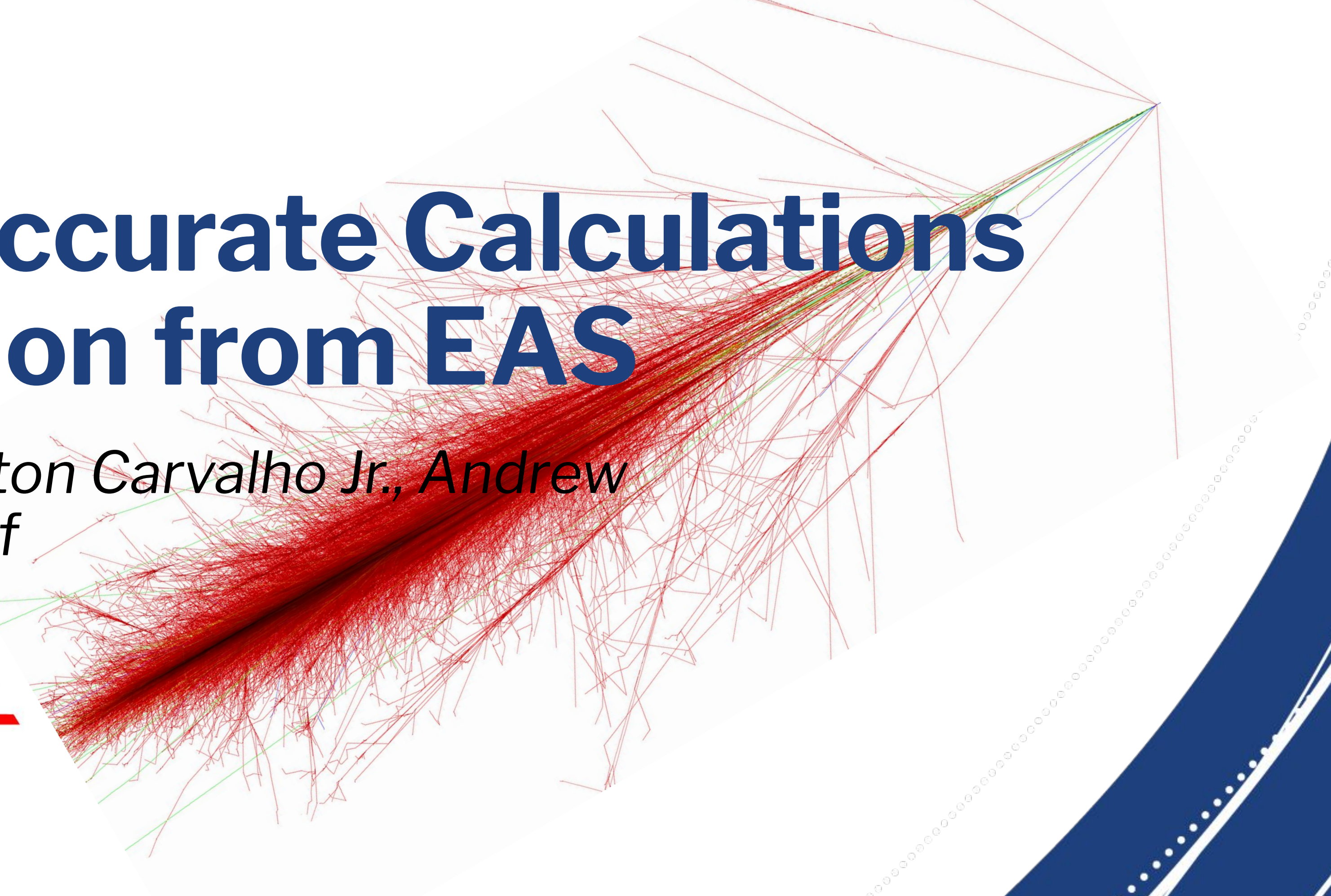
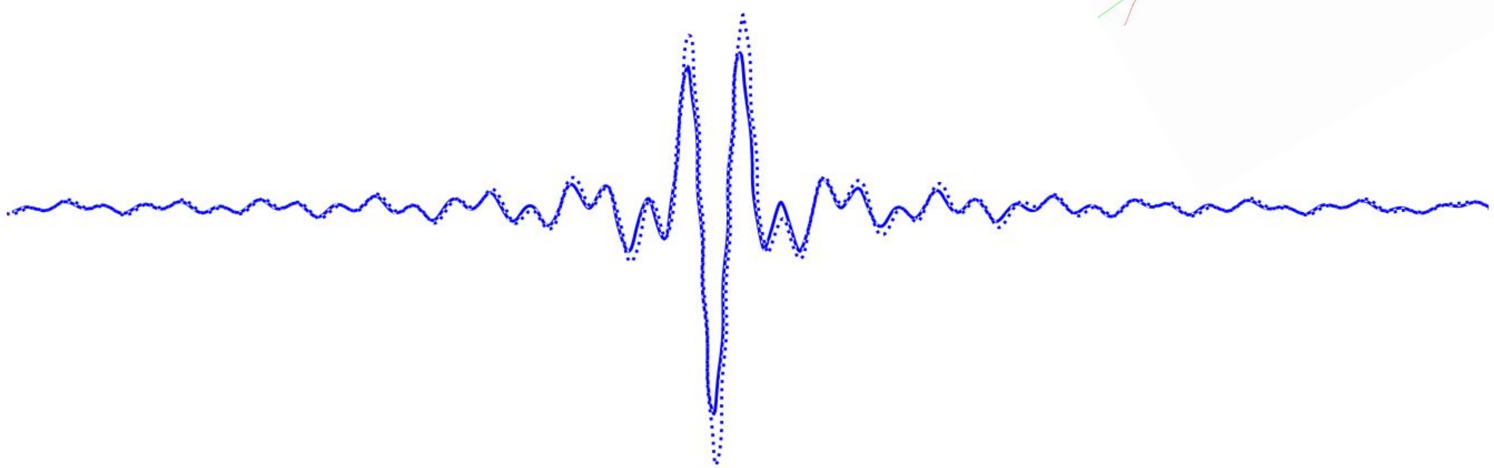
Practical and Accurate Calculations of Radio Emission from EAS

Austin Cummings, Washington Carvalho Jr., Andrew Ludwig, Andres Romero-Wolf*





ARENA

CHICAGO 2024

How Do We Get Radio Emission?

Step 1: Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu \mathbf{J} + \mu\epsilon \frac{\partial \mathbf{E}}{\partial t}$$

Step 2: Introduce scalar and vector potentials

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi$$

Step 3: Choose transverse gauge

$$\nabla \cdot \mathbf{A} = 0$$

Step 4: Equations to solve

$$\nabla^2 \phi = -\frac{\rho}{\epsilon}$$

$$\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial^2 t} = -\mu \mathbf{J}_\perp$$

J. Alvarez-Muniz, A. Romero-Wolf, E. Zas,
Phys.Rev.D81:123009

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General Solutions

$$\phi = \frac{1}{4\pi\epsilon} \int \frac{\rho(\mathbf{x}', t')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}'$$

$$\mathbf{A} = \frac{\mu}{4\pi} \int \frac{\mathbf{J}_\perp(\mathbf{x}', t')}{|\mathbf{x} - \mathbf{x}'|} \delta\left(\sqrt{\mu\epsilon}|\mathbf{x} - \mathbf{x}'| - (t - t')\right) d^3\mathbf{x}' dt'$$

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Assume contributions from straight line travel

$$\mathbf{J}_\perp(\mathbf{x}', t') = e\mathbf{v}_\perp \delta^3(\mathbf{x}' - \mathbf{x_0} - \mathbf{v}t')\left[\Theta(t' - t_1) - \Theta(t' - t_2)\right]$$

Account for projection

$$|\mathbf{x} - \mathbf{x_0} - \mathbf{v}t'| \simeq R - \mathbf{v} \cdot \hat{\mathbf{u}} t'$$

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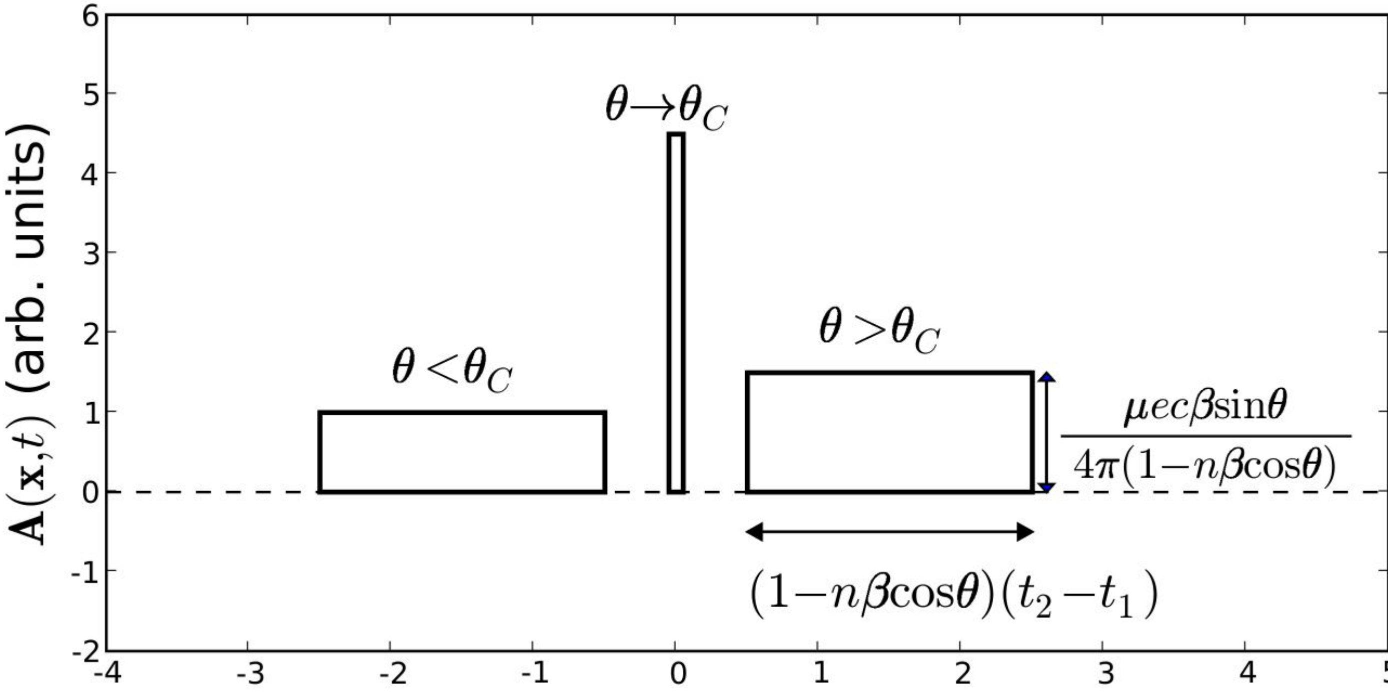

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- Constant emission along travel (box function)
- Proportional to electric charge
- Amplification when measured at: $\theta = \theta_{\mathrm{ch}} = \cos^{-1}\left(\frac{1}{n\beta}\right)$

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Microscopic Simulations

- Vector potential **A** calculated for every electron in the EAS:

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  - Calculation of *n*: index of refraction
  - Calculation of n_{eff}: average line of sight integral of (1-n):

$$n_{\mathrm{eff}} = 1 + \frac{1}{L}\int_0^L [n(z) - 1]\, dl$$

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  - Time binning of **A** at observer

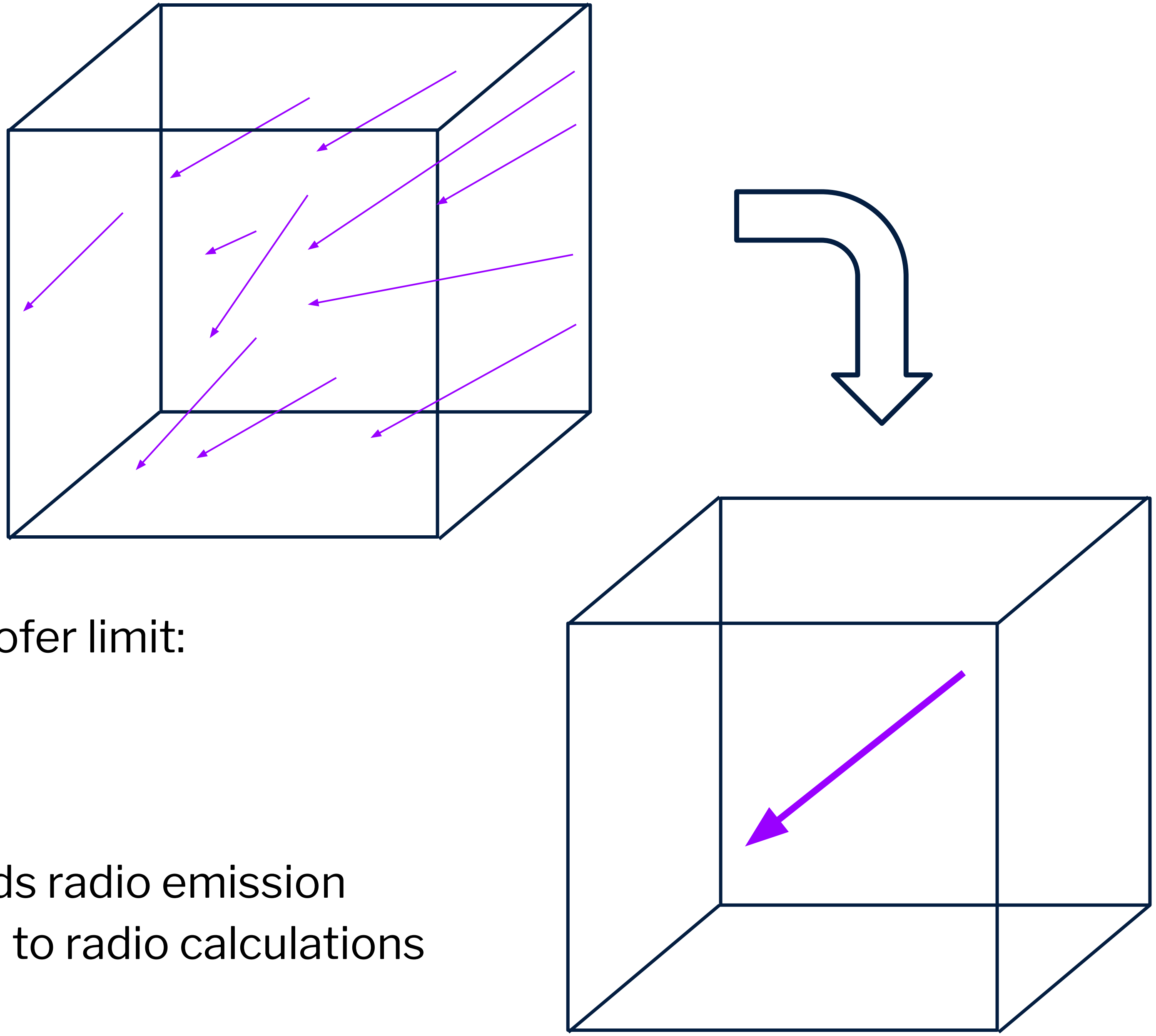
Computationally expensive!

Practical and Accurate Approach

4-D Binning

- Treat $e^{\pm}$ tracks within 4-D volumes as effective single tracks

- Reduces number of computations by a factor N, the number of tracks contained in an average volume



- Preliminary cell size based on Fraunhofer limit:

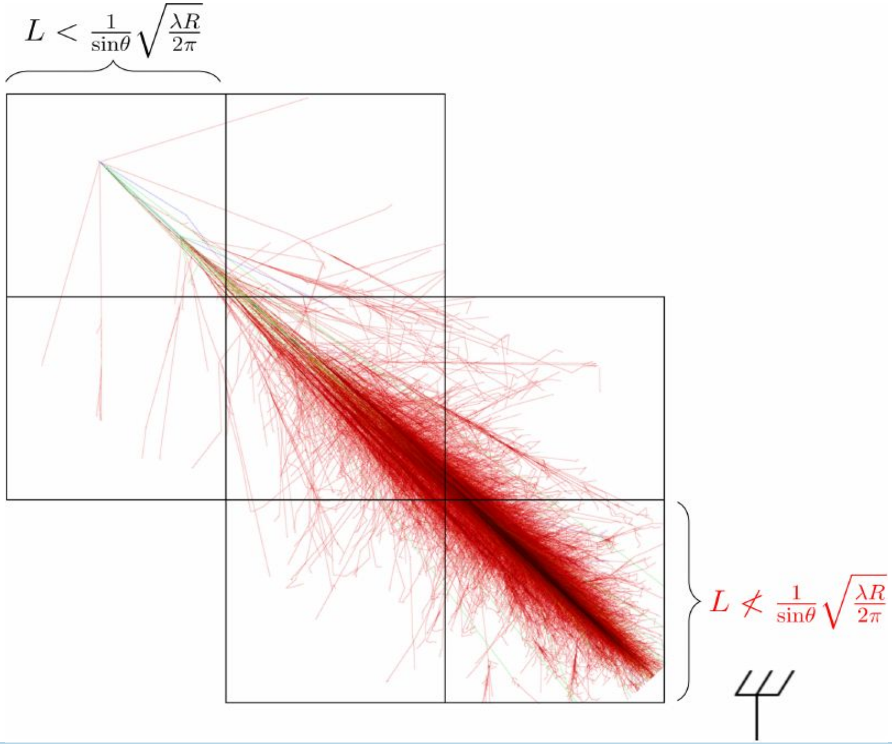
$$L < L_F = \frac{1}{\sin\theta}\sqrt{\frac{\lambda|\vec{R}|}{2\pi}}$$

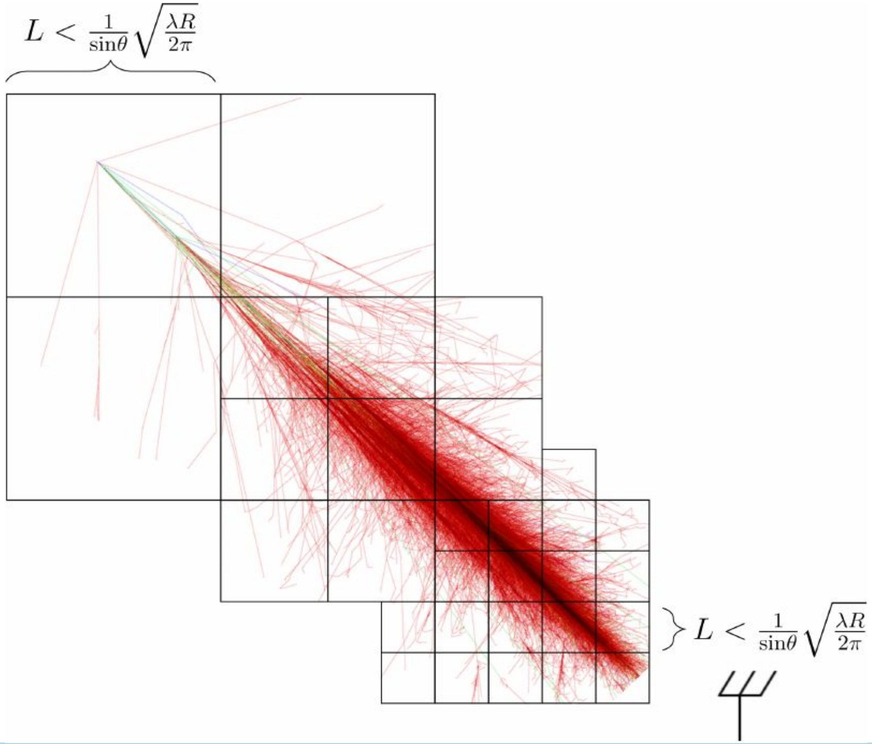
- Essentially, a thinning oriented towards radio emission
    - Standard particle thinning is blind to radio calculations

OctTree Binning

- Fraunhofer condition dependent on both frequency and distance to observer
  - Less efficiency for precision at higher frequencies/closer observation

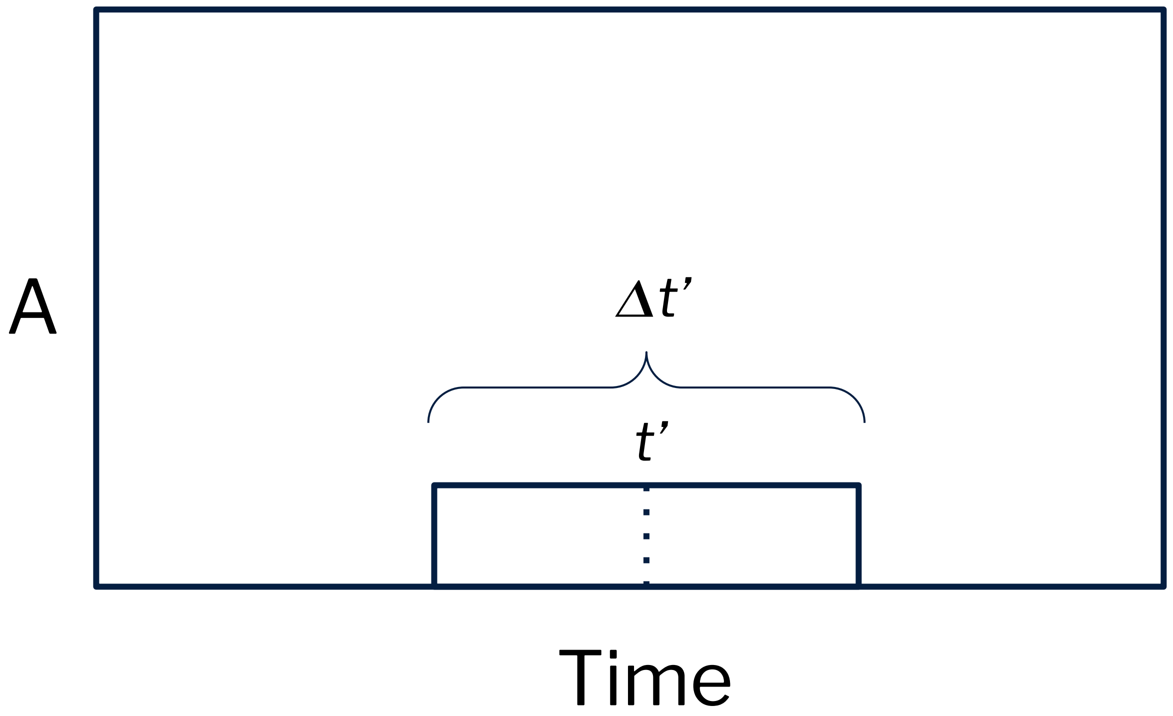
- Cells are bisected until they pass the Fraunhofer limit





Effective Treatment

- Let's define emission from the center of a track:
  - $t' = \frac{n_{\text{eff}} R}{c} + t$
  - $\Delta t' = \Delta t(1 - n\beta \cos\theta)$



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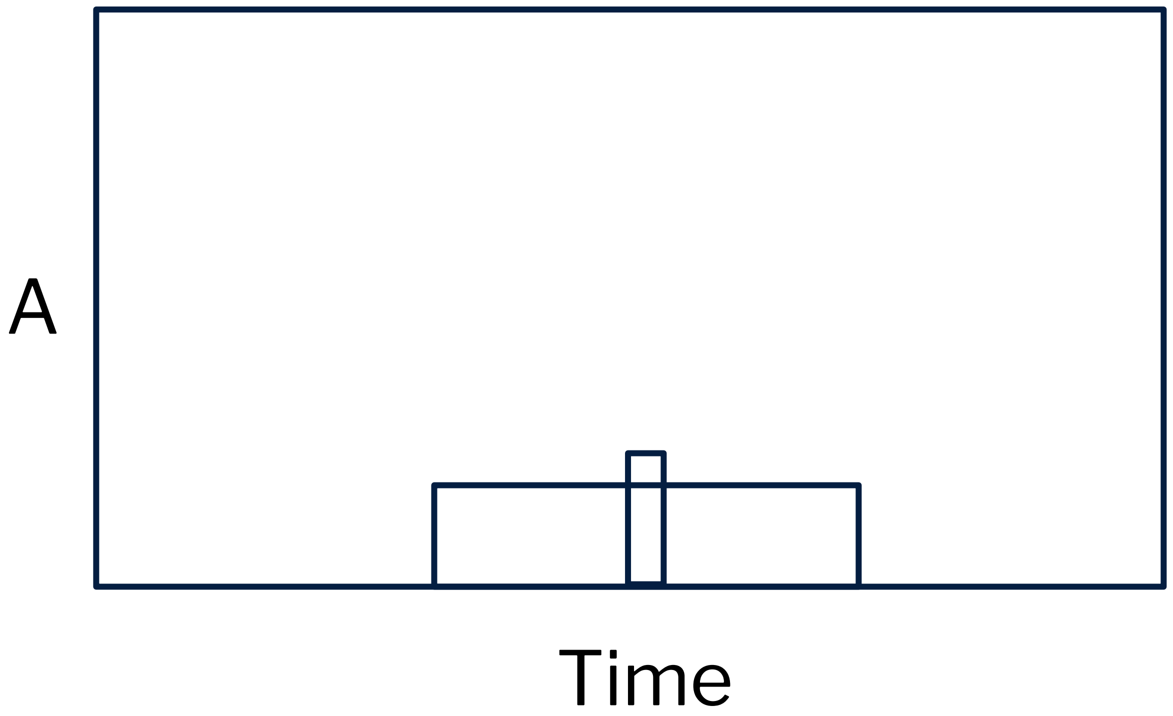

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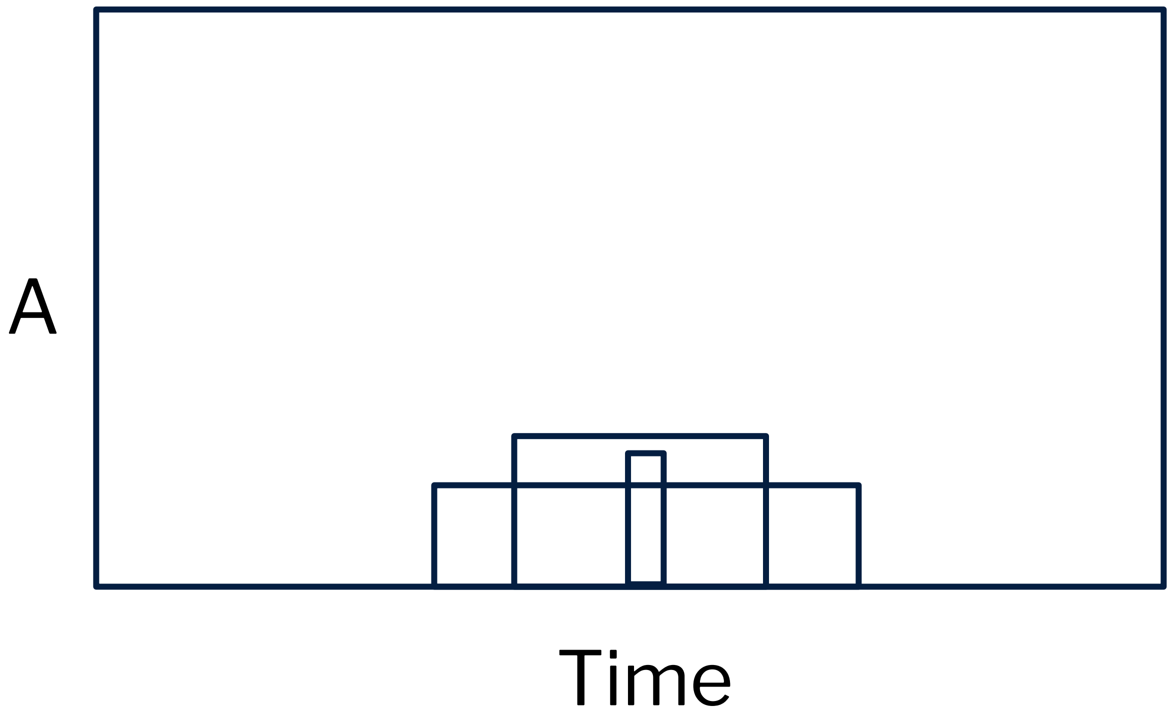

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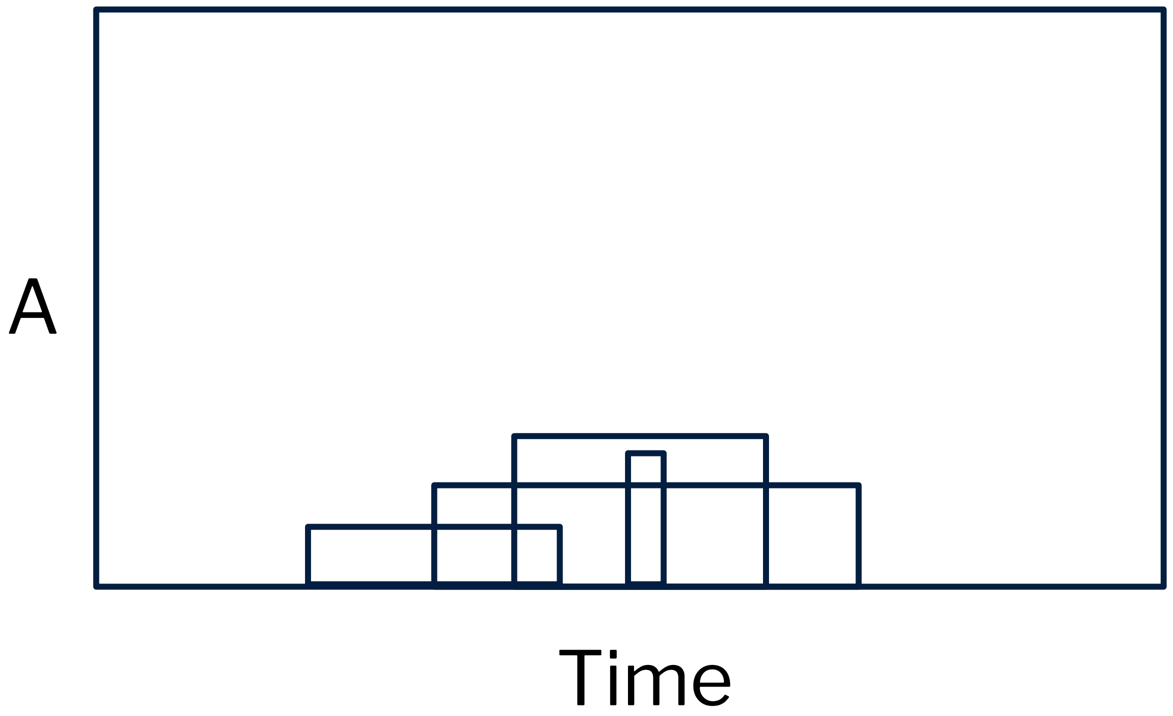

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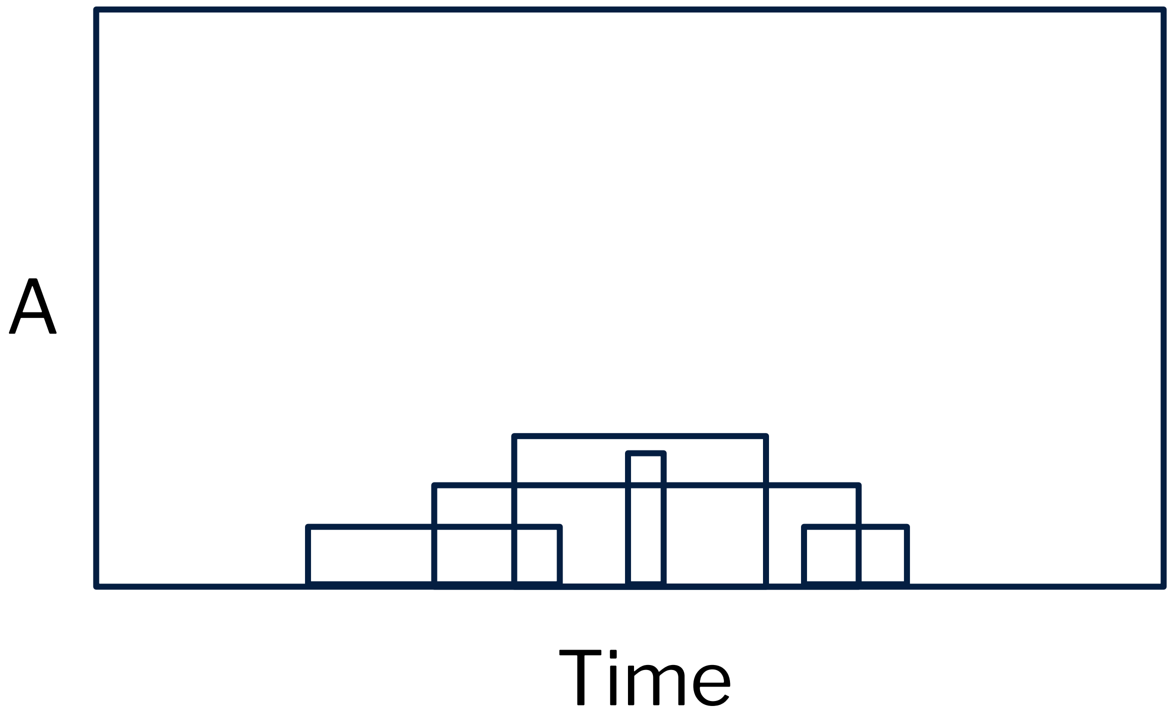

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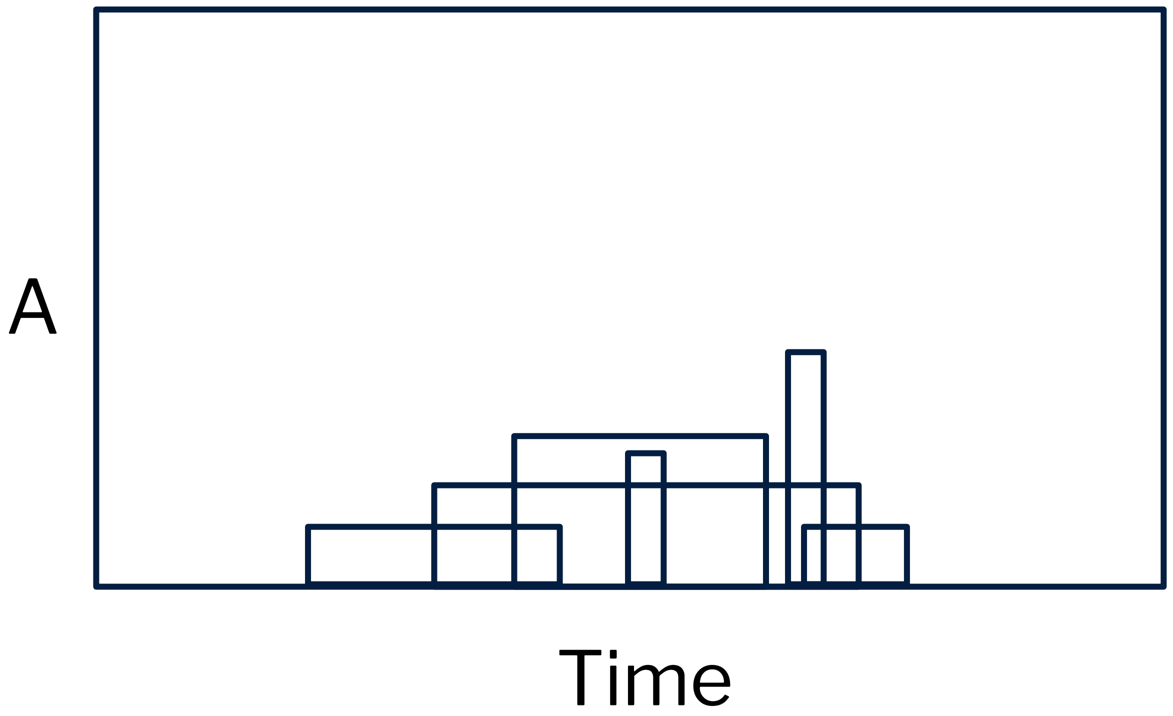

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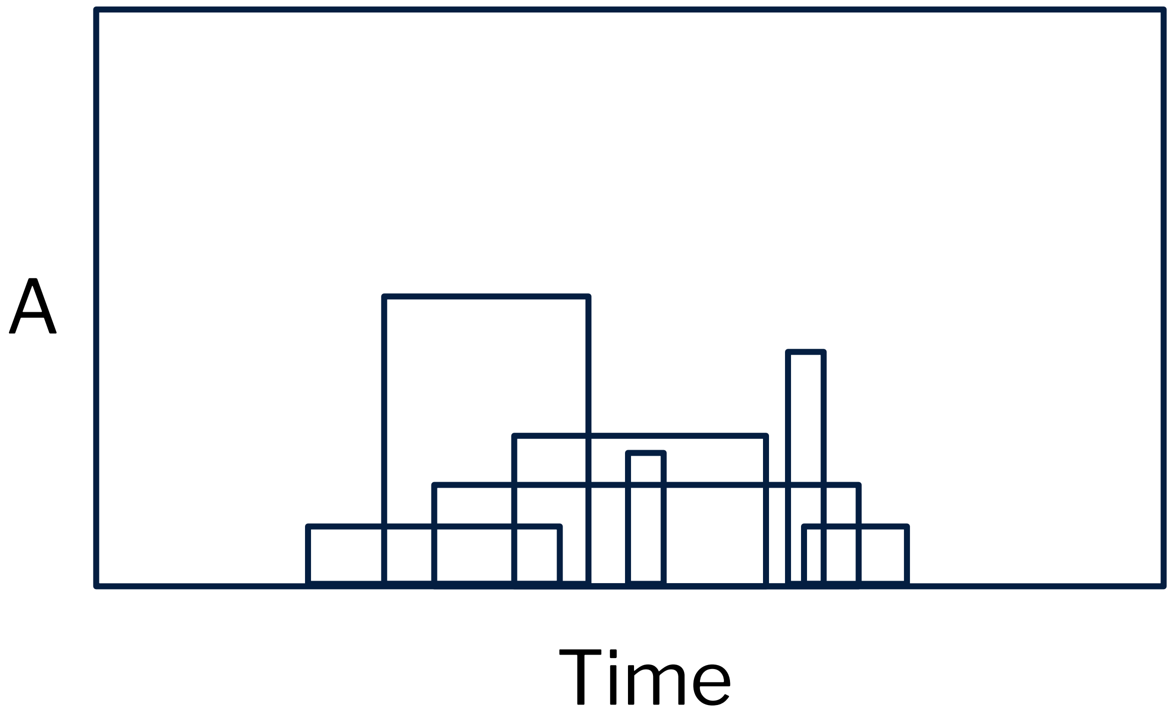

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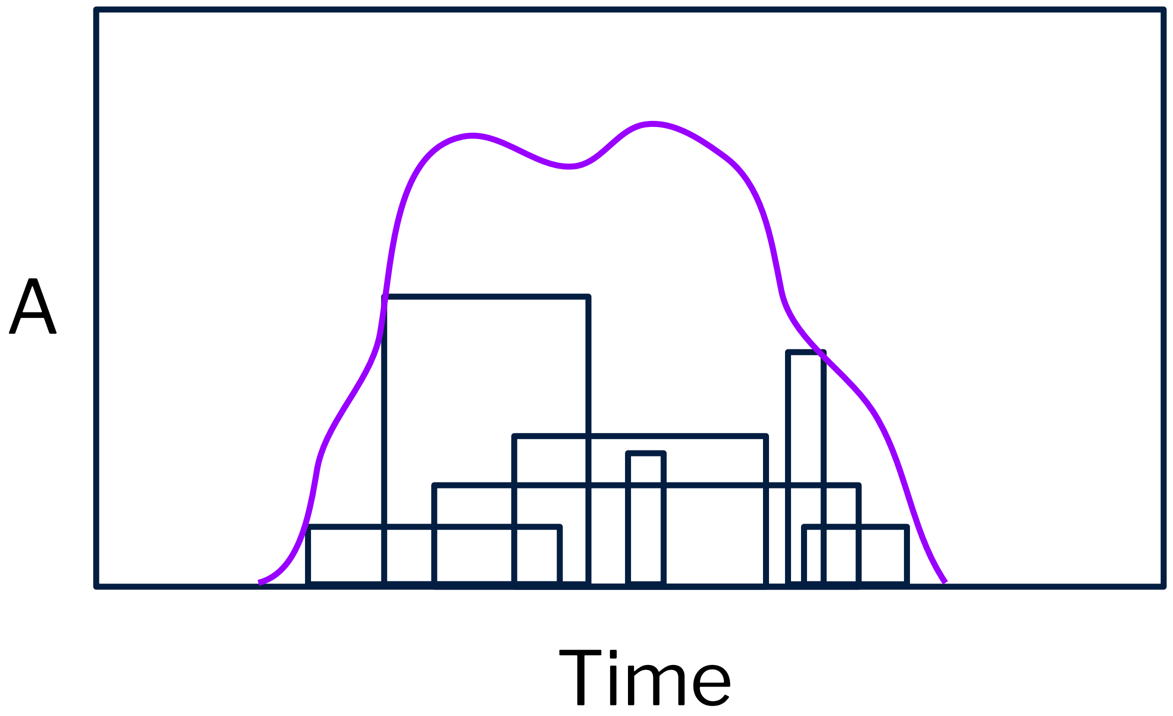

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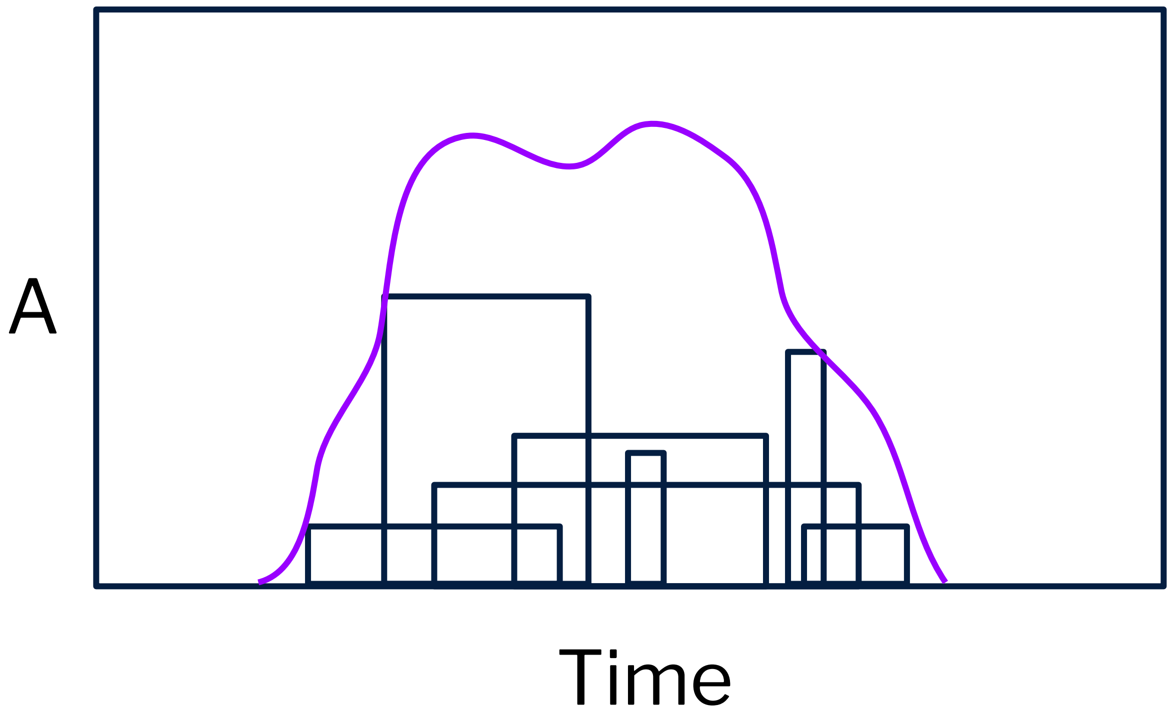

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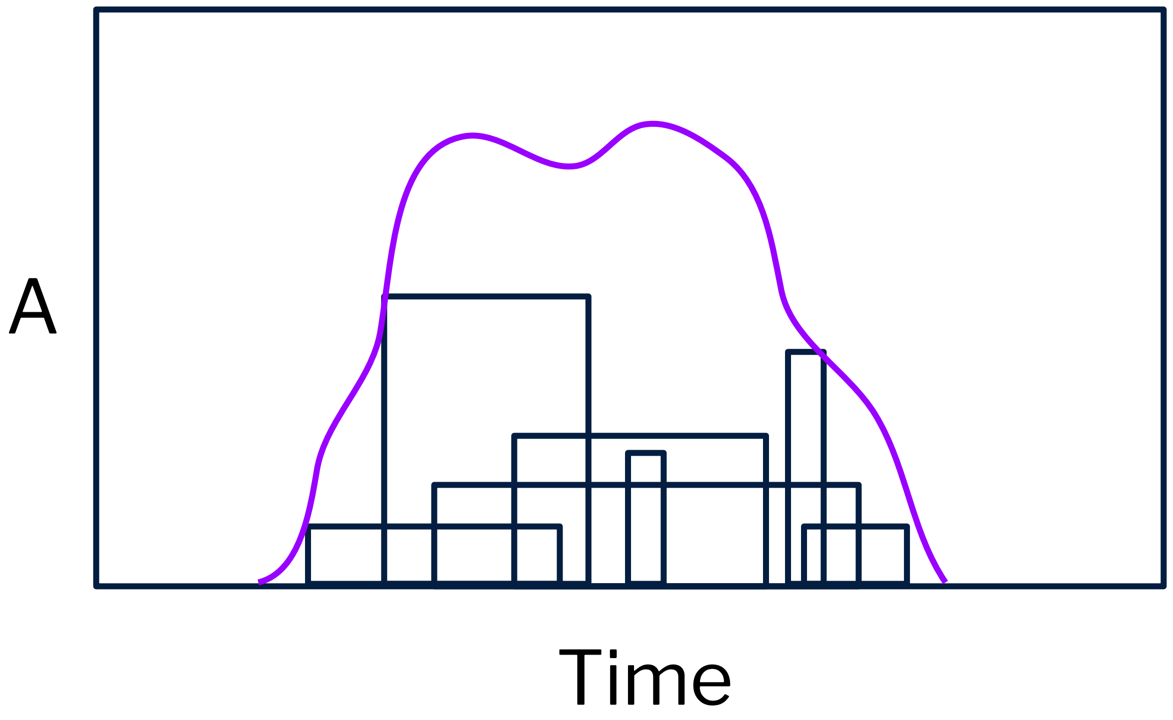

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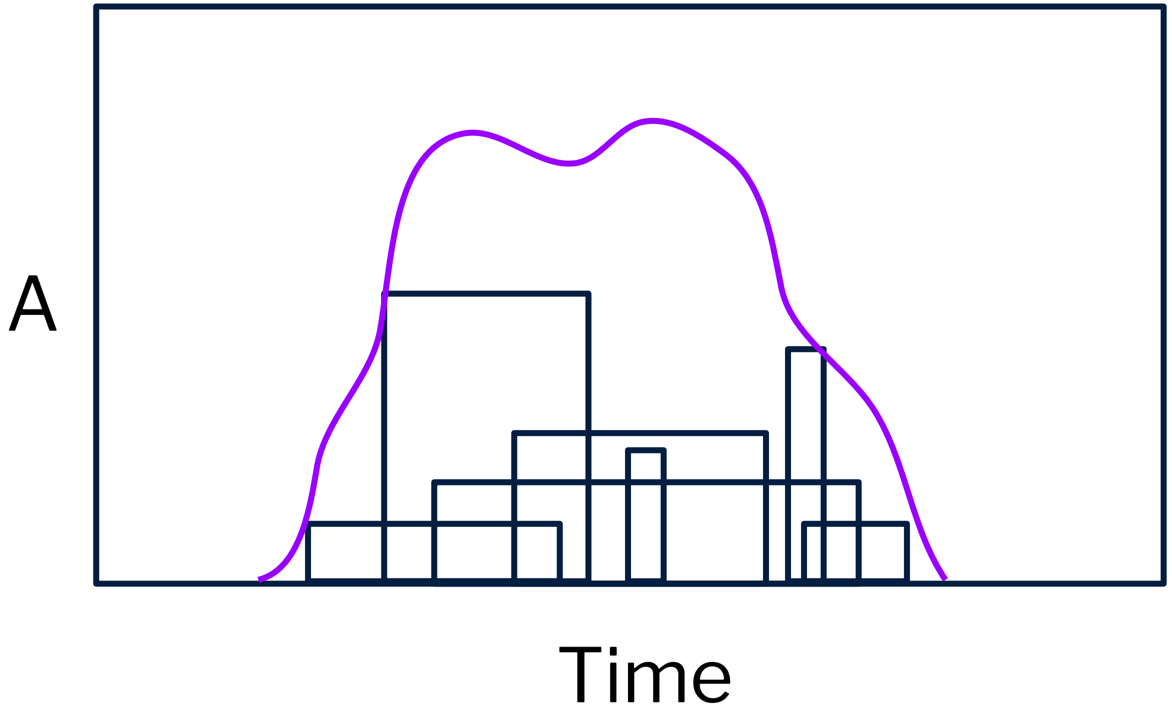

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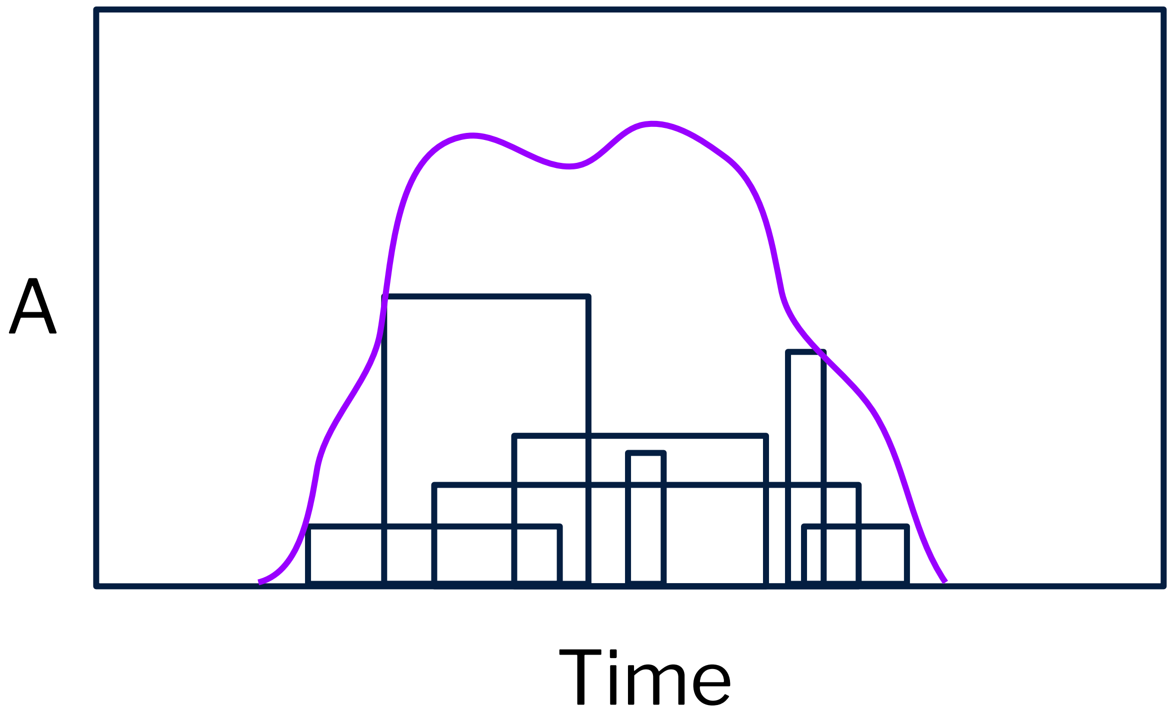

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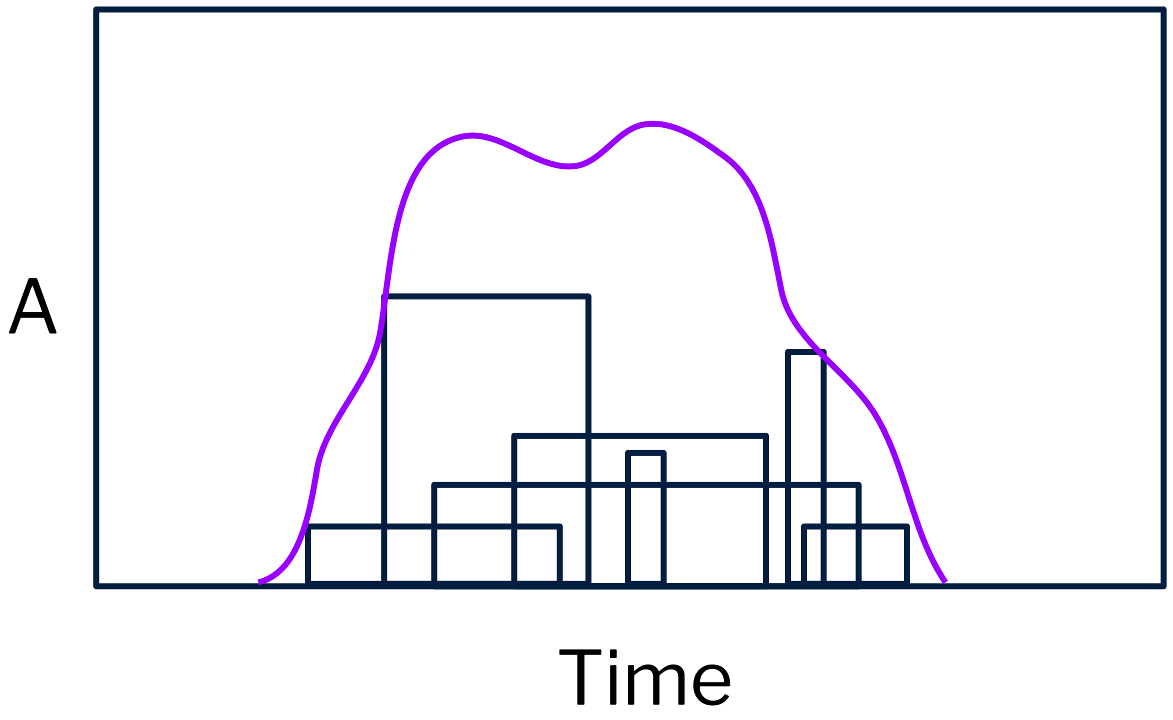

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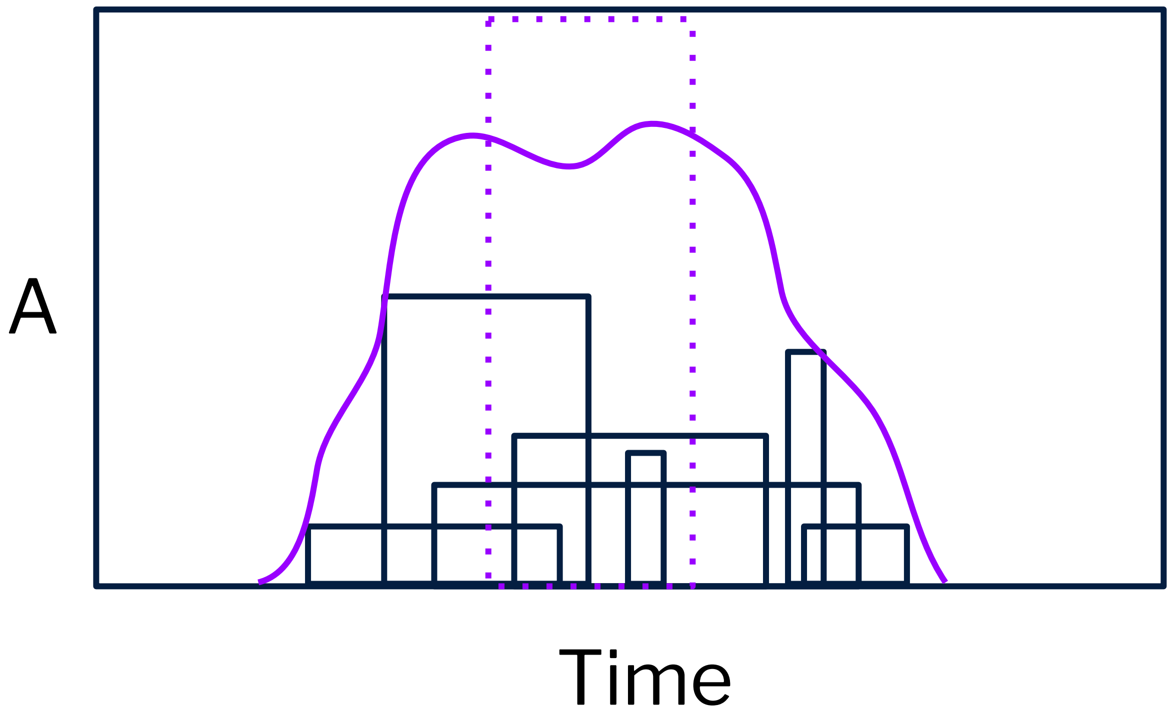

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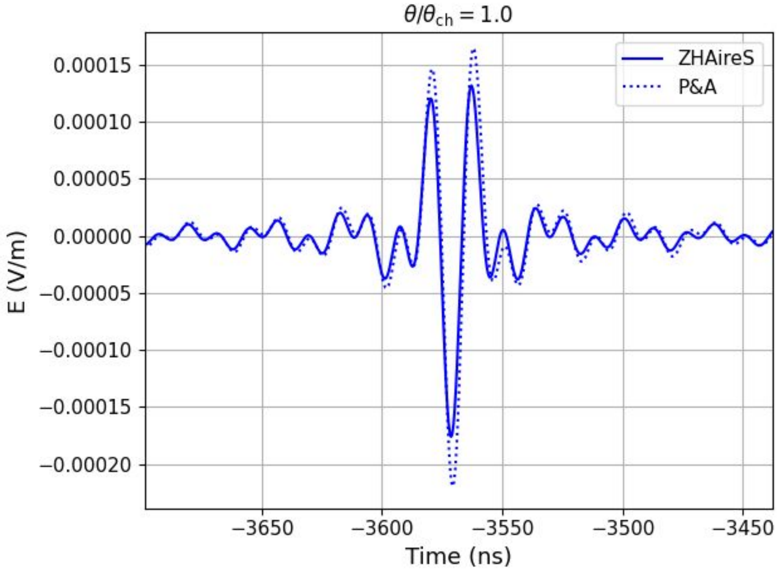
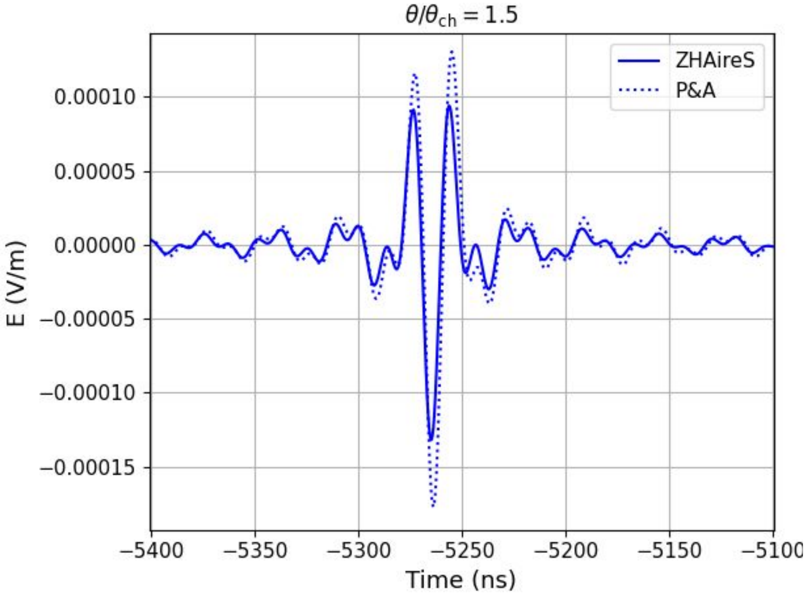
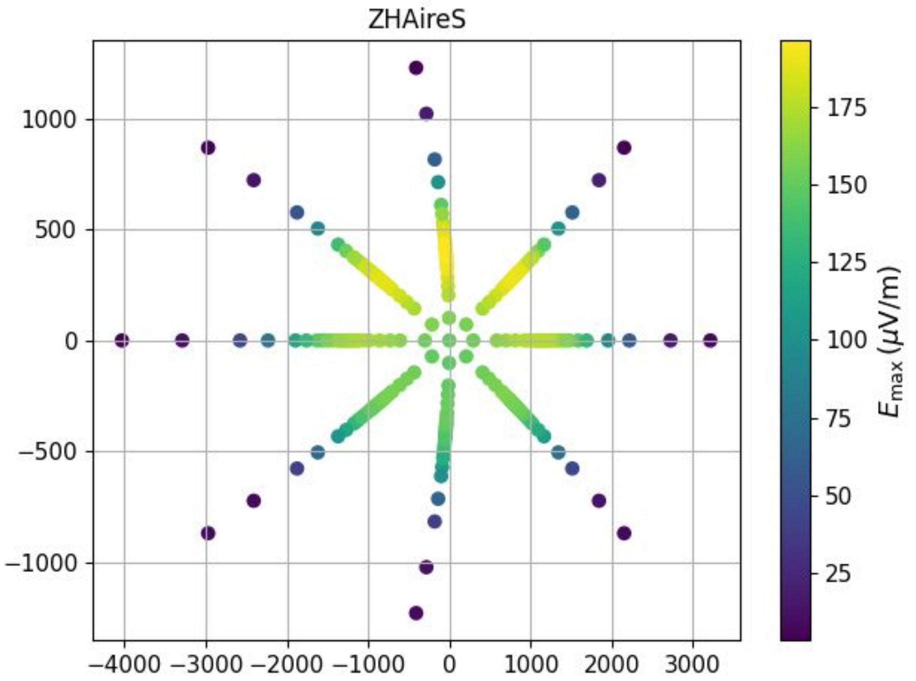
Example Performance



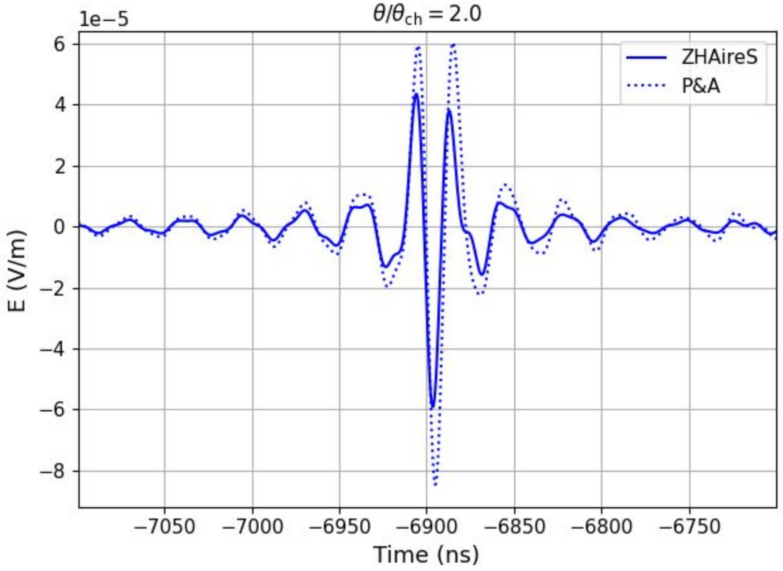


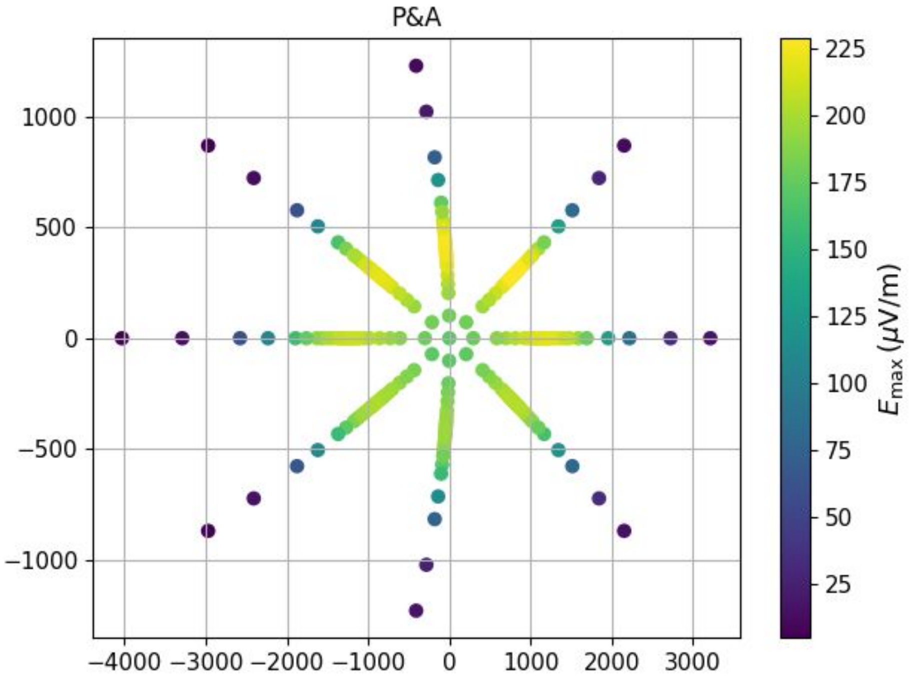


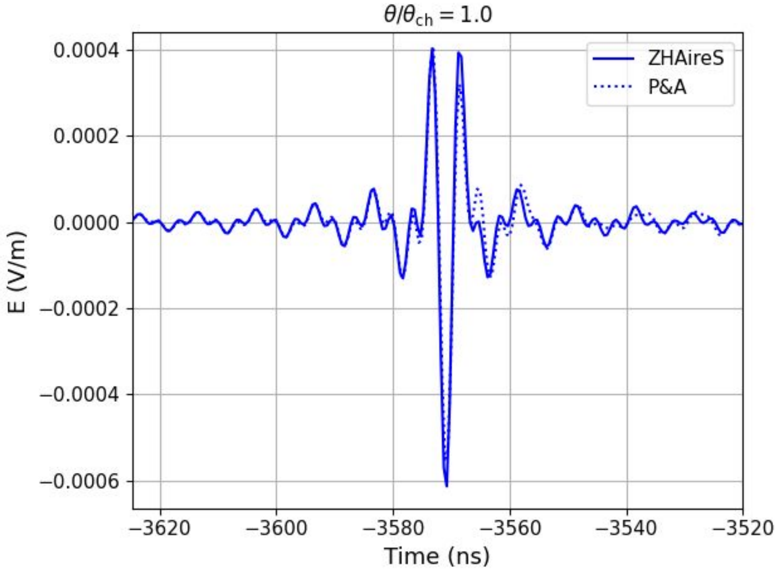
Shower Parameters:

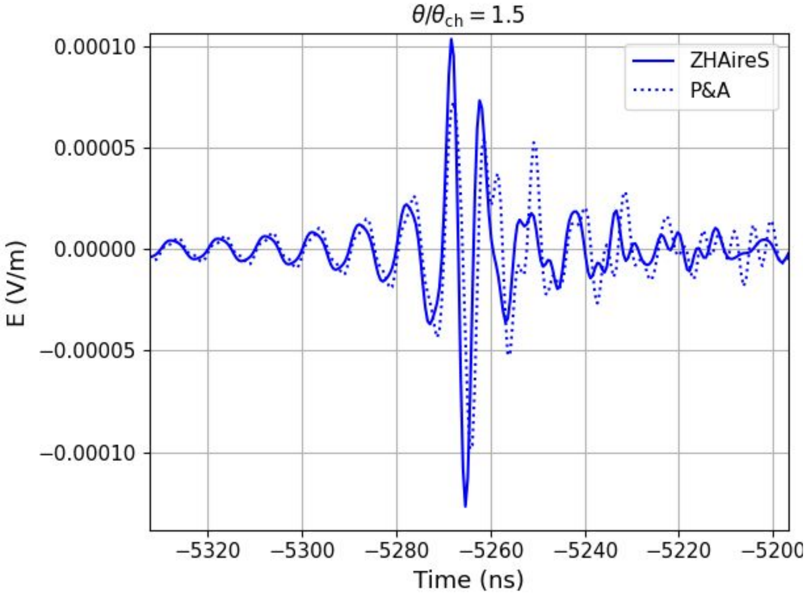
- Primary: proton
- E: 1EeV
- **B**: 43μT, 23° inclination
- θ: 70°, φ: 0°
- 10^{-6} thinning
- L = 0.5L_F

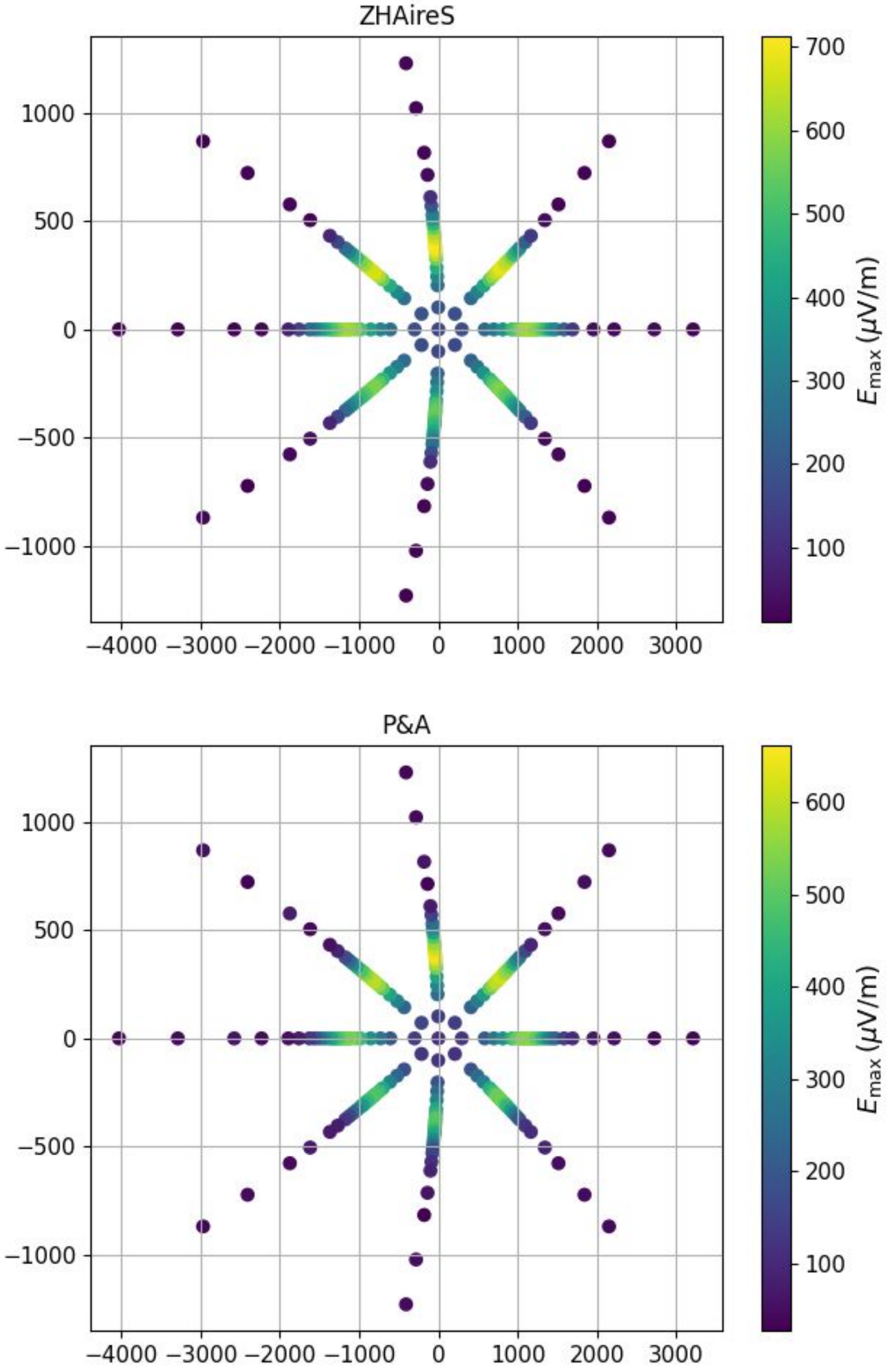
30-80MHz











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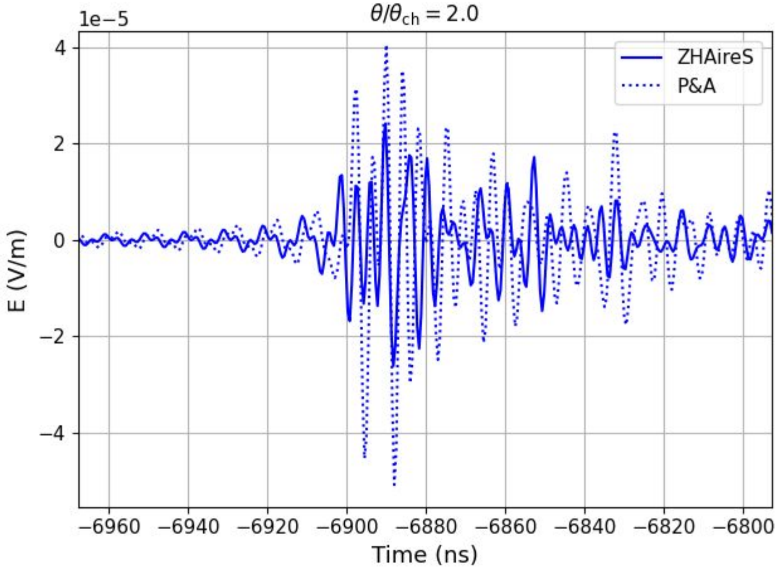

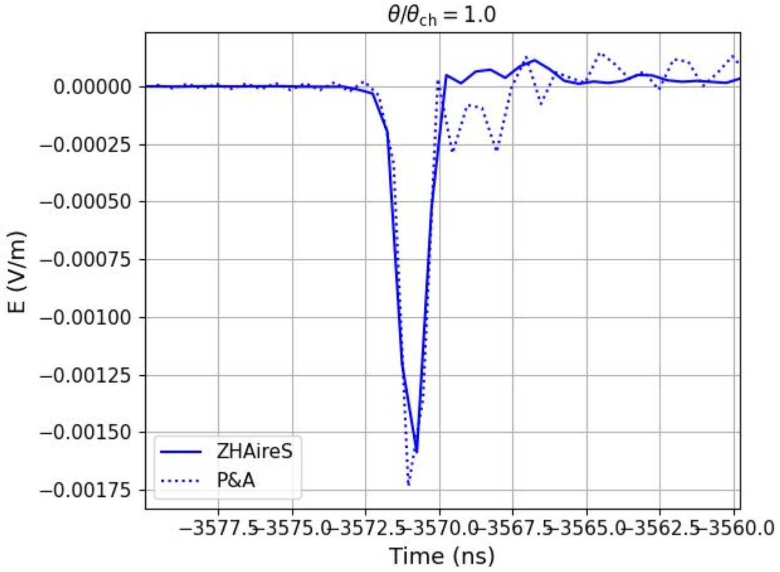

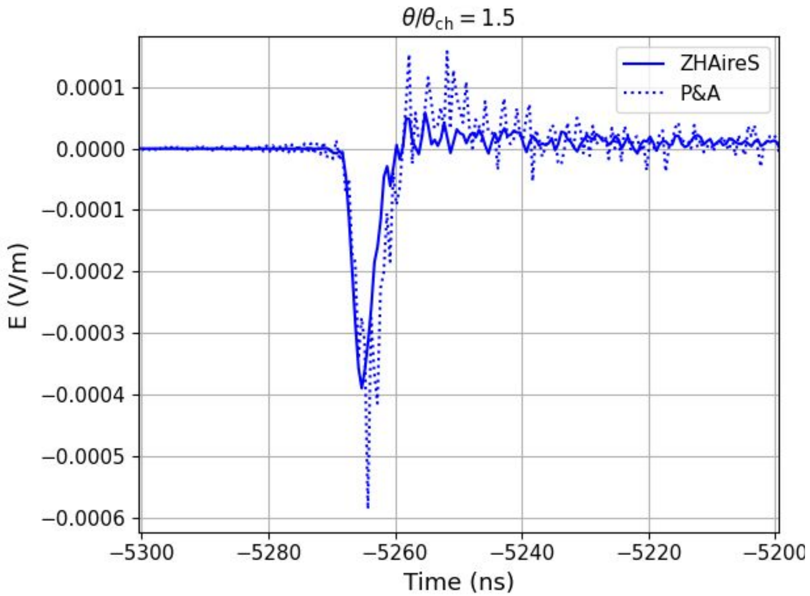

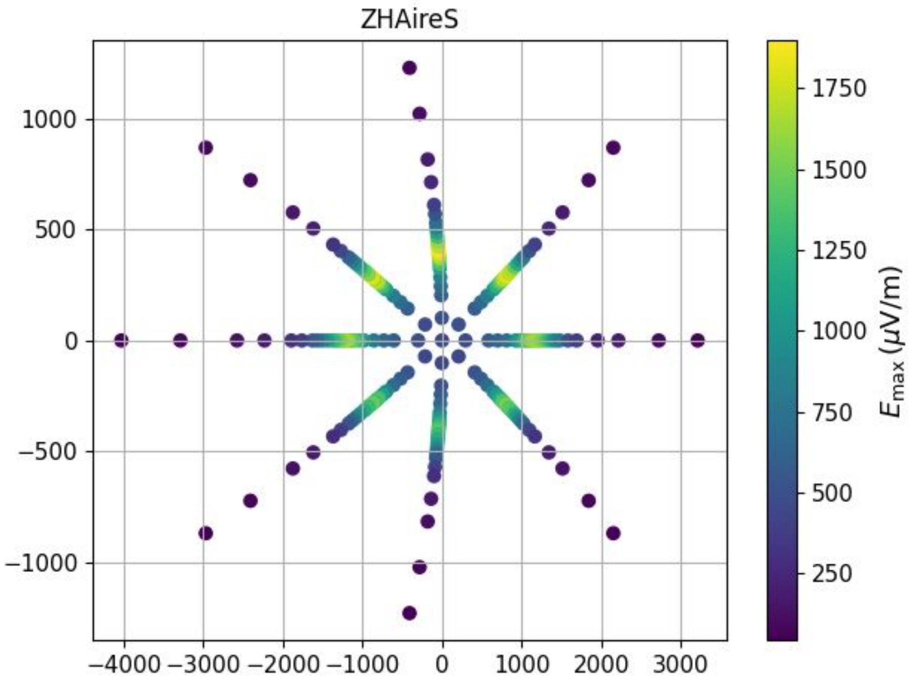

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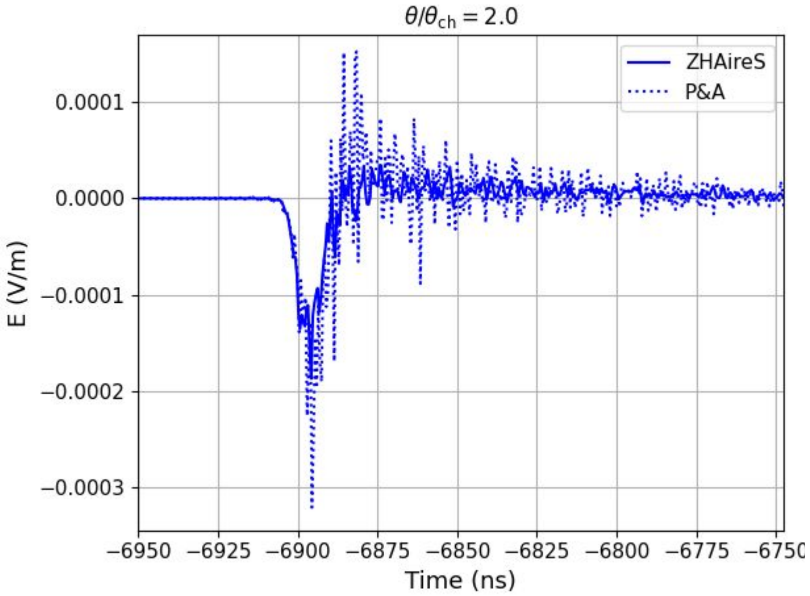

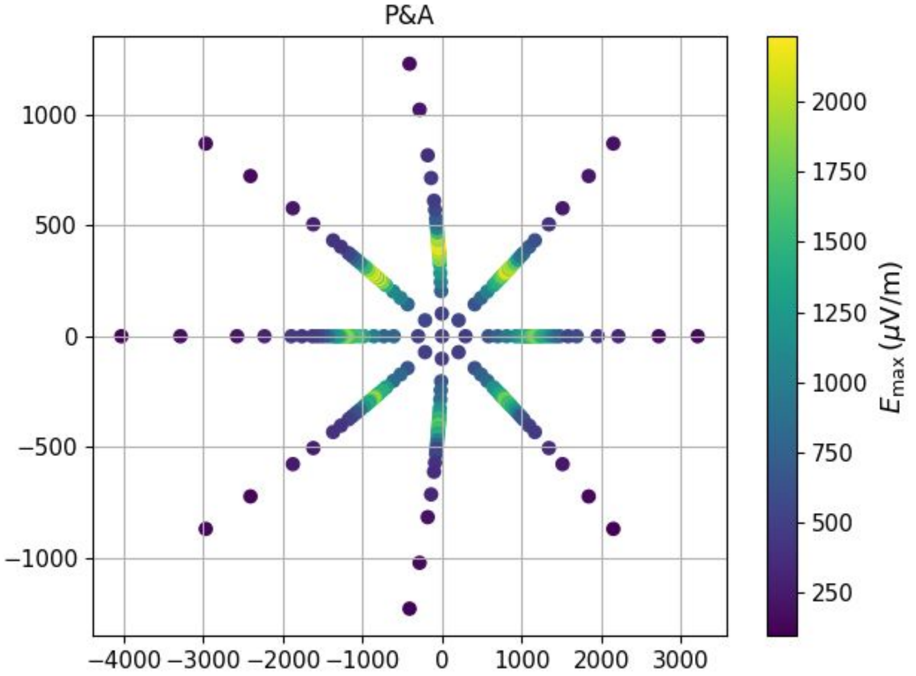

Performance Status

- Basic methodology works well for low frequencies
  - Peak E field within 10%
  - Beam pattern captured well (geomagnetic/Askaryan)

- Significant performance improvements
  - Average tracks/cell: ~20
  - Maximum tracks/cell: 200,000

- Lower accuracy at high frequencies
  - In particular for large viewing angles

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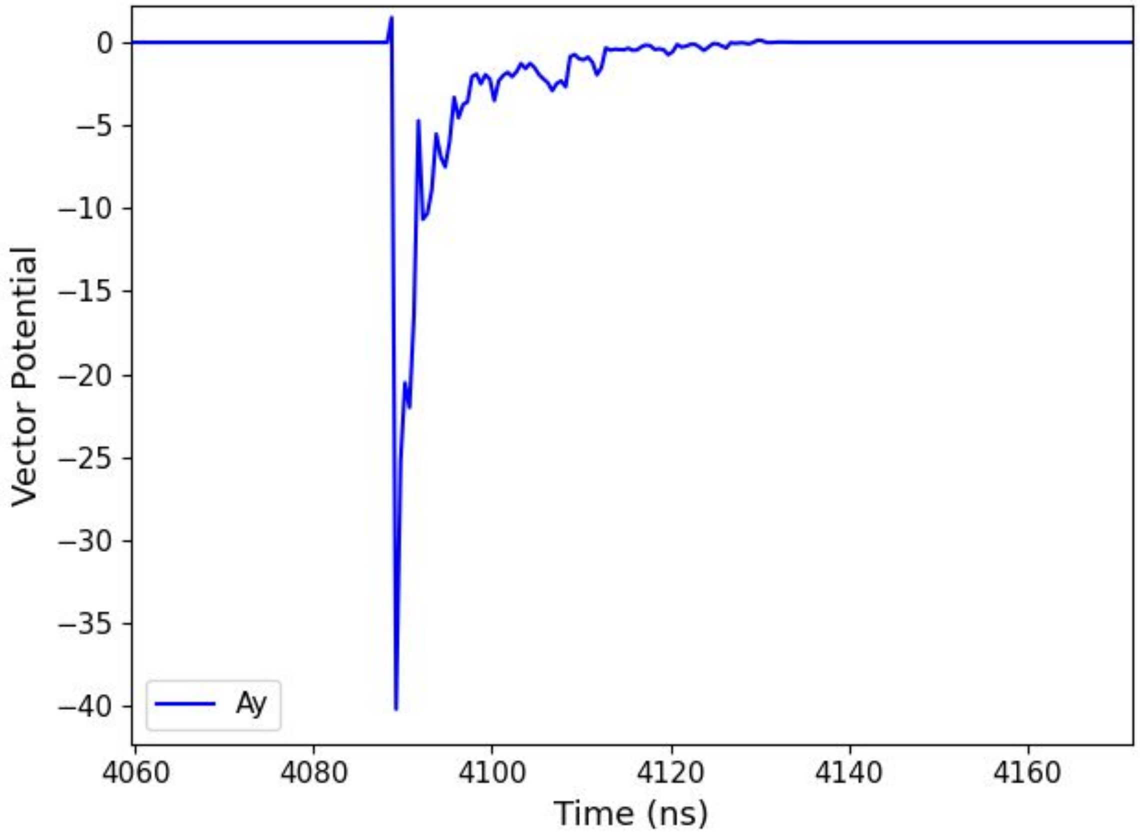

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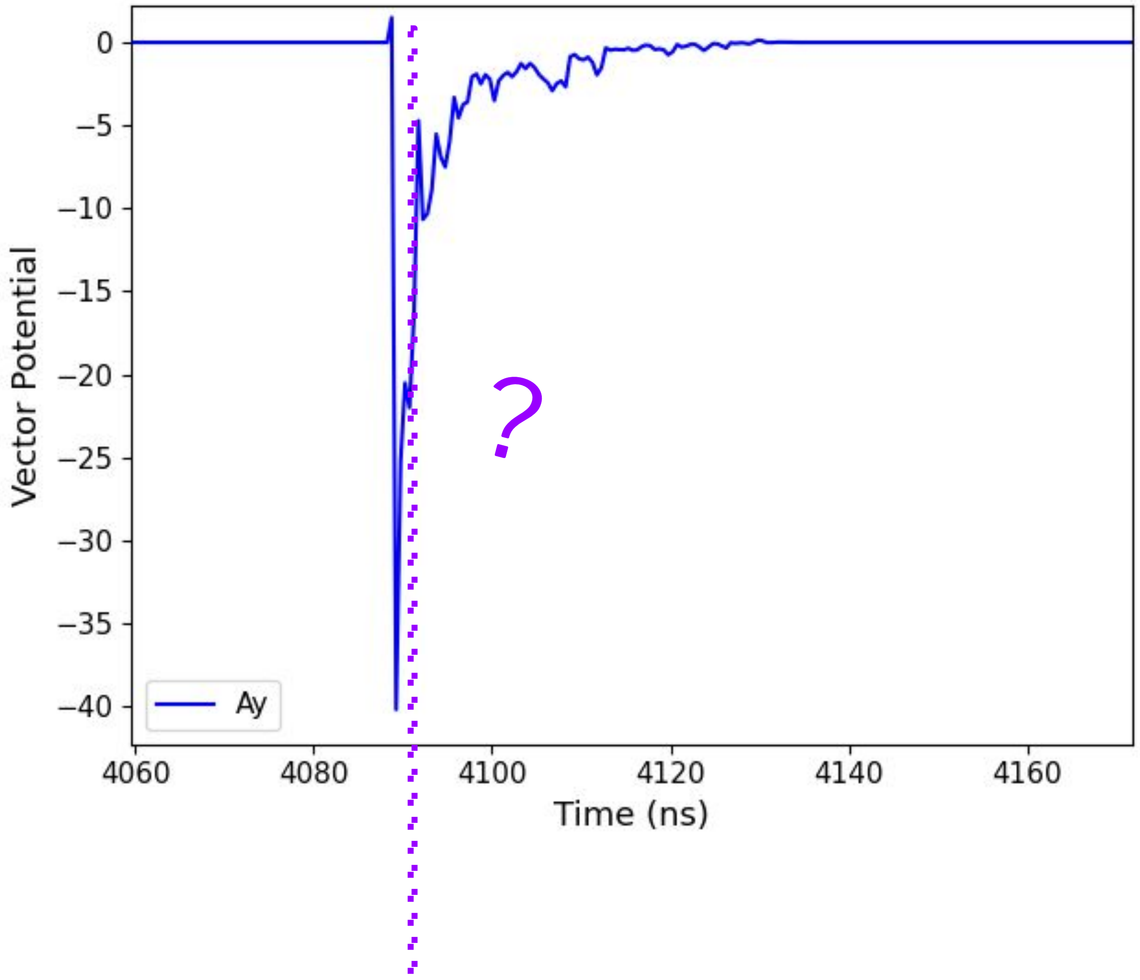
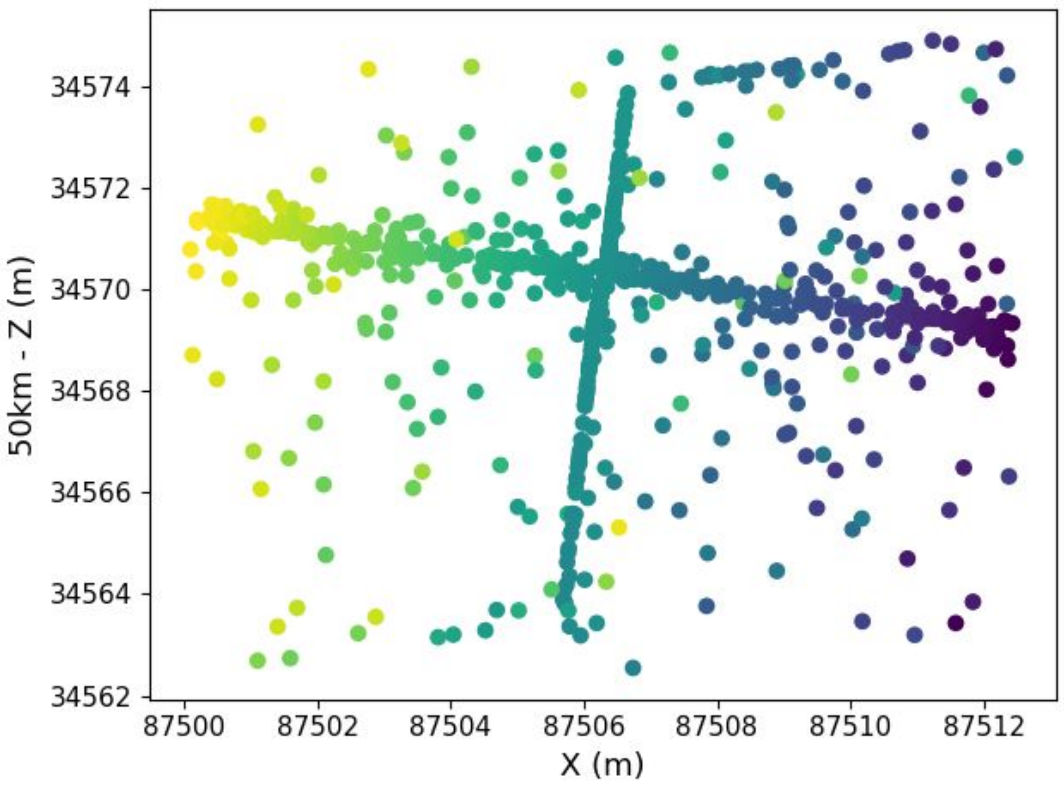


Improvements

Simplest Improvement

- Smaller cells



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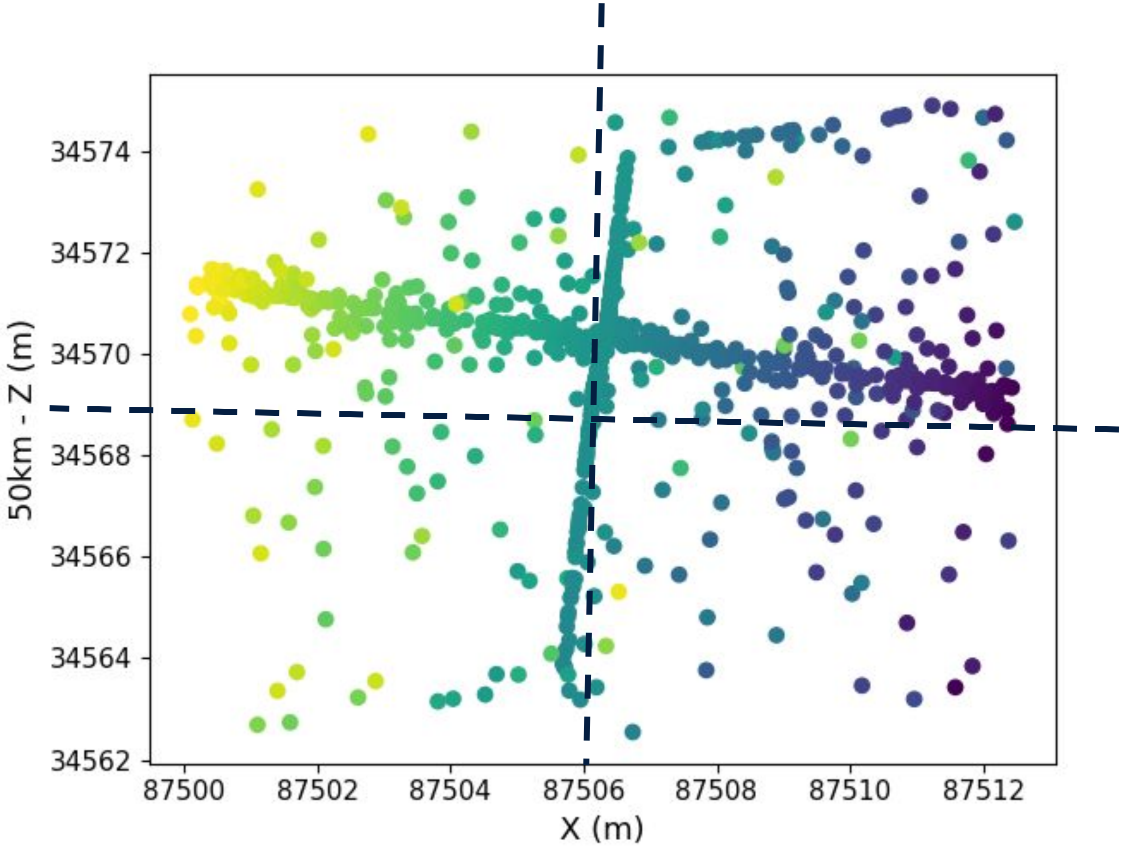

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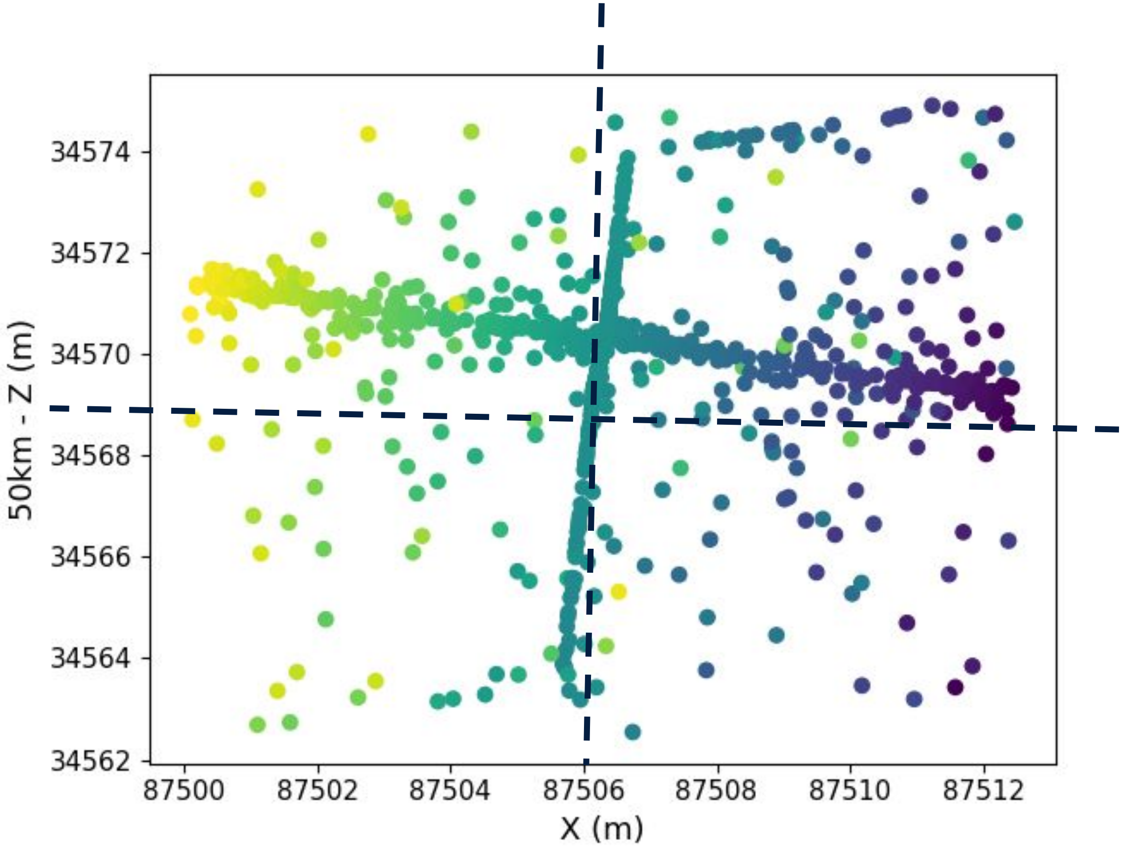
- Smaller cells
  - Less spread in vector potential (average treatment works better)
  - Considerable computational resources
    - Expect #tracks ∝ L^3, but closer to L in practice



Simplest Improvement

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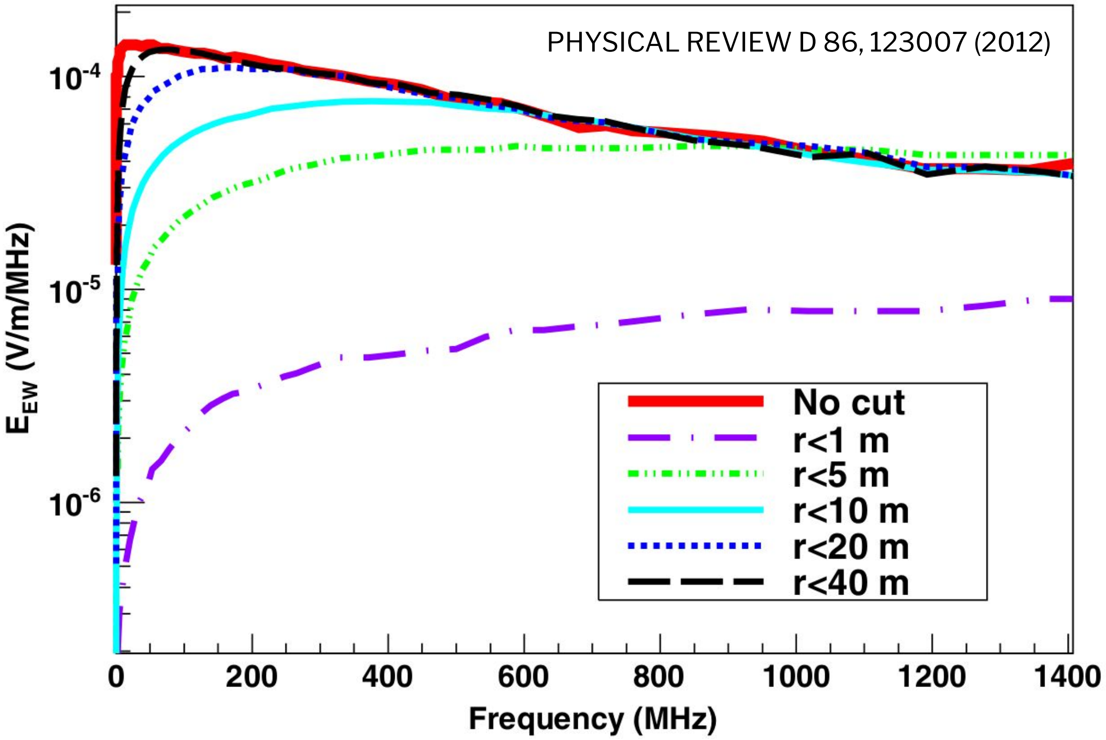
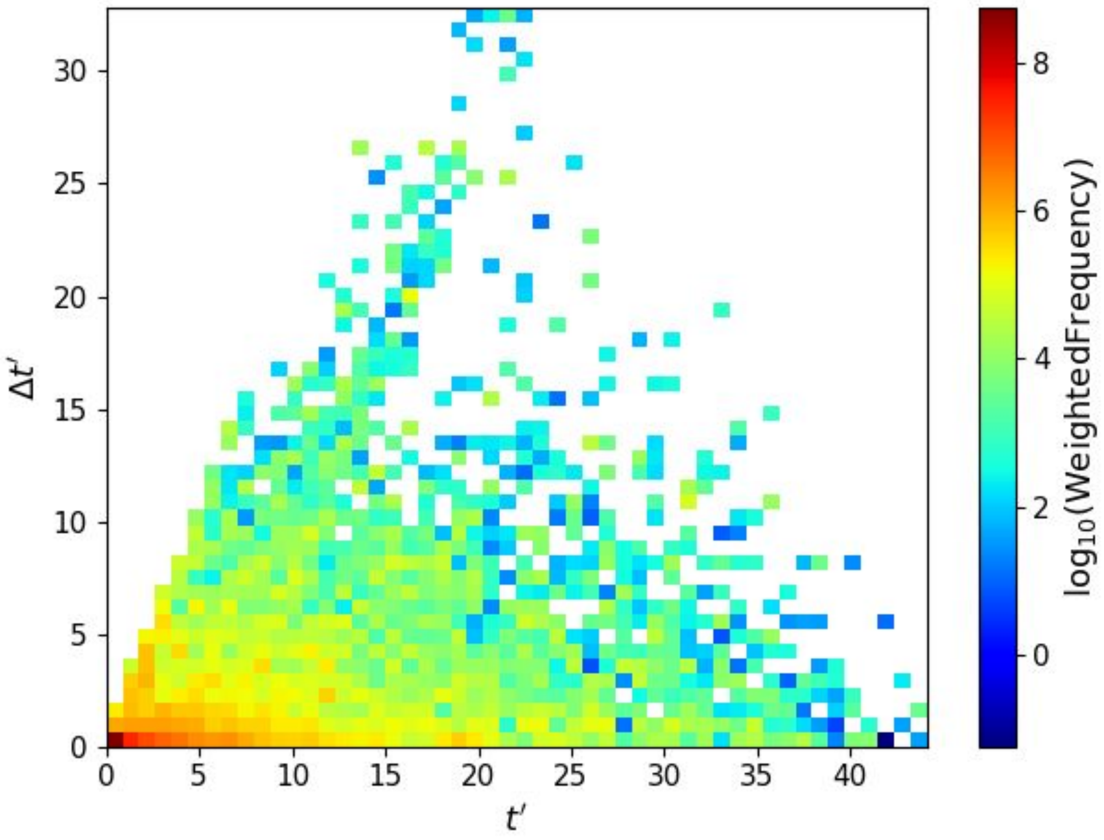
- To combat this, consider cuts in lateral distance



Shape Parameterization

- Geometric separation of tracks determines shape of vector potential, **NOT** time compression
  - Track μ, σ of X, Y, Z, T within cell



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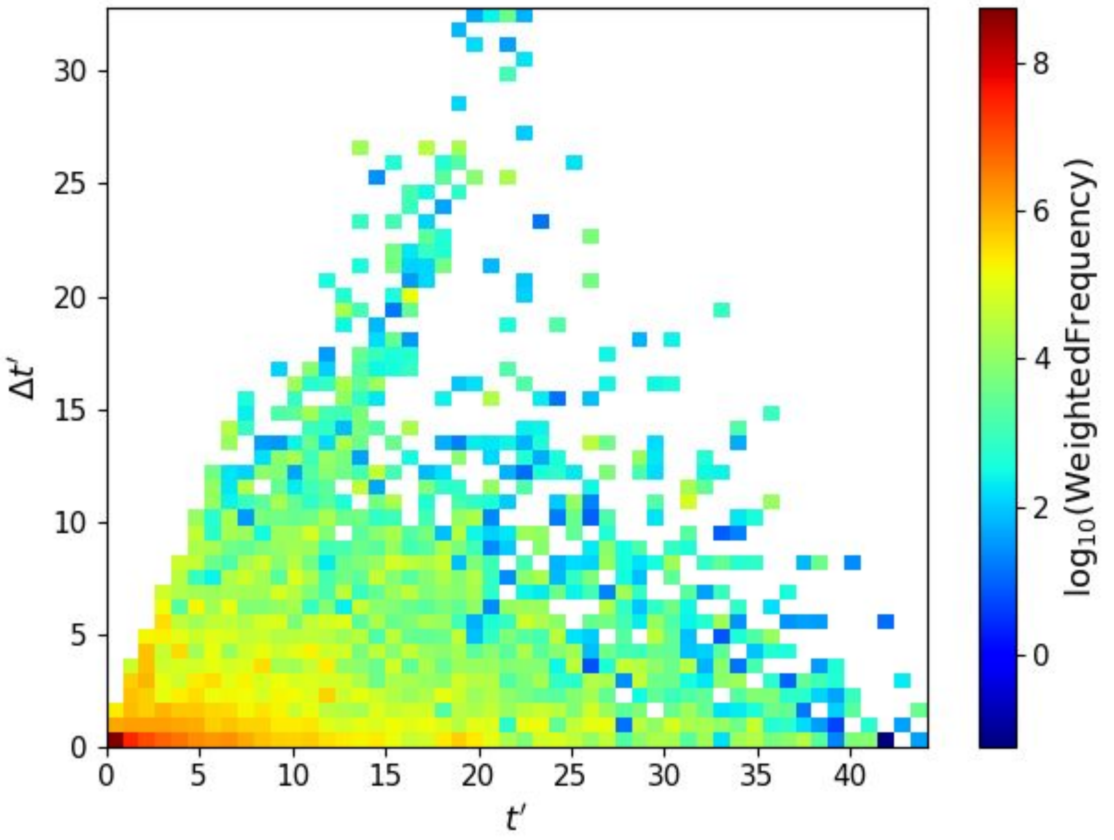
$$\Delta t' \sim \frac{1}{2} \frac{X\Delta X + Y\Delta Y + Z\Delta Z}{R} \frac{n}{c} \pm \Delta T$$



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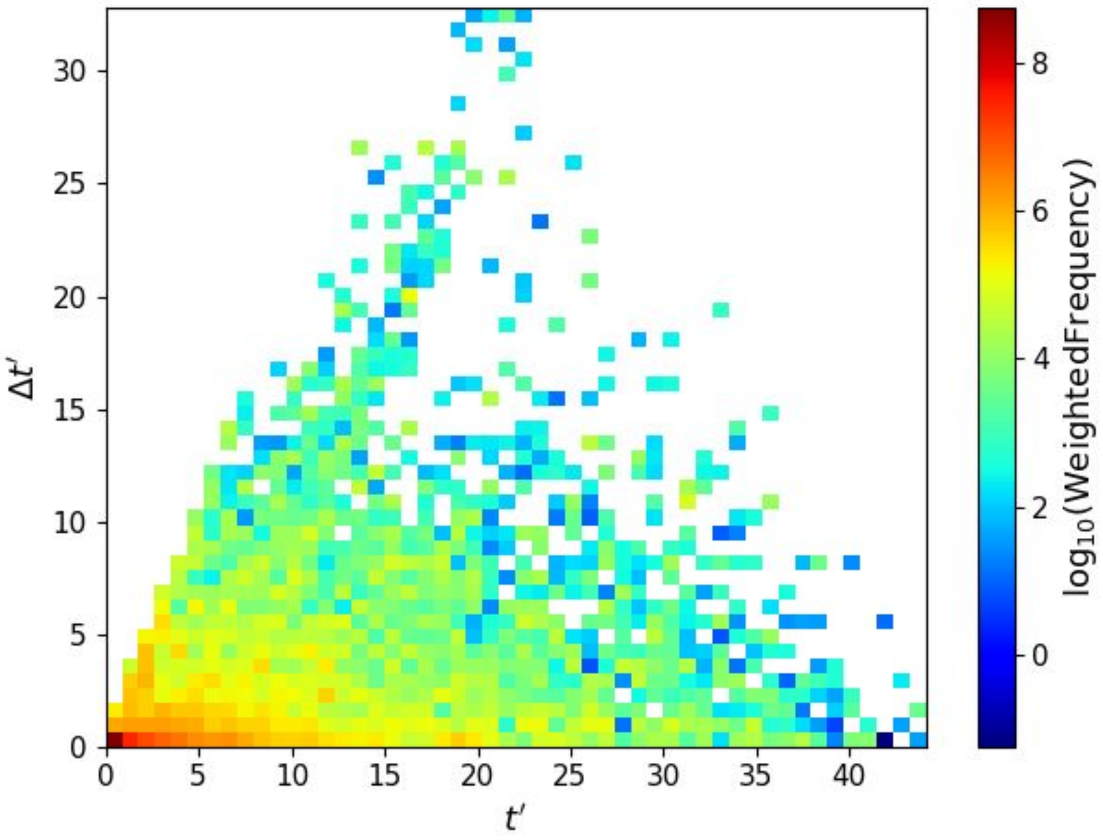
- Consider maximum and minimum observer times



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$$VP(t) = \begin{cases} 0 & t \leq t_{\text{min}} \\ A(t - t_{\text{min}})/(\bar{t} - t_{\text{min}}) & t_{\text{min}} \leq t \leq \bar{t} \\ Ae^{-(t-\bar{t})/\sigma_t} & \bar{t} \leq t \end{cases}$$

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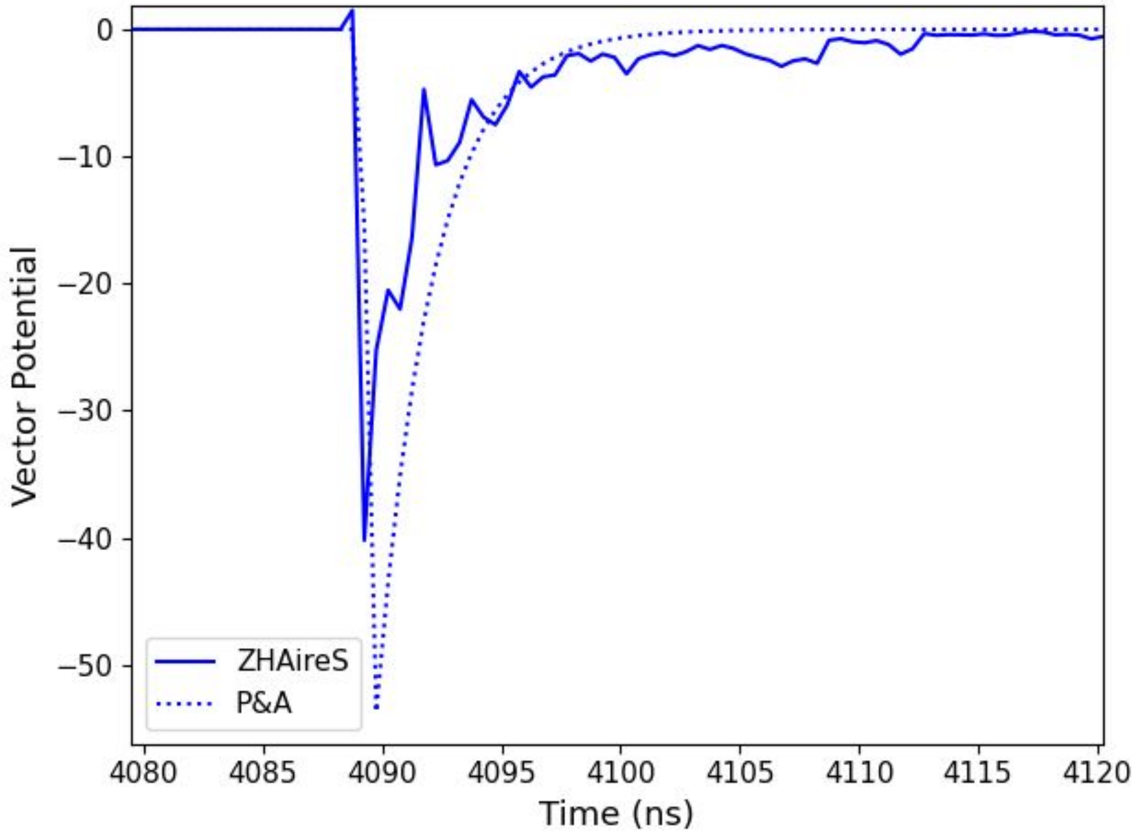
- Seems to model the vector potential shape well, capturing particularly well the rising edge



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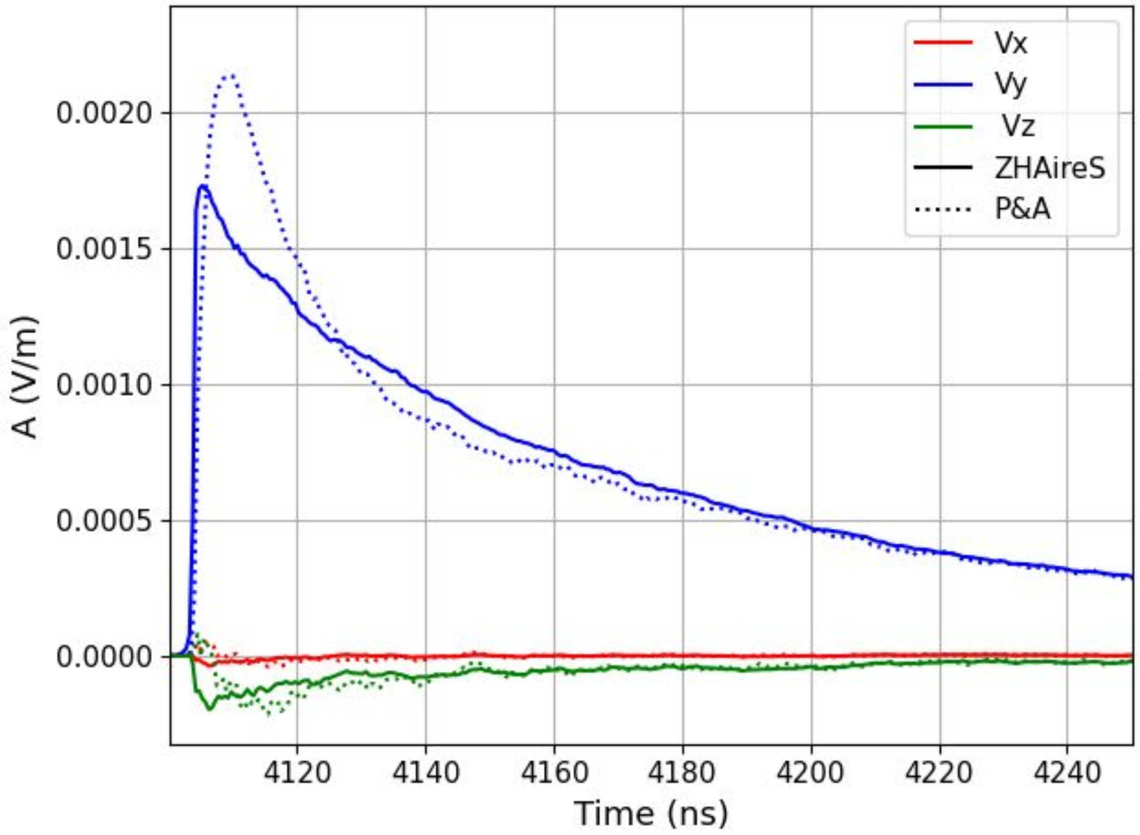

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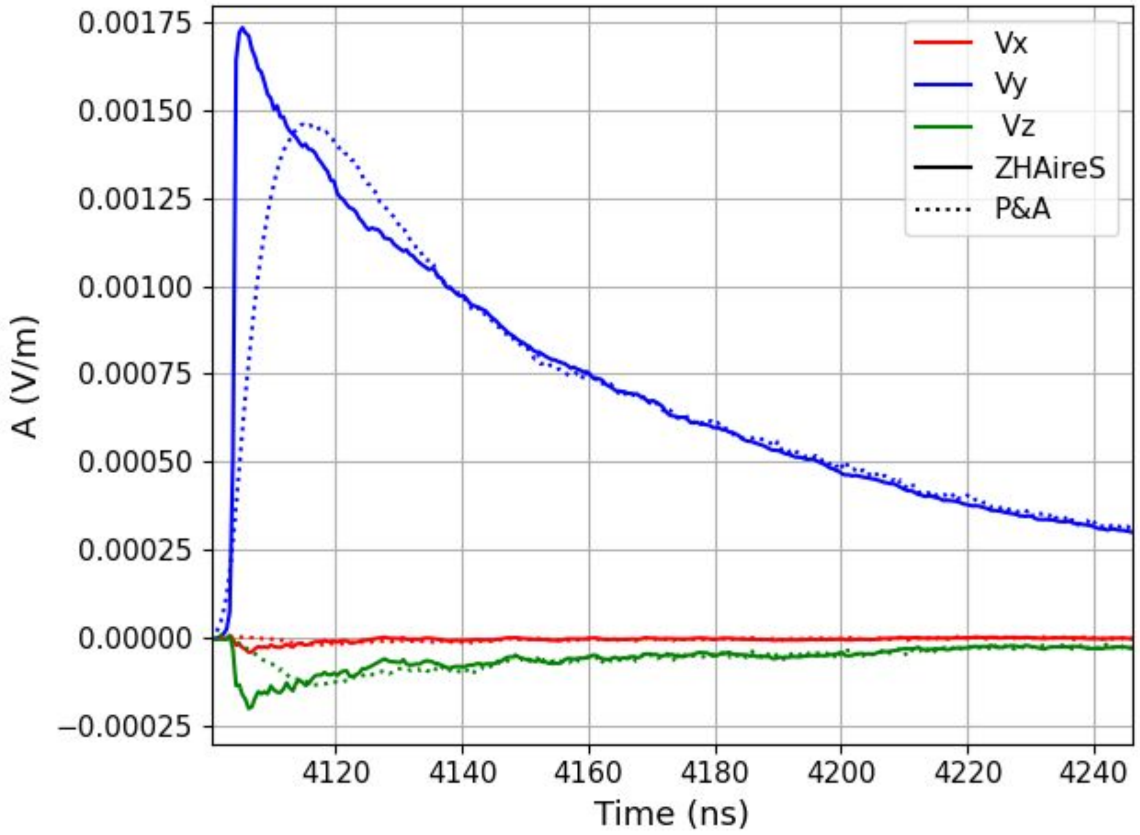

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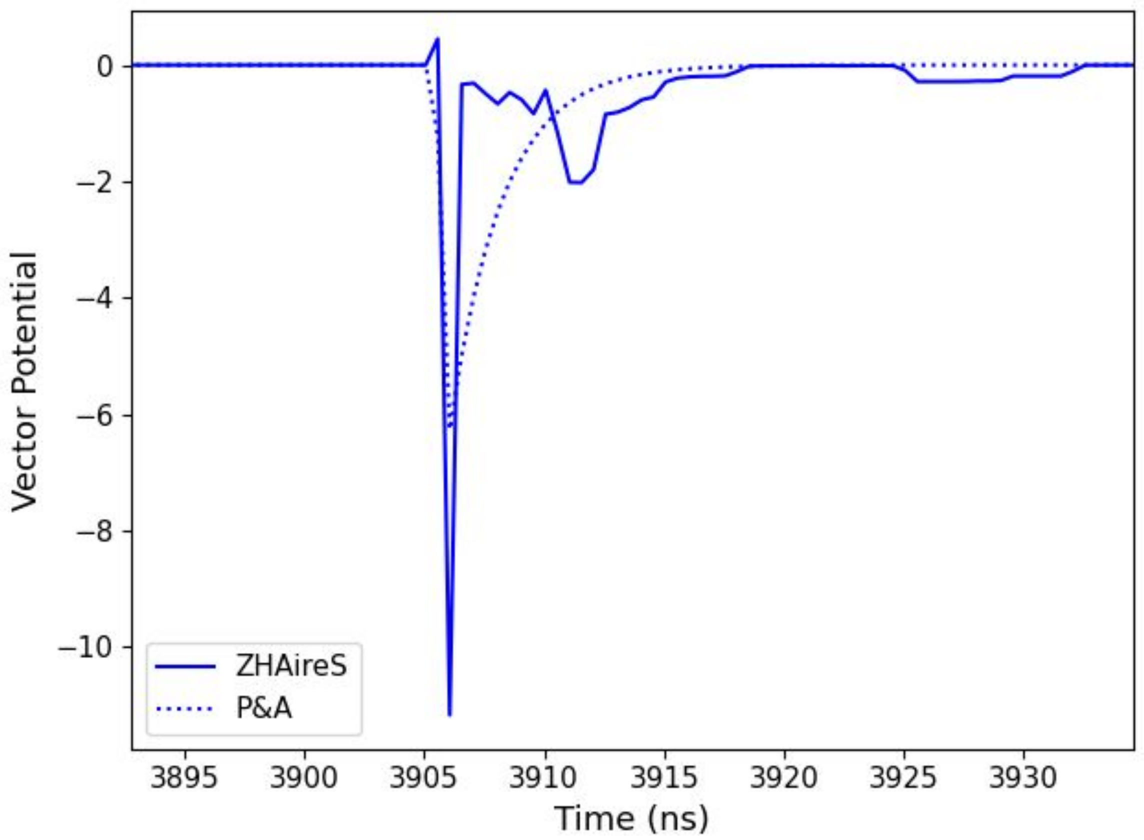

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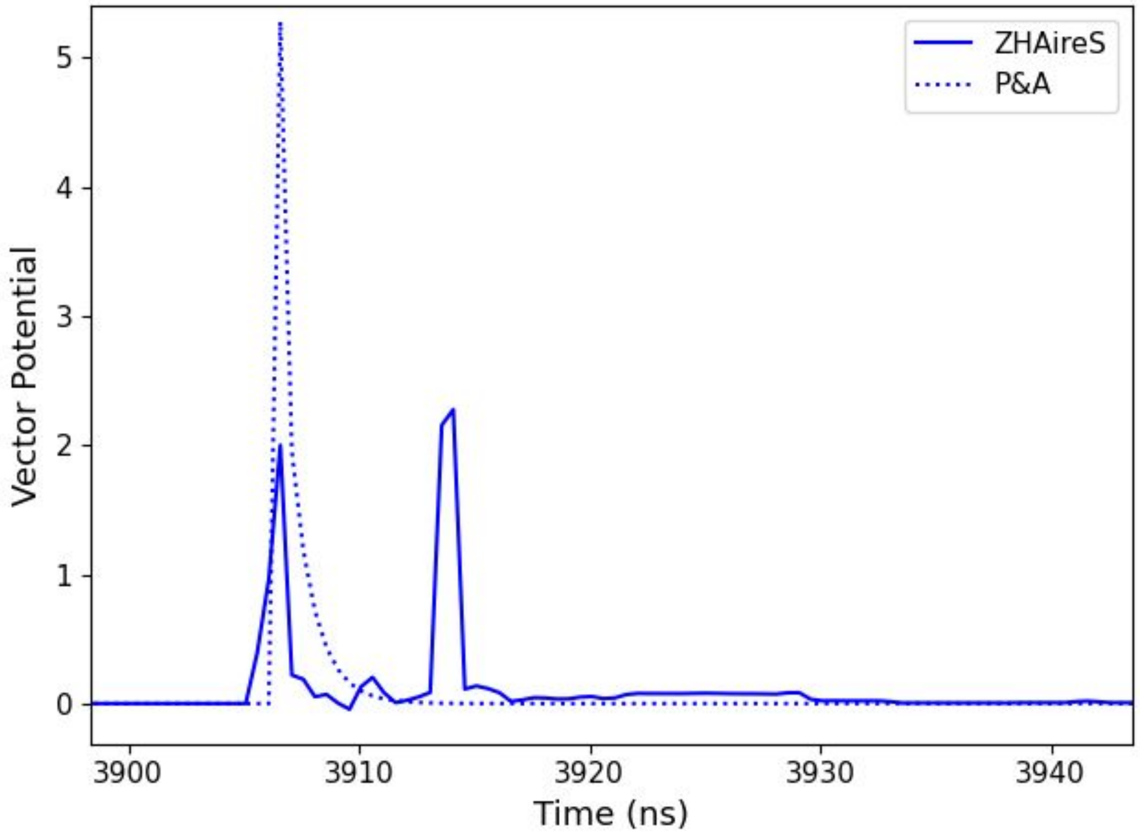

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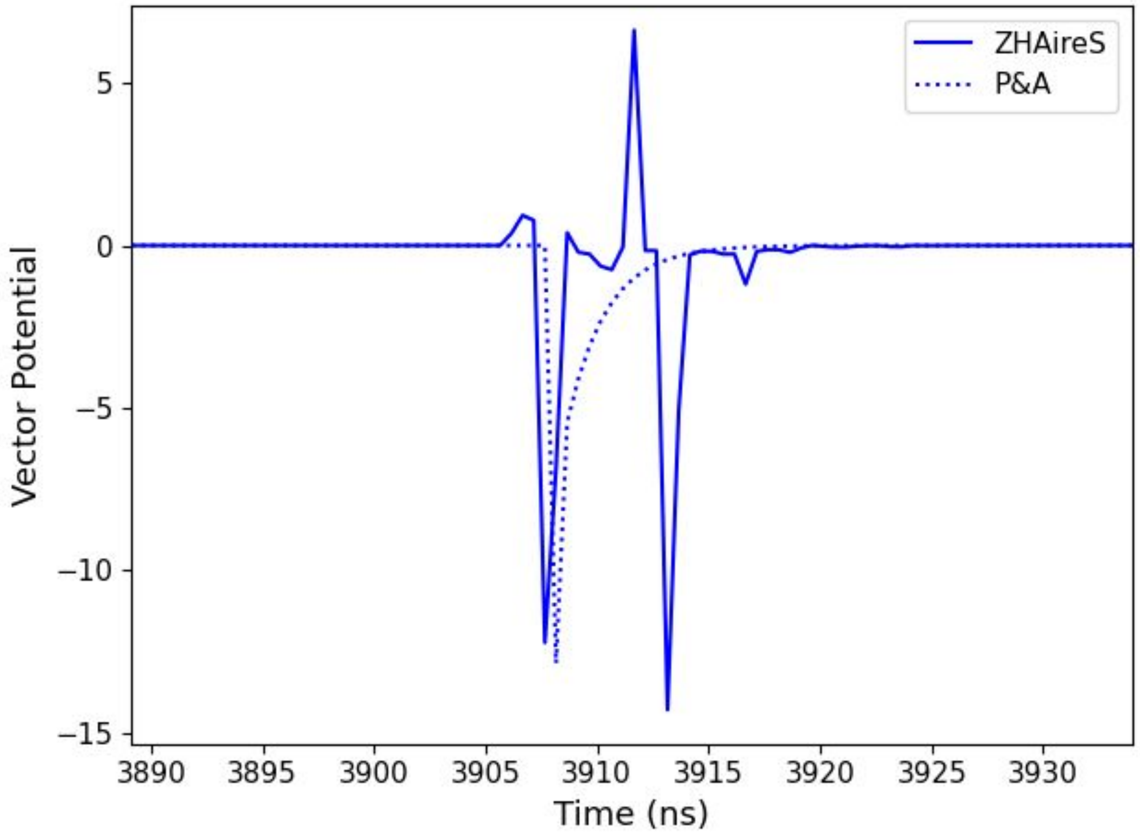
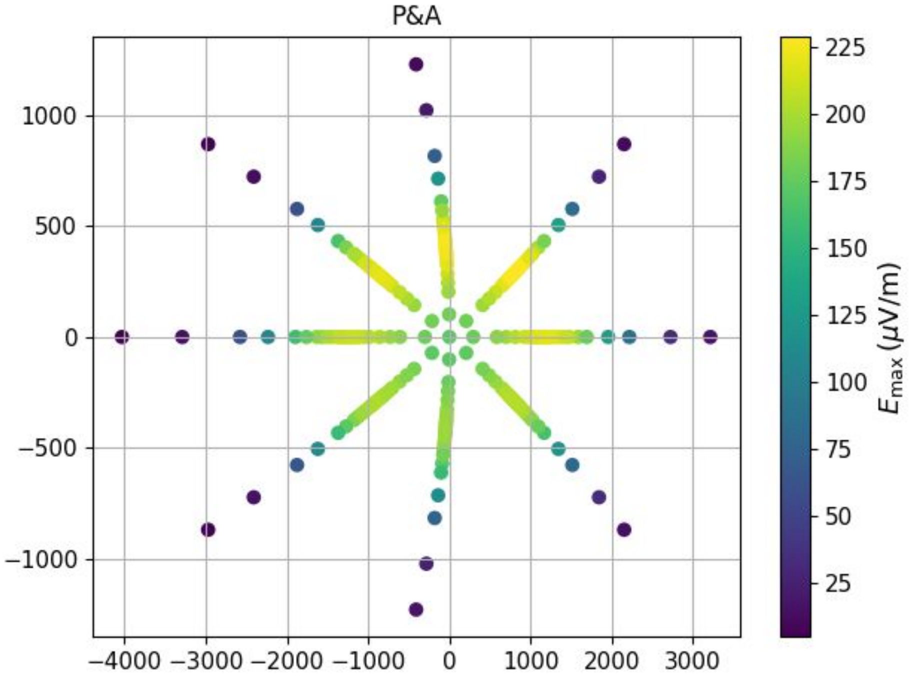
- Breaks down when the number of tracks decreases

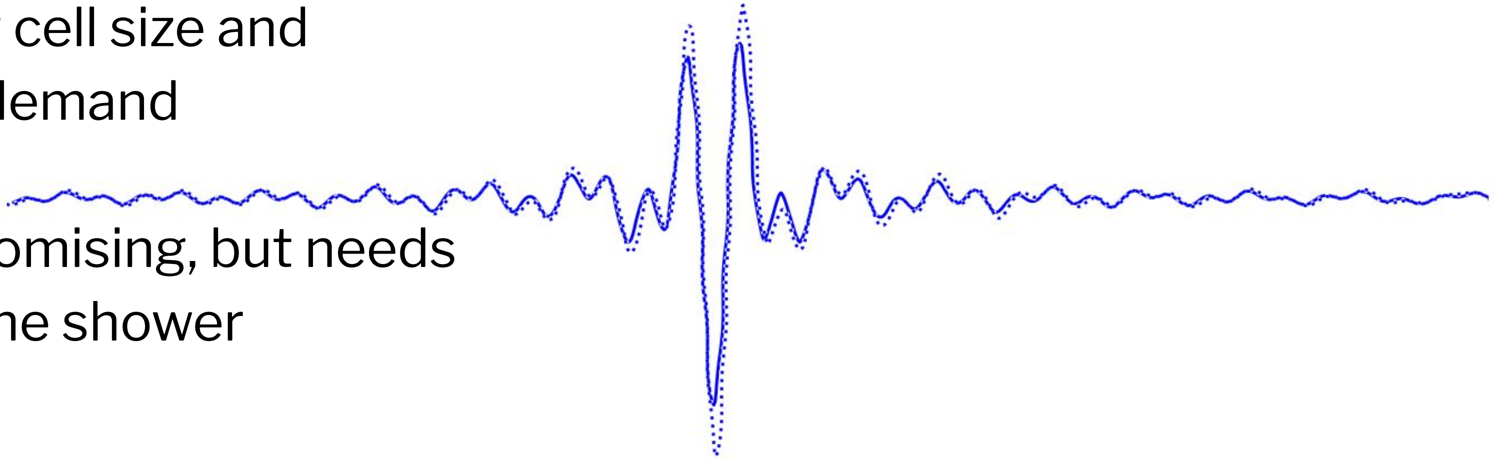


Summary

- The practical and accurate methodology calculates averaged vector potentials in 4-D cells by summing currents
  - Reproduces expected beam patterns
  - Peak E field within 10% for low frequencies
  - Expected performance gain factor >20



- High frequency behavior is largely not right
  - Can be fixed by decreasing cell size and increasing computational demand



- Profile shaping seems to be promising, but needs to be applied properly across the shower

Bonus Slides

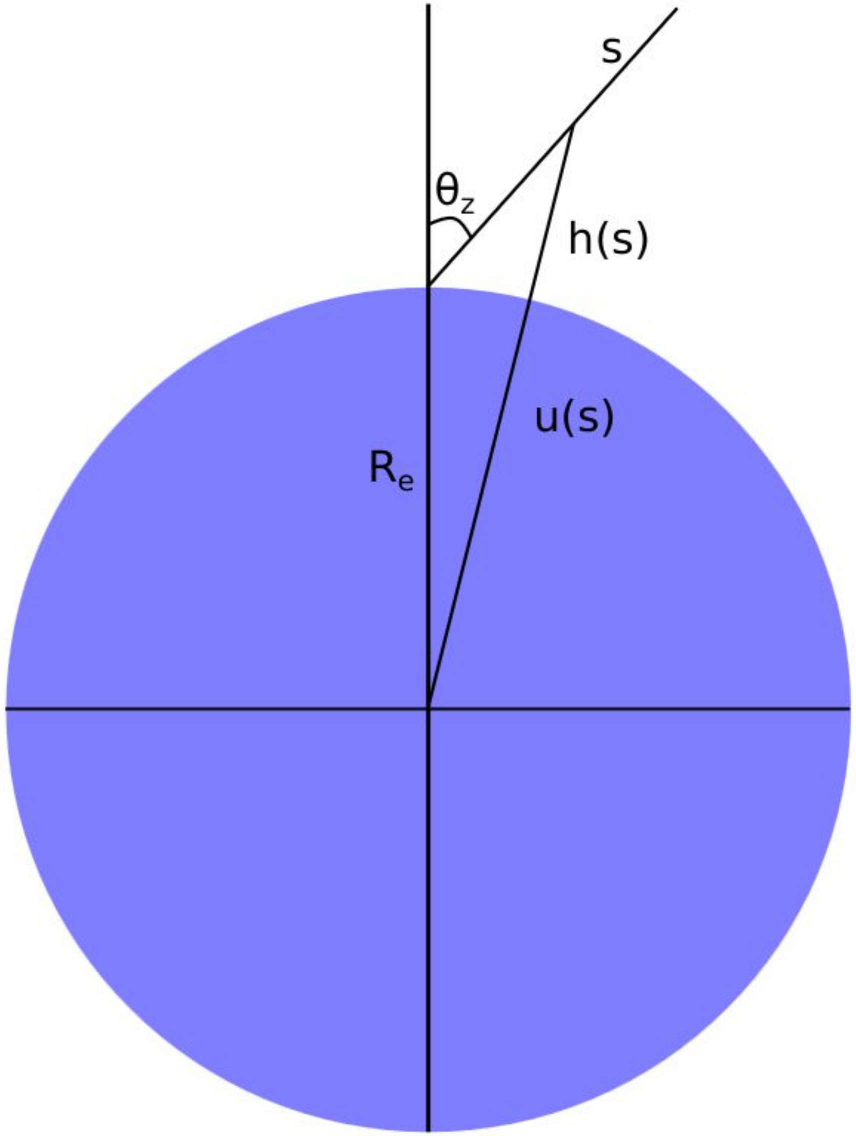
Analytical n_{eff} calculation

$$\mathcal{R}(h) = \mathcal{R}_s \exp(-K_r h) \quad \mathcal{R}_{eff} = \frac{1}{R}\int_0^R \mathcal{R}(h)\, dl$$

$$u^2 = s^2 + R_e^2 + 2sR_e\cos\theta_z$$

$$h(s) \approx s\cos\theta_z + \frac{s^2\sin^2\theta_z}{2R_e} + \mathcal{O}(s^3)$$

$$\mathcal{R}_{eff} = \frac{\mathcal{R}_s}{R}\frac{\sqrt{\pi R_e}}{\sqrt{2K_r}\sin\theta_z}\exp\left(\frac{K_r R_e cos^2\theta_z}{2\sin^2\theta_z}\right)$$
$$\mathrm{erf}\left(\frac{\sqrt{K_r R_e}}{2sin\theta_z}\left(\cos\theta_z + \frac{s\sin^2\theta_z}{R_e}\right)\right)\Bigg|_0^R$$



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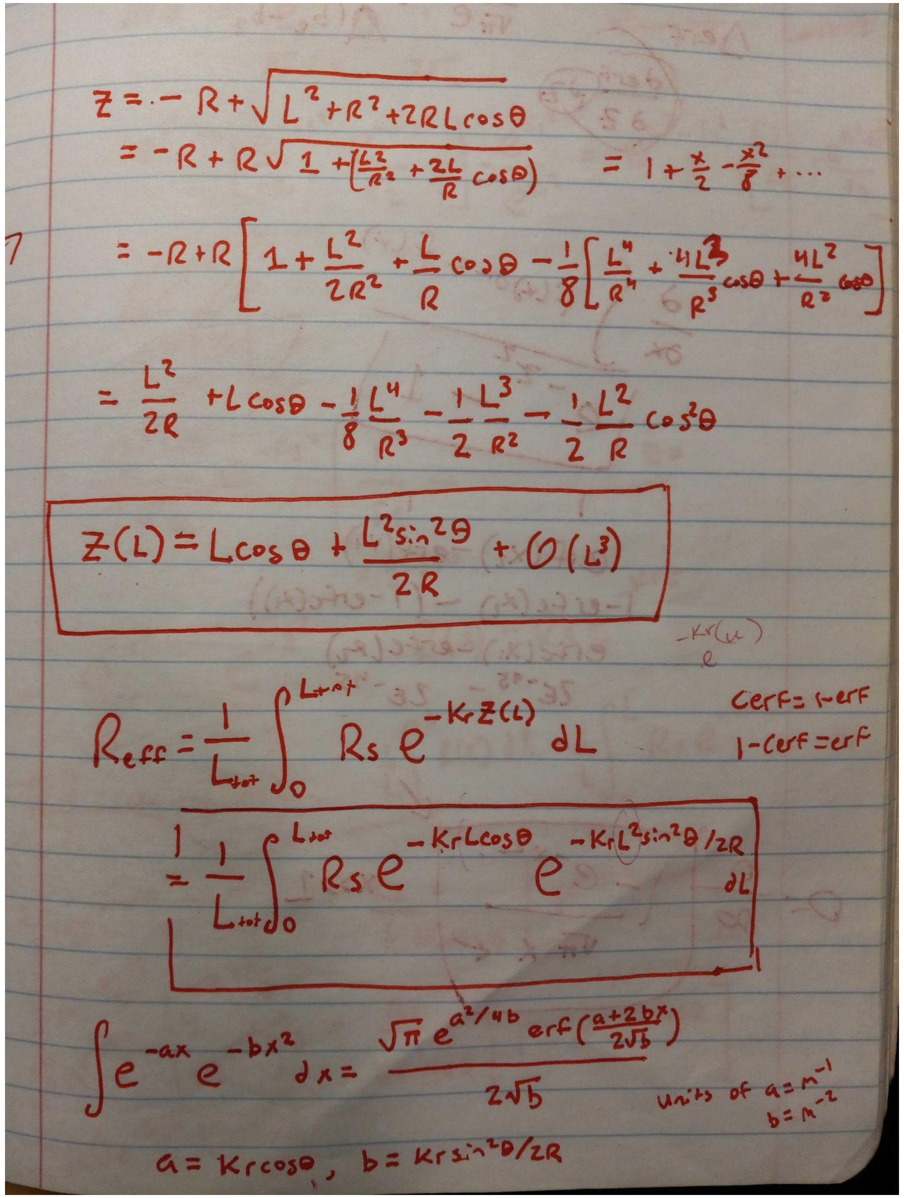

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