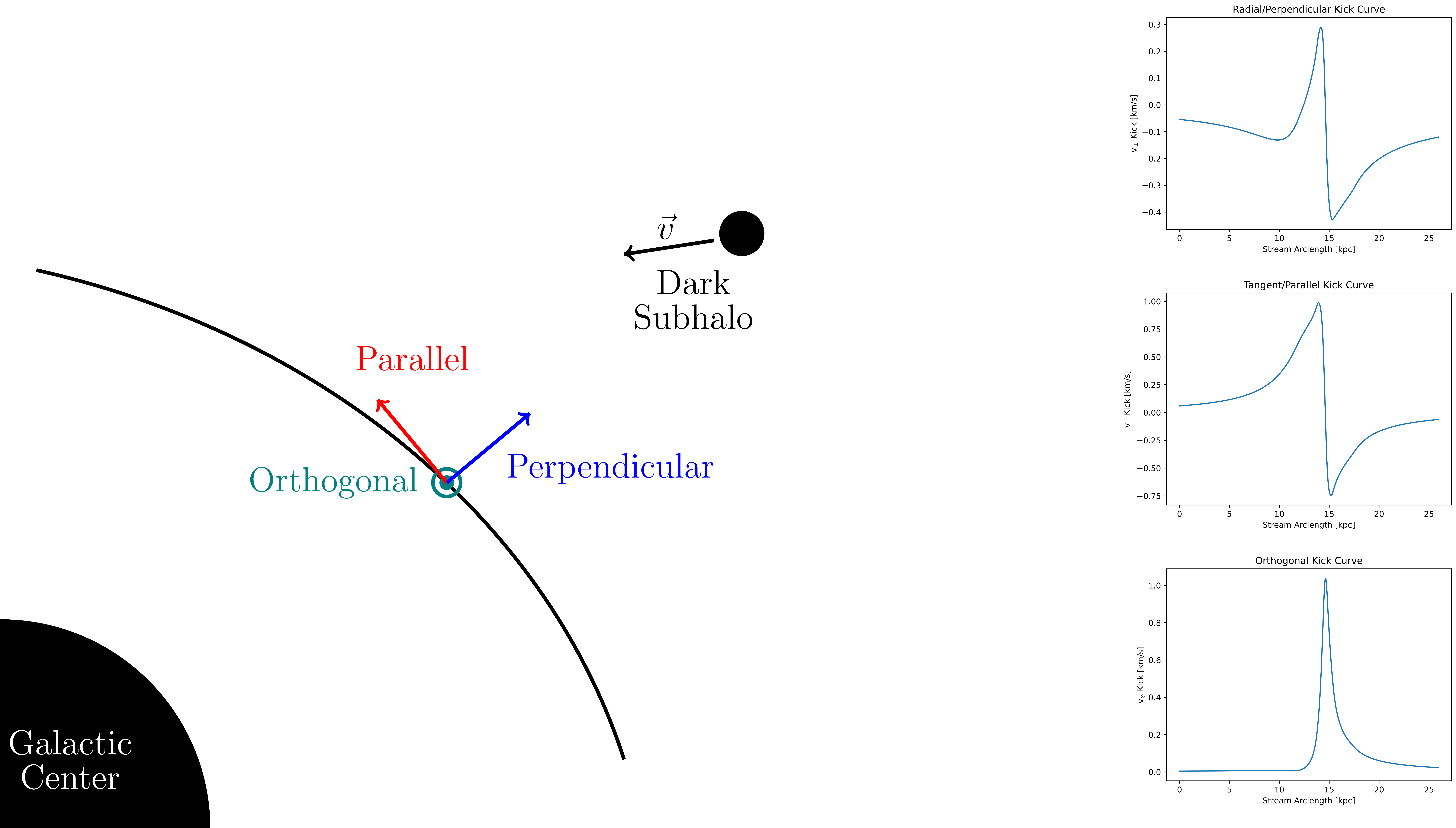
Streams as Detectors?



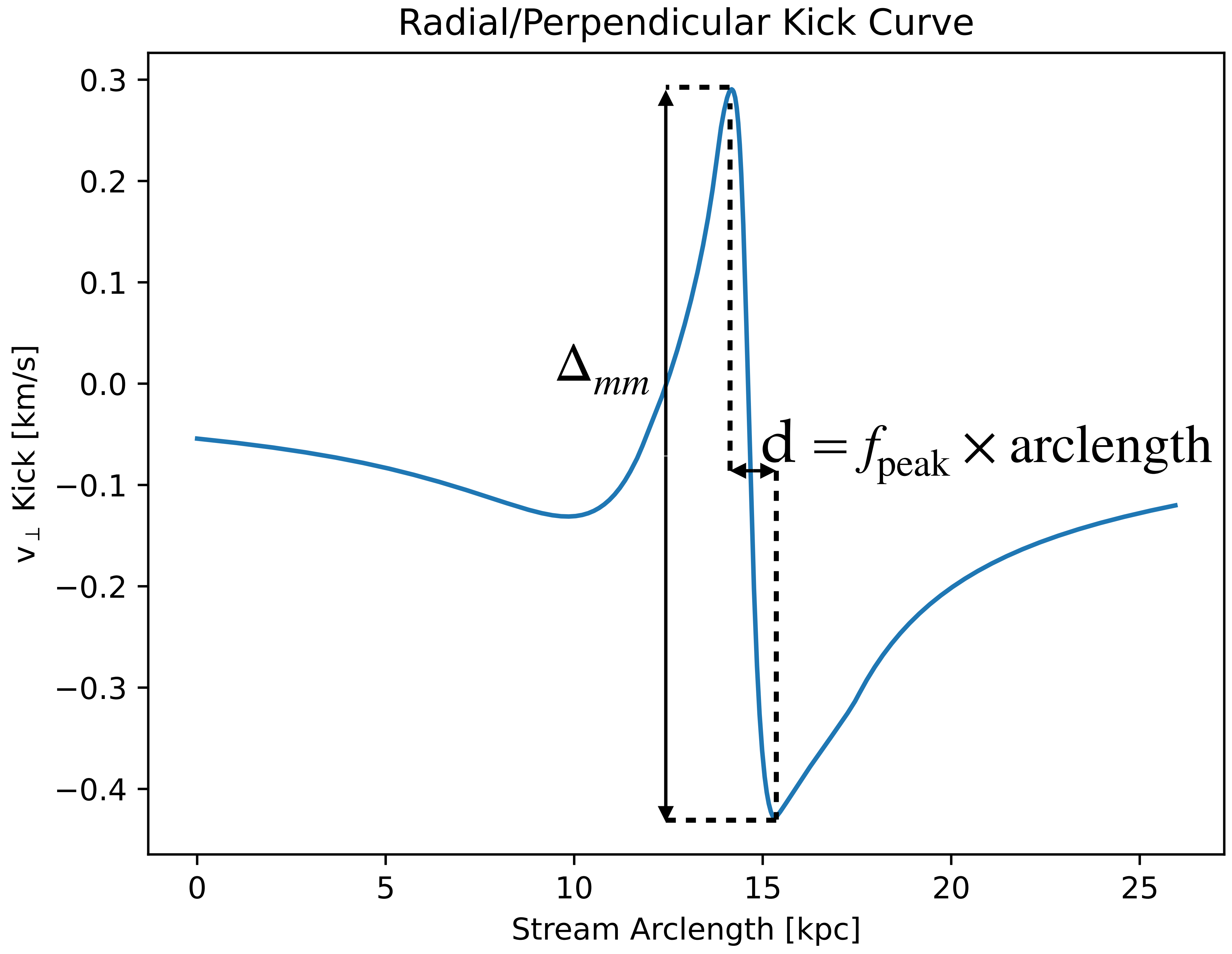
[2405.19410]

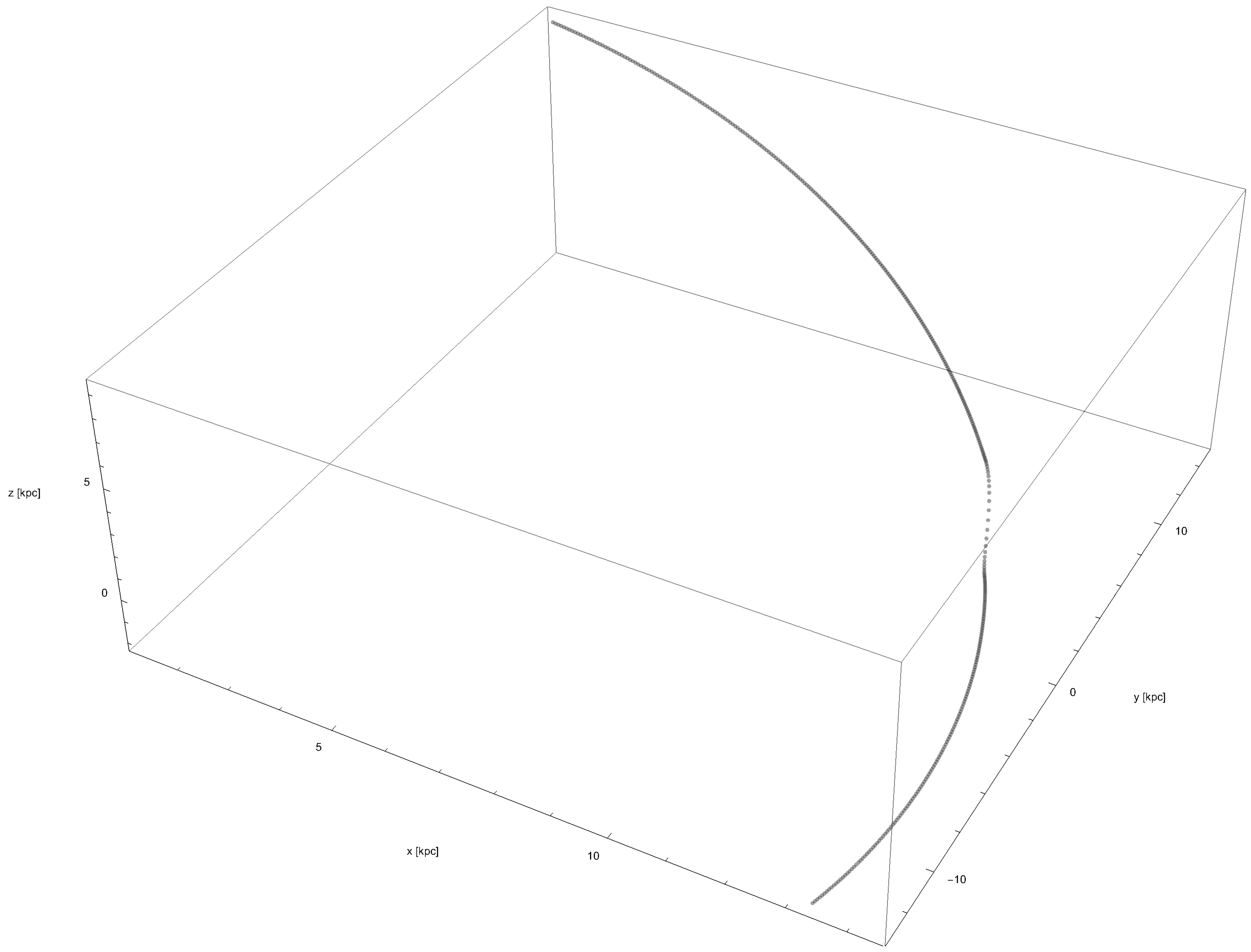


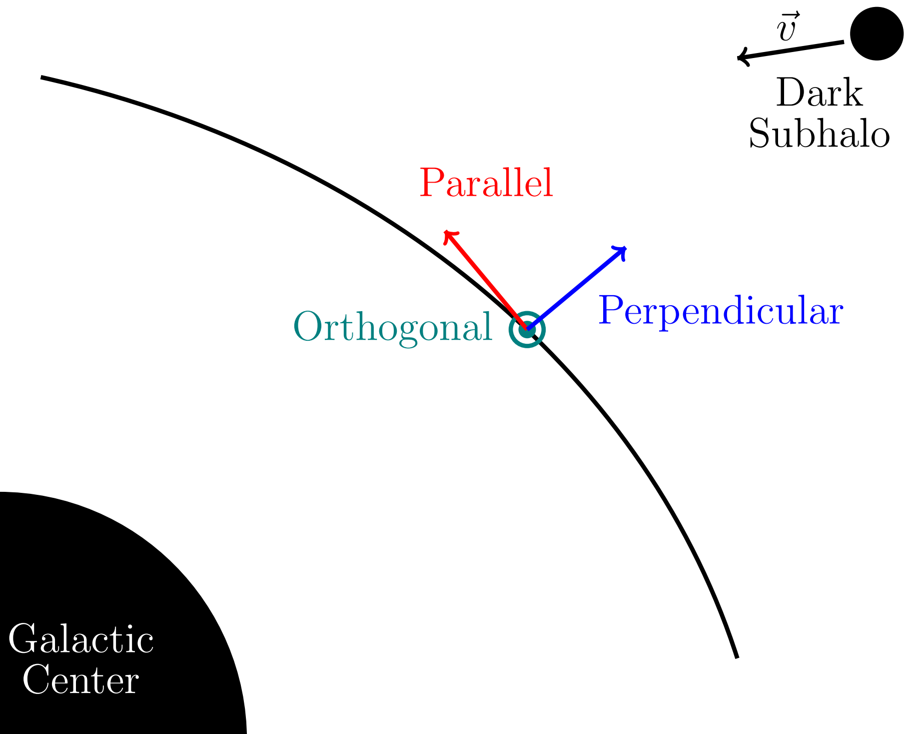
$\vec{v}$
Dark
Subhalo
Parallel
Perpendicular
Orthogonal
Galactic
Center



$\vec{v}$
Dark Subhalo
Parallel
Perpendicular
Orthogonal
Galactic Center
Radial/Perpendicular Kick Curve
$v_\perp$ Kick [km/s]
Stream Arclength [kpc]
Tangent/Parallel Kick Curve
$v_\parallel$ Kick [km/s]
Stream Arclength [kpc]
Orthogonal Kick Curve
$v_\odot$ Kick [km/s]
Stream Arclength [kpc]







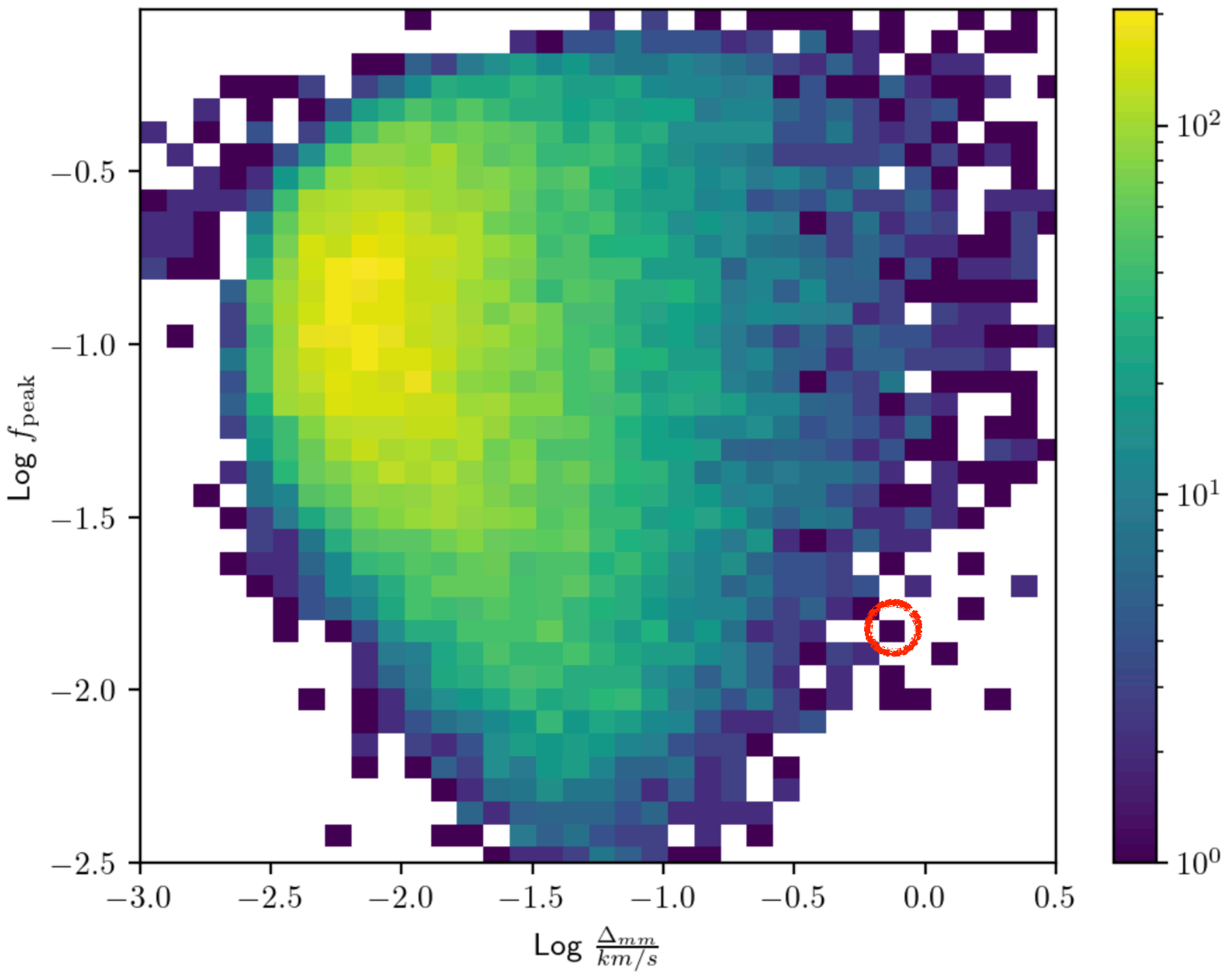
- Δ_{mm} and f_{peak} characterize* the velocity kicks imparted by the perturber.
- What are the distribution of these variables? How many such events do we expect?

The Semi-Analytic Approach

- Semi-analytic tools initialize and generate various Milky Way-like realizations, by seeding an initial distribution of subhalos of various masses around the galaxy and then evolving them in the host potential.

- Semi-analytic approaches allow us to run lots of samples quickly which helps combat effects such as halo-to-halo variance, etc.

- Performing robust population studies is crucial in critically evaluating the extent to which CDM subhalos can create the observed features in streams.



Summary

- SIDM can alter the dark matter density profiles of subhalos, especially for lower mass objects. Streams provide a useful detection avenue for dark substructure.

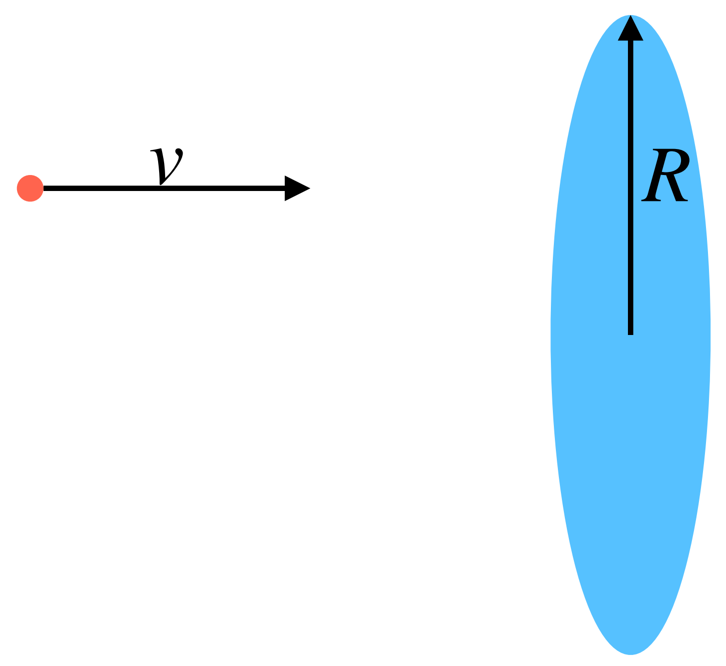
- We are currently in the process of establishing the CDM baseline expectation for the effects on streams. This includes both the rare interactions which can produce gaps and the more common interactions which increase the inherent velocity dispersion and lead to heating.

- Our method allows us to explore the effects of various density profiles of the perturbing subhalos on streams as well which can lead to complementary constraints on the dark matter microphysics.

Backup

Sommerfeld Enhancement

- A Classical Analogy
  - w/o gravity $\sigma_0 = \pi R^2$
  - w/ gravity $\sigma = \pi b_{max}^2 = \sigma_0 \left(1 + \frac{v_{esc}^2}{v^2}\right)$



- Non-perturbative effect that can be treated quantum mechanically
  - Match a field theory calculation onto a quantum mechanical potential and solve the corresponding Schrödinger equation

Case Studies: Scalar & Pseudoscalar Exchange

$$V_{\text{scalar}}(r) = -\frac{\lambda^2}{4\pi r}\mathrm{e}^{-m_\phi r}$$

$$V_{\text{pseudoscalar}}(r) = \frac{\lambda^2}{4\pi}\left(\frac{4\pi\delta^3(\vec{r})}{4m_\chi^2 - m_\phi^2}\left(\frac{1}{2} - 2S_1\cdot S_2\right) - \frac{4\pi\delta^3(\vec{r})}{3m_\chi^2}\mathrm{e}^{-m_\phi r}S_1\cdot S_2 + \frac{\mathrm{e}^{-m_\phi r}}{m_\chi^2}\left[\frac{m_\phi^2}{3r}S_1\cdot S_2 + \boxed{\frac{3(S_1\cdot\hat{r})(S_2\cdot\hat{r}) - S_1\cdot S_2}{r^3}\left(1\right.}\left. + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)\right]\right)$$

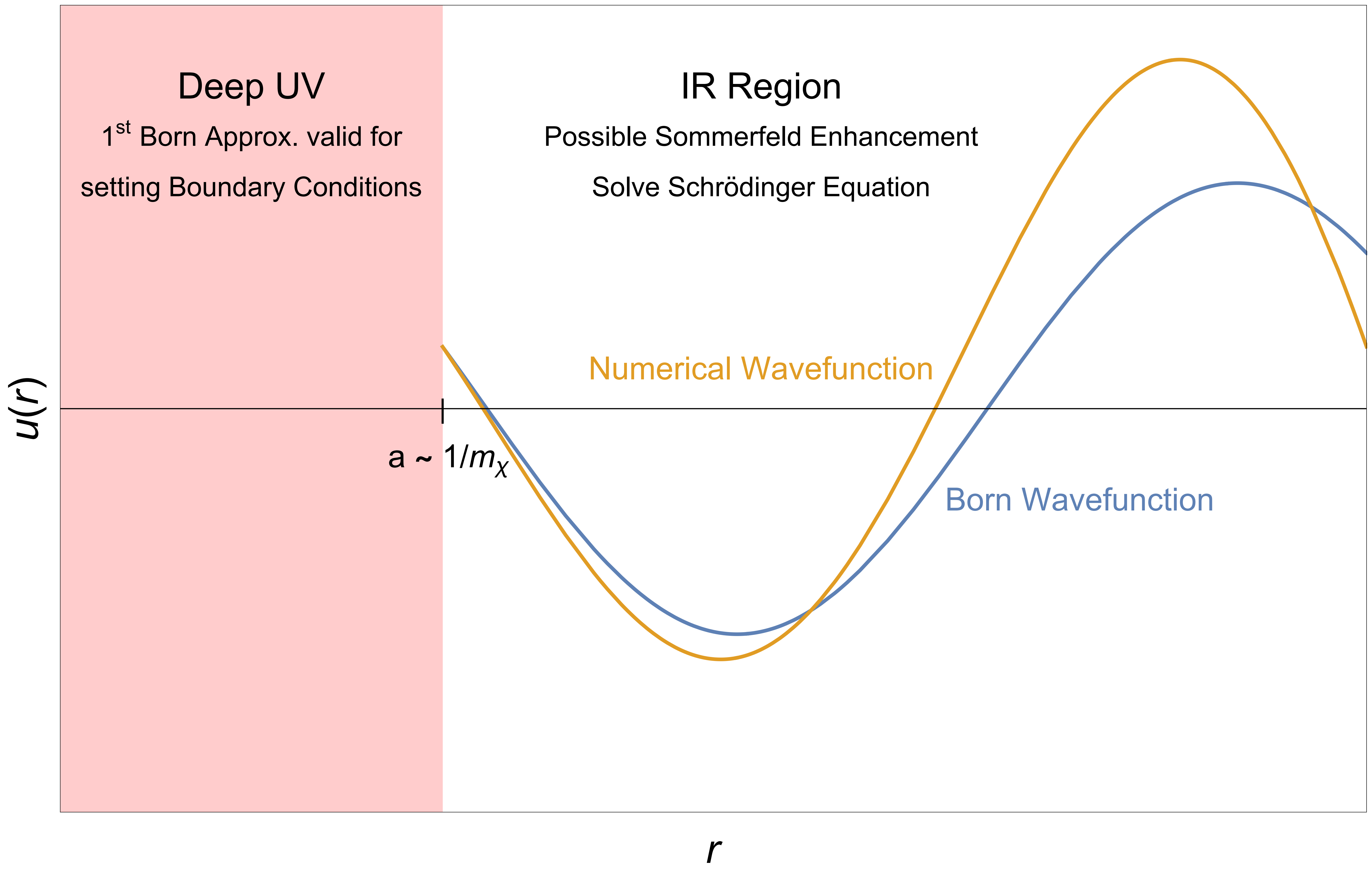
How do we set well-defined boundary conditions for r^{-3} potentials?

Matching Prescription

- Short distances correspond to semi-relativistic momenta

- QFT is a better description than the effective QM potential

- Sommerfeld enhancement is important at large distances



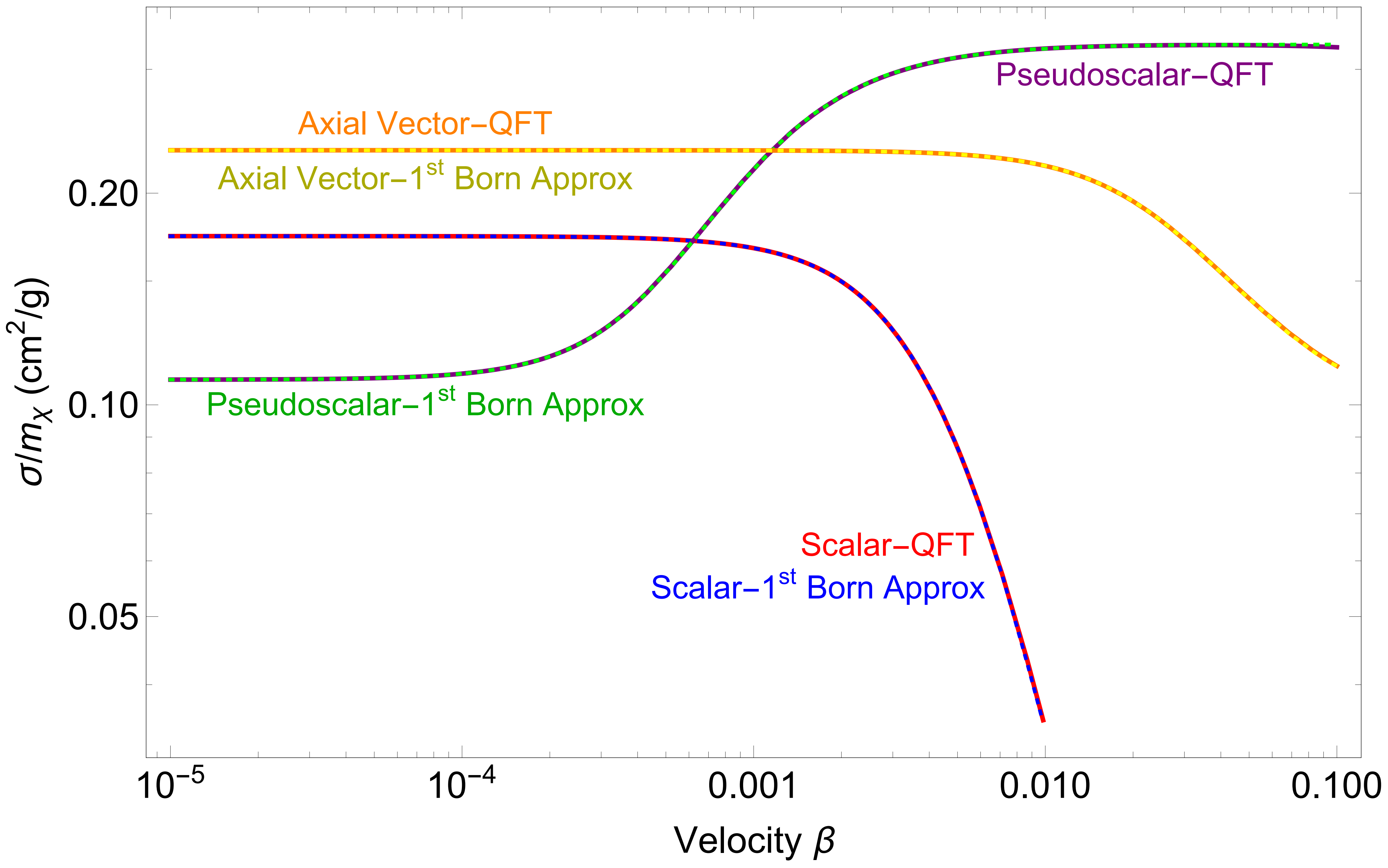
Setting Boundary Conditions

- How do we access the QFT information?
  - The first Born approximation in quantum mechanics faithfully reproduces tree-level QFT

$$K^a_{\ell s,\ell' s'} = -\frac{2\mu}{k}\int_0^a dr s_{\ell'}(kr)V_{\ell s,\ell' s'}(r)s_\ell(kr)$$

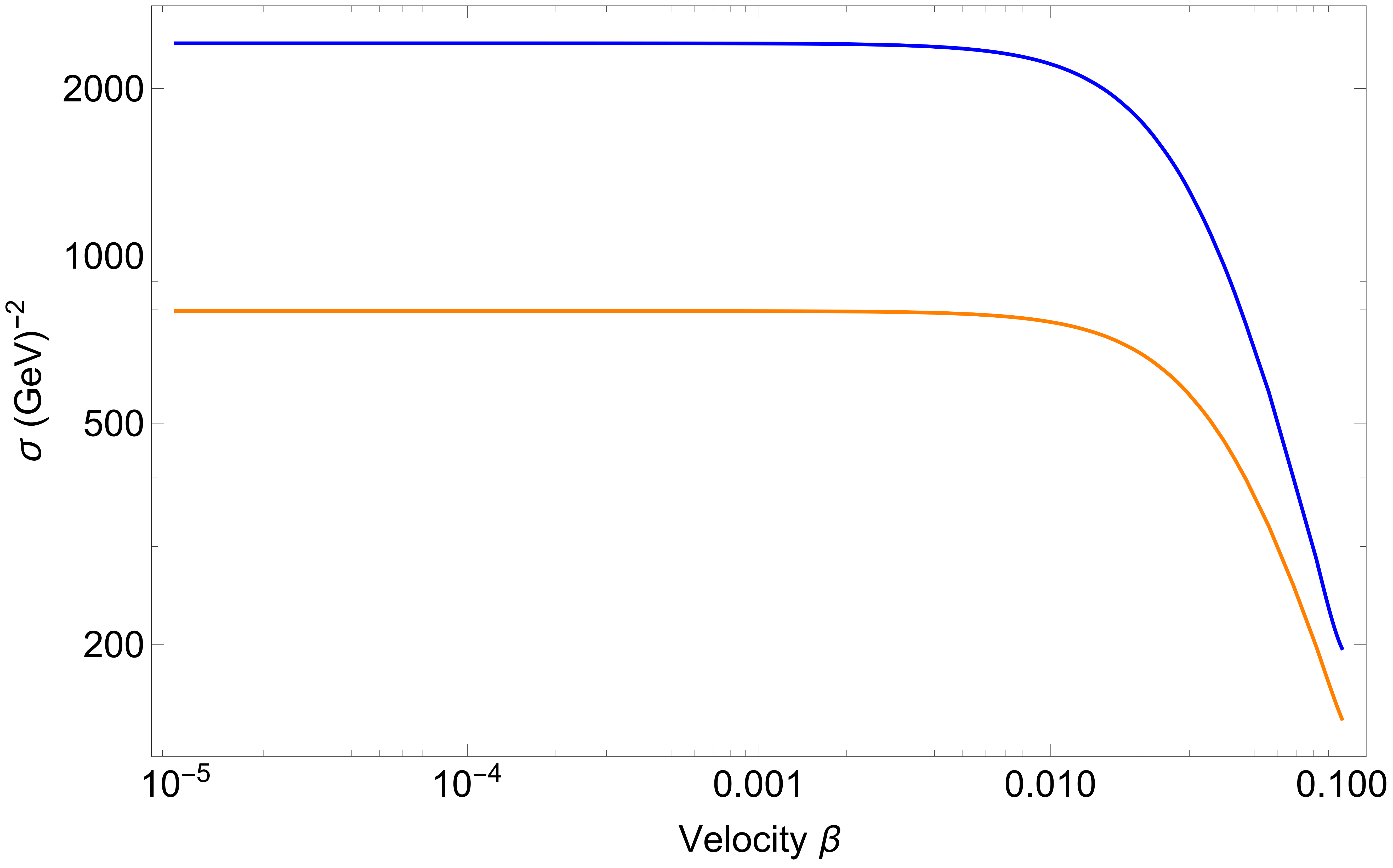
  - This alters the wavefunction and its derivative

$$u_{\ell s,\ell' s'}(a) \sim \delta_{\ell s,\ell' s'} s_\ell(ka) + K^a_{\ell s,\ell' s'} c_\ell(ka)$$



Numerical Results: Scalar Exchange

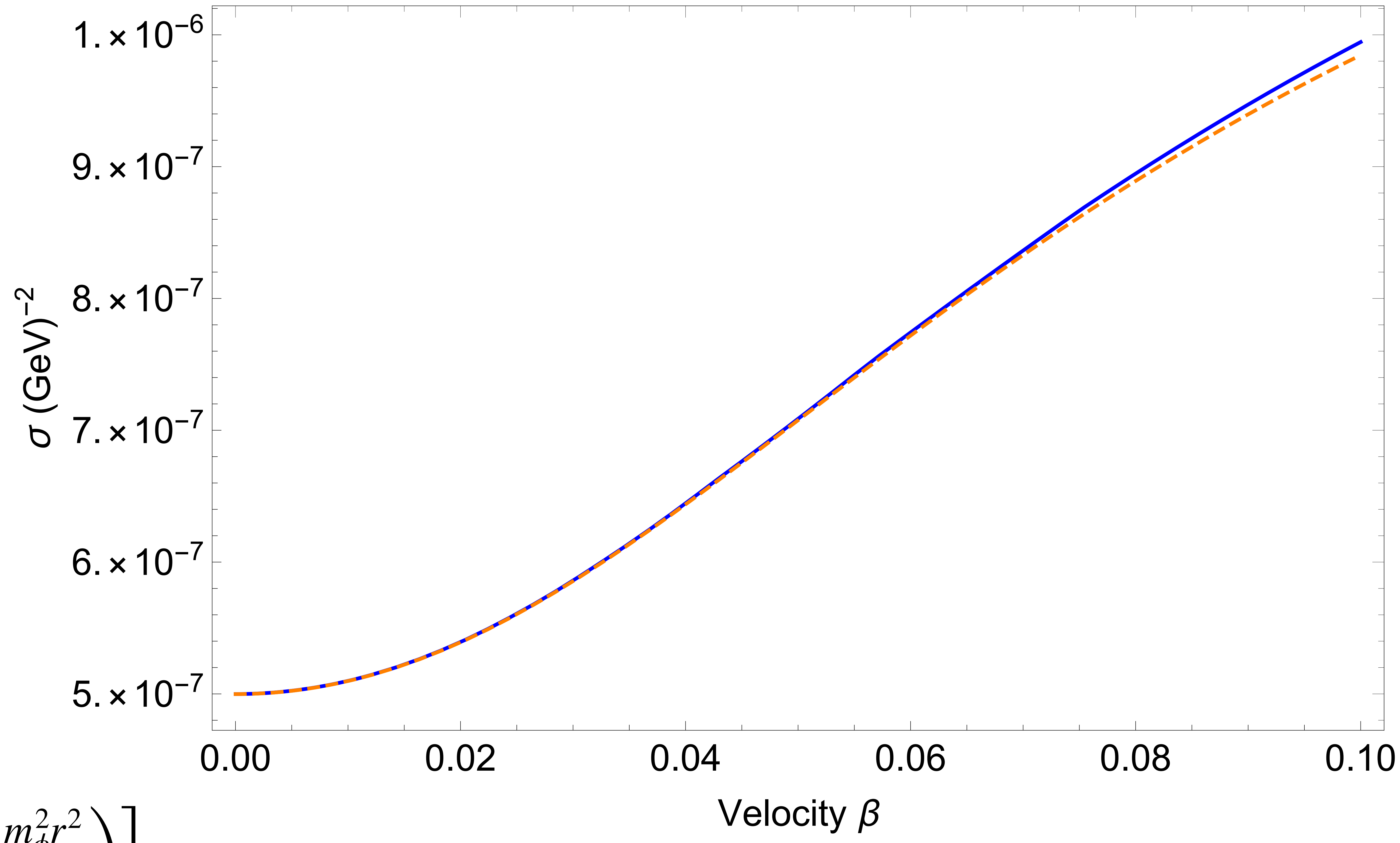
- Numerical cross section vs. QFT tree-level cross section
- Sommerfeld enhancement at low velocities
- Numerical results agree with and without our matching procedure!



Mediator Mass = 0.1 GeV; Dark Matter Mass = 1 GeV; Coupling = 1

Numerical Results: Pseudoscalar Exchange

- Numerical cross section vs. QFT tree-level cross section
- No Sommerfeld enhancement at low velocities
- Effectively a short-range potential

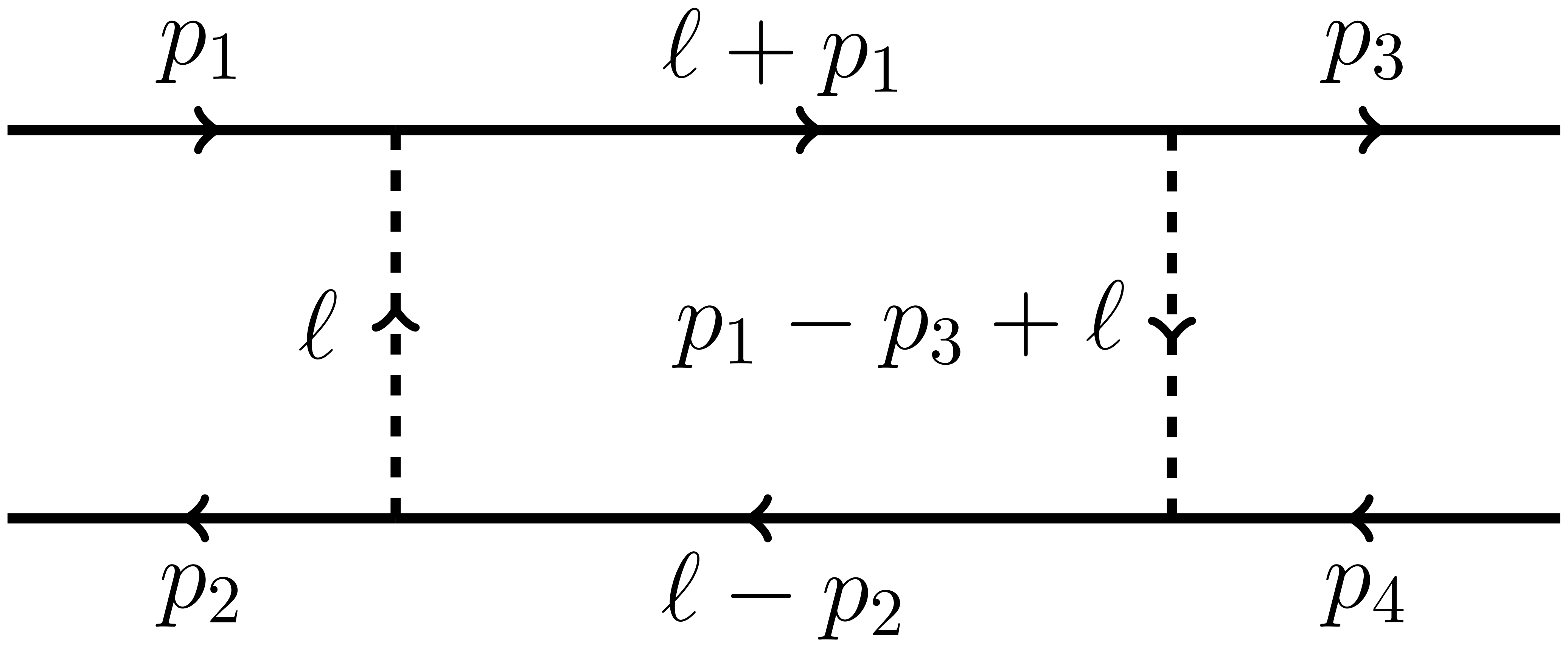


$$V \supset \frac{\lambda^2}{4\pi}\frac{e^{-m_\phi r}}{m_\chi^2}\left[\frac{m_\phi^2}{3r} S_1 \cdot S_2 + \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)\right]$$

Mediator Mass = 0.1 GeV; Dark Matter Mass = 1 GeV; Coupling = 0.1

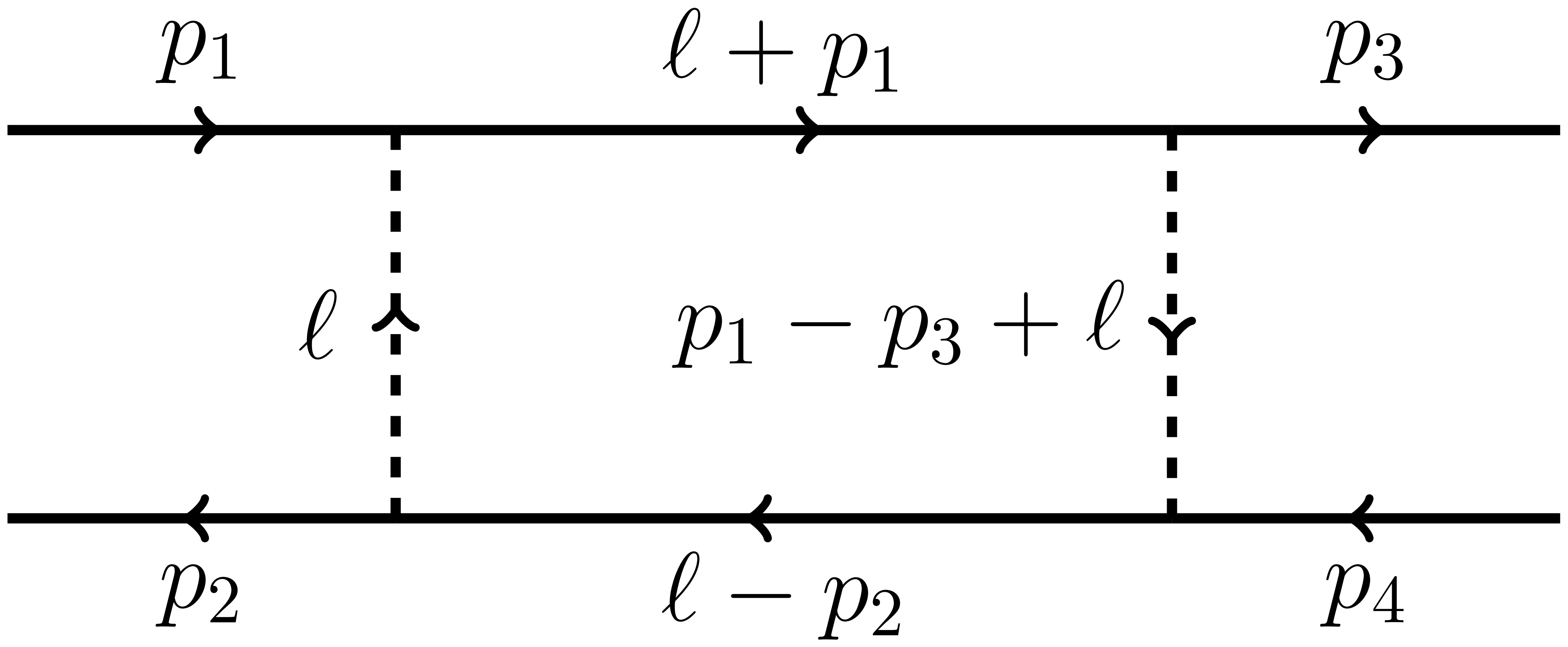
Sommerfeld Enhancement from Feynman Diagrams: Pseudoscalar

- Tree-level has s- and t-channel diagrams
- Box diagram for the 1-loop process
- Pseudoscalar case
  - t-channel velocity suppressed in the NR limit
- $\frac{M_{1-loop}}{M_{tree}} \sim \frac{\lambda^2}{32\pi^2} \log \frac{m_\chi^2}{m_\phi^2}$



Sommerfeld Enhancement from Feynman Diagrams: Scalar

- Tree-level has s- and t-channel diagrams
- Box diagram for the 1-loop process
- Scalar case
  - t-channel dominant in the NR limit
  - $\dfrac{M_{1-loop}}{M_{tree}} \sim \dfrac{\lambda^2 m_\chi}{4\pi m_\phi} \quad \rightarrow \quad m_\phi \lesssim \dfrac{\lambda^2 m_\chi}{4\pi}$



Singular or Not?

- Is the pseudoscalar potential singular?

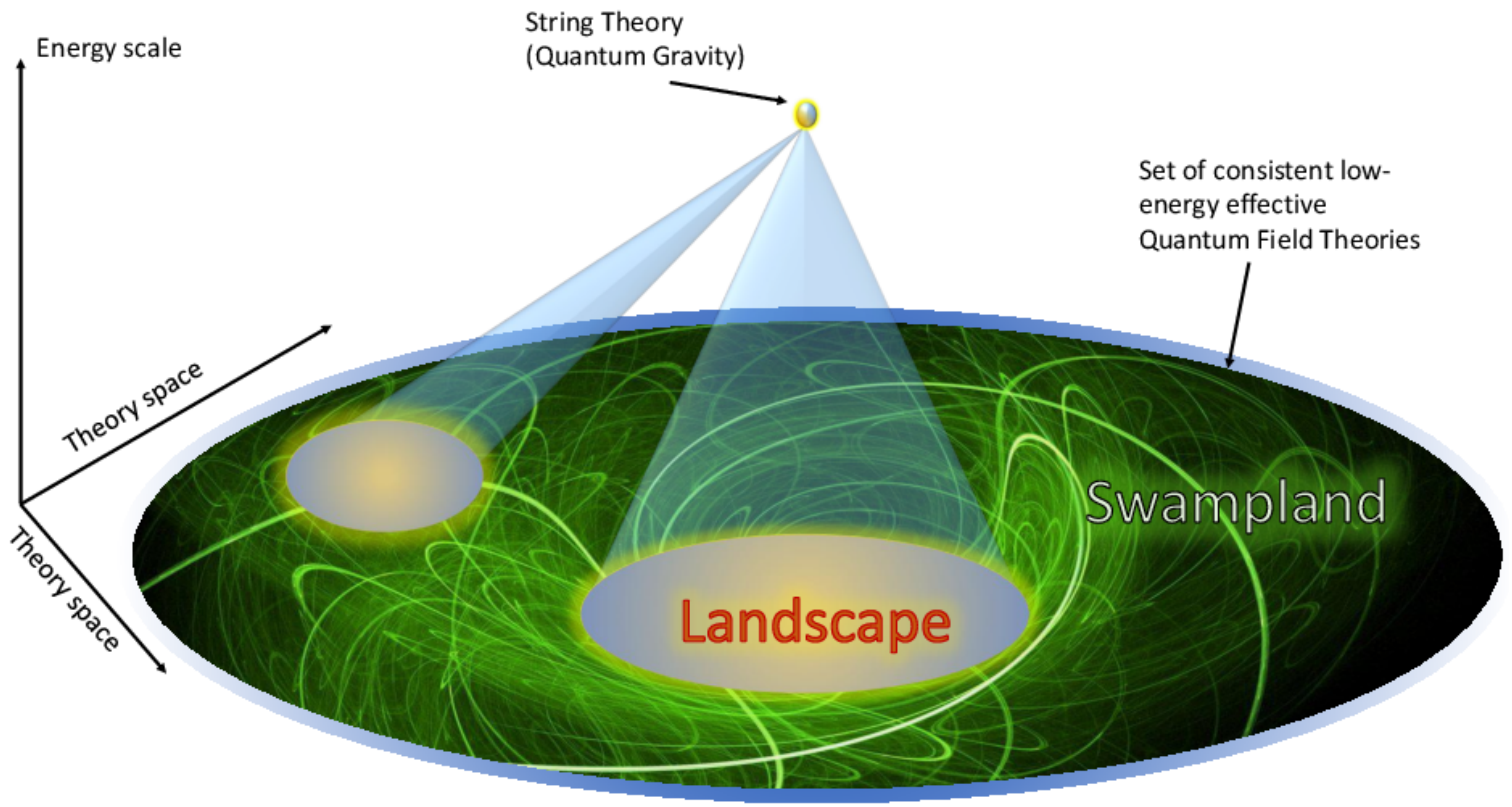
- Do singular potentials have phenomenological implications?

    - Extensive reviews in the literature exploring singular potentials. Two primary classification schemes exist. The first classifies potentials as singular if they diverge faster than r^{-2} at the origin. The second classifies potentials as singular if they arise from non-renormalizable operators in QFT.

    - More recently, it has been claimed that this has implications for SIDM and Sommerfeld enhancement for pseudoscalars. In particular, they introduce square-well regulators, but treat the depth of the square well and the coupling strength as free parameters, instead of matching to a perturbative QFT.

An Aside: Swampland Overview

- Consistency is the key to the Swampland program
- Frame conjectures in terms of criteria that low energy QFTs must satisfy to reside in the Landscape
- Apply this philosophy to the study of QM potentials



Palti [1903.06239]

The Quantum Mechanics Swampland

- The Landscape consists of all quantum mechanical potentials that can be derived from well-defined tree-level QFTs. A potential resides in the Swampland if it is singular.

- **Diagnostic:** If the first Born approximation diverges for *any* combination of incoming and outgoing states, then the potential is singular.

$$K_{\ell s,\ell' s'} \propto \int_0^a dr s_{\ell'}(kr) V_{\ell s,\ell' s'}(r) s_\ell(kr) \approx \int_0^a dr (kr)^{\ell'+1} V_{\ell s,\ell' s'}(r)(kr)^{\ell+1}$$

- As an example, we'll evaluate the pseudoscalar potential and the four-fermion version of it.

Pseudoscalar Potentials

- Mediated by renormalizable operators, we get
  - $\mathscr{L}_{int} = i\lambda\phi\overline{\psi}\gamma^5\psi \quad \rightarrow \quad V \supset \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)\frac{e^{-m_\phi r}}{m_\chi^2}$
- Diagnostic check - the only singular term has a vanishing matrix element!
  - $K_{\ell s,\ell' s'} \supset \int_0^a dr s_{\ell'}(kr)\frac{N_{\ell,\ell'}}{r^3}s_\ell(kr) \approx N_{\ell,\ell'}\int_0^a dr r^{\ell+\ell'-1}$
  - $N_{0,0} = \langle \ell' = 0 | 3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2 | \ell = 0 \rangle = 0$

Pseudoscalar Potentials

- Mediated by non-renormalizable operators, we get
  - $\mathscr{L}_{int} = \frac{\lambda}{\Lambda^2}\overline{\psi}_1\gamma^5\psi_1\overline{\psi}_2\gamma^5\psi_2 \quad \rightarrow \quad V \supset \frac{\lambda}{m_1 m_2 \Lambda^2}(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla})\delta^3(\vec{r})$
- Diagnostic check
  - $\int_0^a dr s_{\ell'}(kr)(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla})\delta^3(\vec{r})s_\ell(kr) \approx S_1^i S_2^j \int_0^a dr \nabla_i \nabla_j \frac{\delta(r)}{r^2}(kr)^{\ell+1}(kr)^{\ell'+1}$
  - $S_1^i S_2^j \nabla_i \nabla_j \delta(r) r^{\ell+\ell'} = \delta(r) r^{\ell+\ell'-2}\left[(\ell+\ell'-1)\delta_{ij} + (3+(\ell+\ell')(\ell+\ell'-4))\hat{r}_i\hat{r}_j\right] S_1^i S_2^j$
- Case I: $\ell = \ell' = 0 \rightarrow \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^2}\delta(r)$ Case II: $\ell + \ell' = 1 \rightarrow 0$

Extensions to Higher Dimensions

- Coulomb potentials in d spatial dimensions
  - No operator structure! Problematic for d > 4?

$$V(r) = \frac{\alpha}{r^{d-2}}$$

- To compute the diagnostic, we need the free particle solutions in d spatial dimensions. Let's turn to solving the free Schrödinger equation in d dimensions, which will give us these solutions.

Solving the Schrödinger Equation in Higher Dimensions

- Consider the free particle Schrödinger equation in d spatial dimensions

$$-\frac{1}{2\mu}\nabla_d^2\Psi(r) = E\Psi(r) \qquad \nabla_d^2 = \partial_r^2 + \frac{d-1}{r}\partial_r + \frac{1}{r^2}\Omega^2$$

- The wavefunction is a product of a radial function and Gegenbauer polynomials. They are the higher dimensional generalization of the spherical harmonics.

$$\partial_r^2 R + \frac{d-1}{r}\partial_r R - \frac{\ell(\ell+d-2)}{r^2}R = -k^2 R$$

- Change variables to cancel the first derivative: $u(r) = r^{(d-1)/2}R(r)$

$$\partial_r^2 u + \left[k^2 - \frac{j(j+1)}{r^2}\right]u = 0 \qquad j = \ell + \frac{d-3}{2}$$

Higher Dimensional Coulomb Potentials

- Coulomb potentials in d spatial dimensions
  - No operator structure! Problematic for d > 4?

$$V(r) = \frac{\alpha}{r^{d-2}}$$

- Free particle solutions in d spatial dimensions

$$s_j(kr) = krj_j(kr) \qquad c_j = -kry_j(kr) \qquad j = \ell + \frac{d-3}{2}$$

- Diagnostic check

$$K_{js,j's'} = \frac{-2\mu}{k}\int_0^a dr s_{j'}(kr)V_{js,j's'}(r)s_j(kr) \approx \frac{-2\alpha\mu}{k}\int_0^a dr r^{j'+1}r^{2-d}r^{j+1} \approx \frac{-2\alpha\mu}{k}\int_0^a dr r^{\ell'+\ell+1}$$

Scalar-Scalar Potentials

- Scalars don't possess any intrinsic spin. This allows us to uniquely fix the non-relativistic limit of the amplitude.

$$\tilde{V}(\vec{q}) = \frac{f(q^2)}{q^2+m^2} = \sum_{n=0}^{\infty} \frac{a_n q^{2n}}{q^2+m^2} = \frac{\tilde{a}_{-1}}{q^2+m^2} + \sum_{n=0}^{\infty} \tilde{a}_n q^{2n}$$

- Every factor of q gives us another derivative, so the potential is the sum of a Yukawa term and even derivatives of delta functions.

- Diagnostic check

$$\nabla^{2n}\delta(r) r^{\ell+\ell'} = (\ell+\ell')(\ell+\ell'-1)\cdots(\ell+\ell'+1-2n)\delta(r) r^{\ell+\ell'-2n}$$