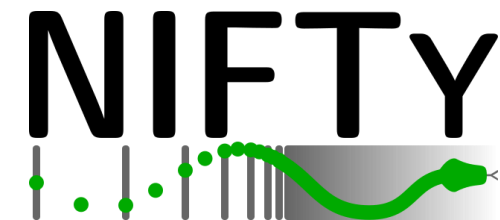
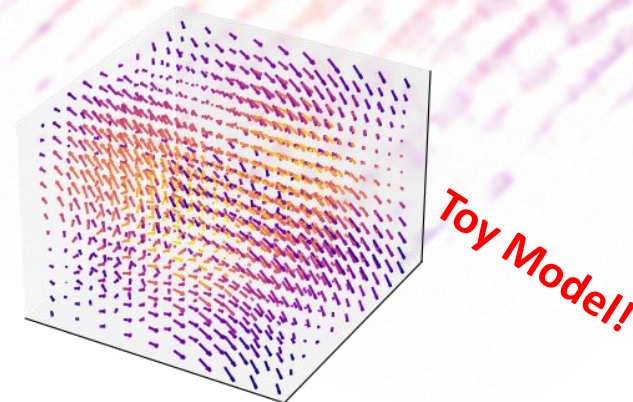




# Semi-parametric Air Shower Shape Reconstruction with Information Field Theory

ARENA 2024 Chicago

**Maximilian Straub**, Torsten Enßlin, Martin Erdmann, Philipp Frank



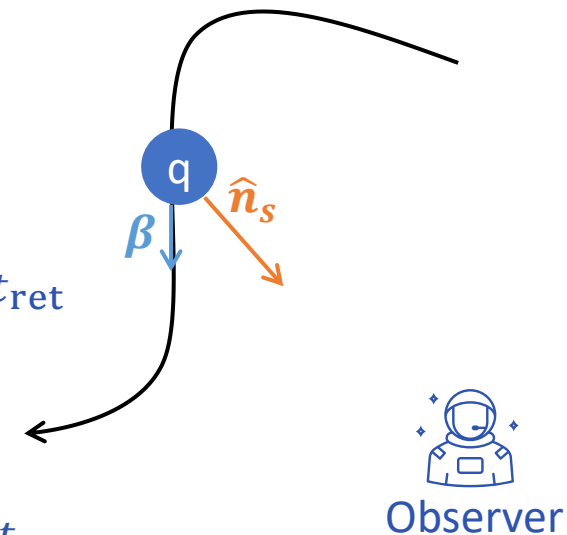
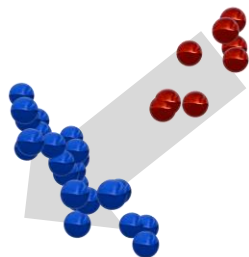
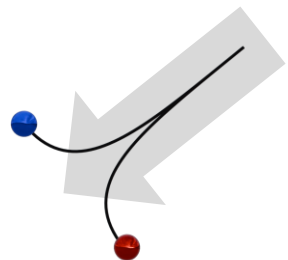
# Liénard–Wiechert potential

- $$A^\mu(\mathbf{x}_{\text{obs}}, t_{\text{obs}}) = \int \frac{j^\mu}{n_{\text{eff}}(\mathbf{x}_{\text{obs}}, \mathbf{x}') |\mathbf{x}_{\text{obs}} - \mathbf{x}'|} \delta\left(t' - t_{\text{obs}} + \frac{n_{\text{eff}}(\mathbf{x}_{\text{obs}}, \mathbf{x}')}{c} |\mathbf{x}_{\text{obs}} - \mathbf{x}'|\right) d^4 x'$$

- Reproduces Liénard-Wiechert potential for point charges:

$$\varphi(\mathbf{x}_{\text{obs}}, t_{\text{obs}}) = \left( \frac{q}{n_{\text{eff}}(1 - n_{\text{eff}} \hat{\mathbf{n}}_s \cdot \boldsymbol{\beta}) |\mathbf{x}_{\text{obs}} - \mathbf{r}_s|} \right)_{t_{\text{ret}}}$$

$$A^i(\mathbf{x}_{\text{obs}}, t_{\text{obs}}) = \left( \frac{q \boldsymbol{\beta}}{n_{\text{eff}}(1 - n_{\text{eff}} \hat{\mathbf{n}}_s \cdot \boldsymbol{\beta}) |\mathbf{x}_{\text{obs}} - \mathbf{r}_s|} \right)_{t_{\text{ret}}}$$



# Differentiable Programming with JAX

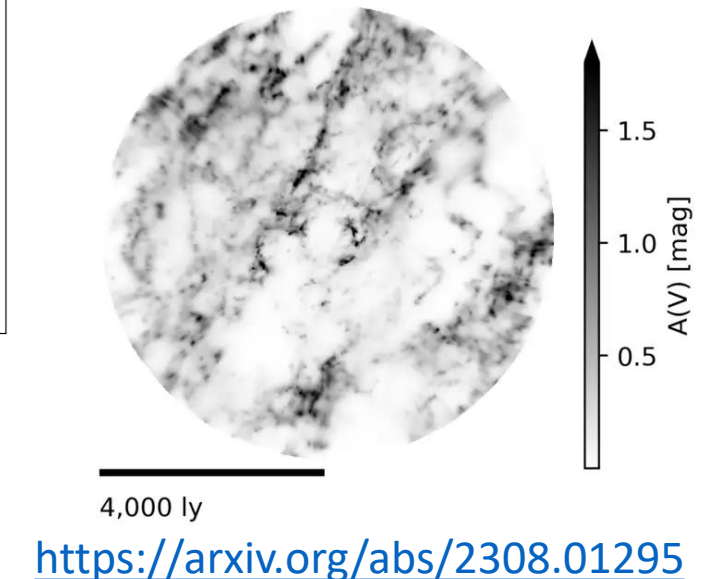
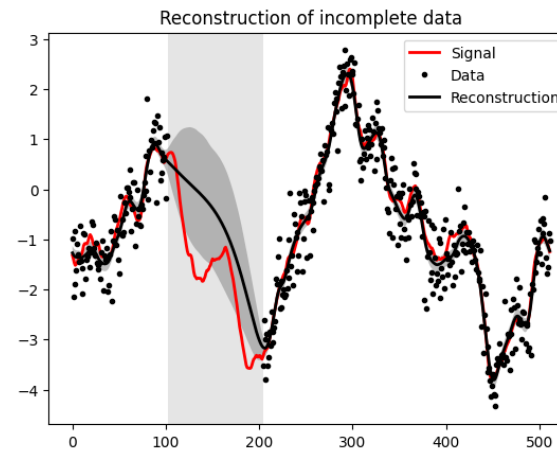
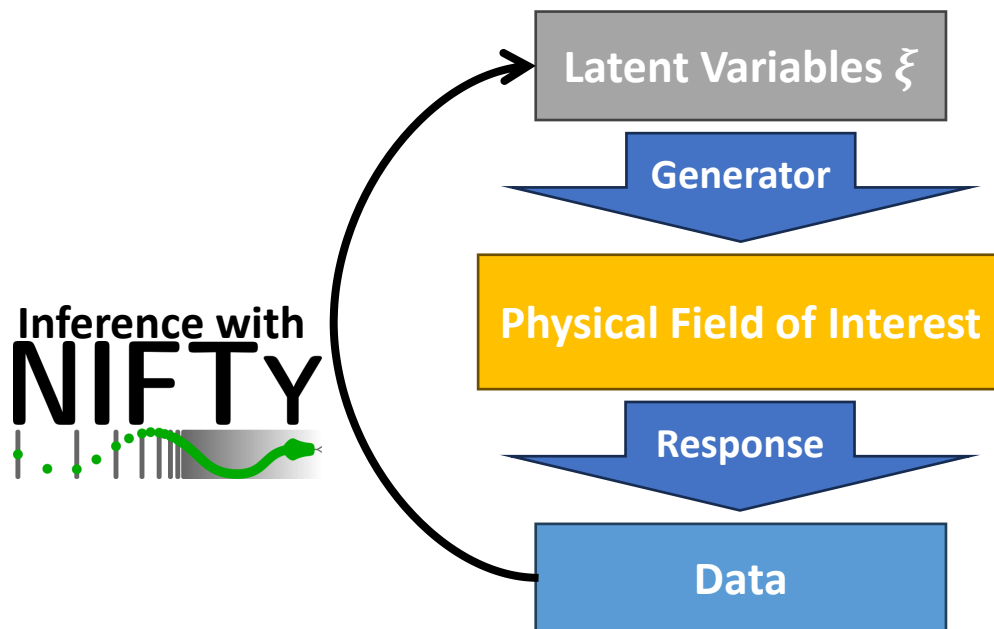
- Python library for high-performance numerical computing
- Numpy-like interface
- Runs on CPU and GPU
- Automatic vectorization
- Autograd: Differentiable python code
- Just-In-Time compilation: Speed-up of 1,000x possible



→ Key component of this analysis

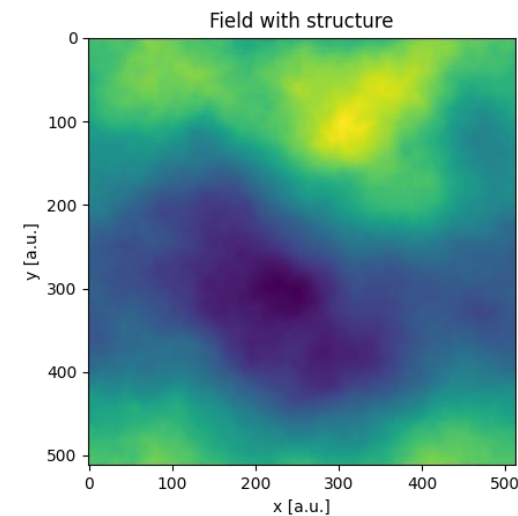
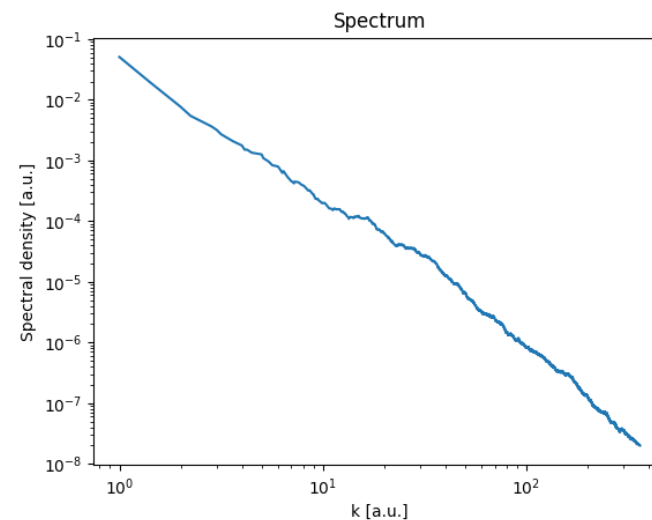
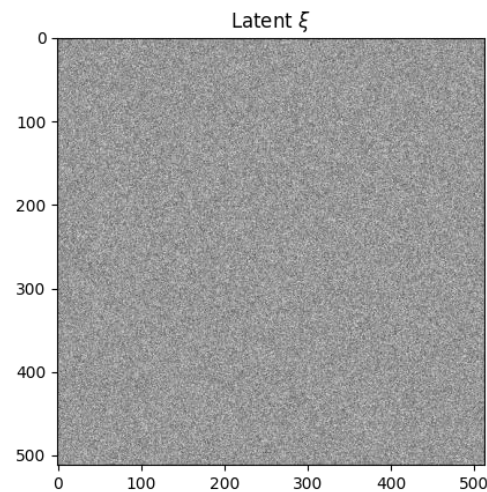
# Information Field Theory

- Bayesian framework for large numbers of degrees of freedom
- Continuous fields  $\leftrightarrow$  discrete representations
- Statistical fit with variational inference
  - Approximate posterior
- Learn correlation structures



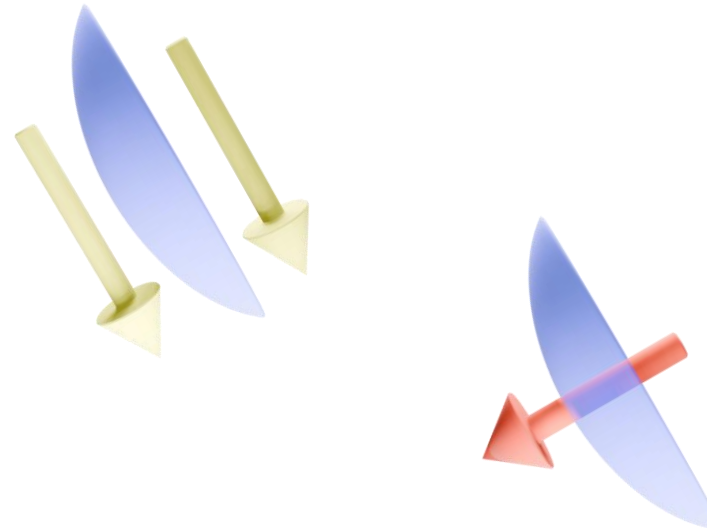
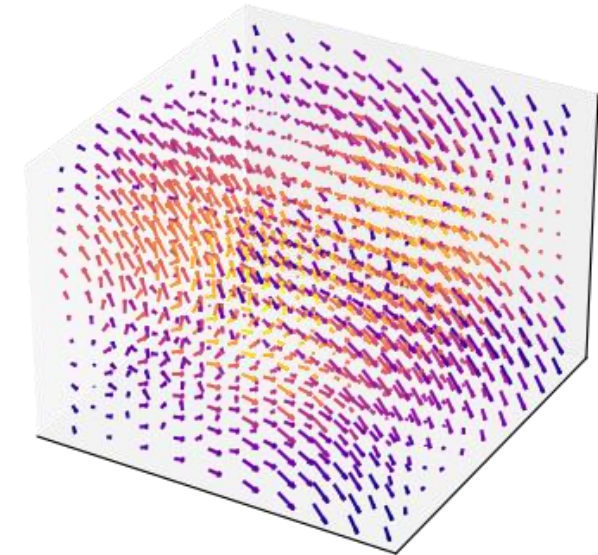
# Modeling structures

- Latent variables  $\xi$ : Zero-centered, unit-width Gaussians
- Impose correlation structure: Neighbors talk to each other
  - Convolve  $\xi$  with spectrum  $\rightarrow$  spectrum can be inferred as well
  - Different spectra for subspaces
- Include other physical priors at this stage



# Shower model

- Model as binned voxels:
  - $q$ : Charge in space, shape (T, X, Y, Z)
  - $\beta$ : Drift velocities, shape (T, X, Y, Z, 3)
  - $\rho$ : Electron density, shape (T)
 } non-parametric
- exploiting e.g., radial symmetry possible
- Scalar parameters
- Drift current:  $q\beta$
- Excess current:  $q\rho c$

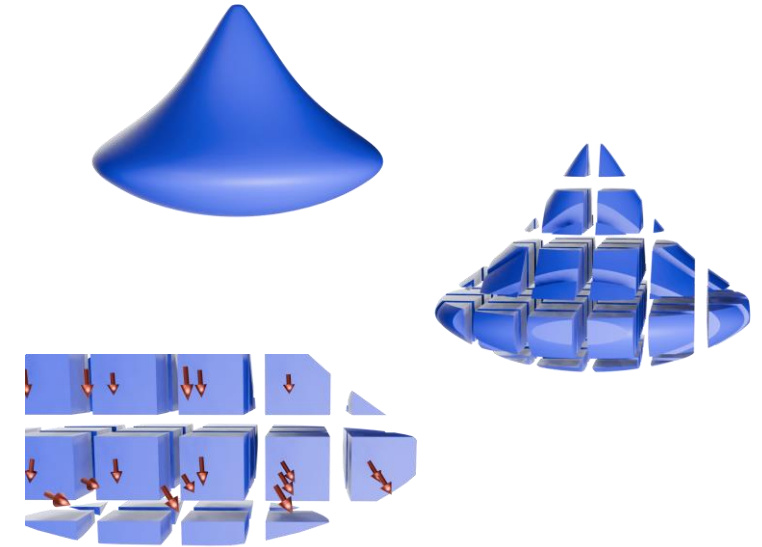




# Forward Process

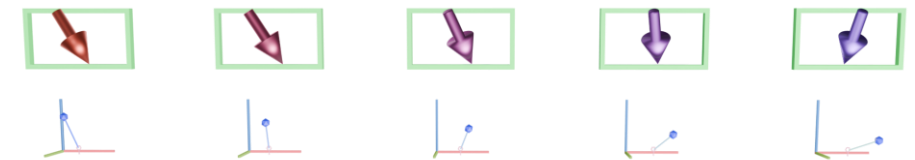
## Shower frame:

- Model shower as a cuboid moving through atmosphere
- Divide cuboid into space-time voxels
- Voxels contents: superposition of point sources
- Bin observed  $A$  in time
- Subsample  $t_{\text{ret}}$



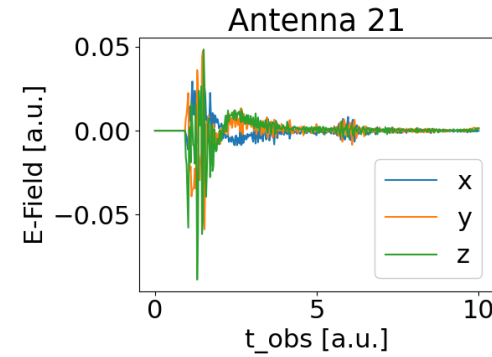
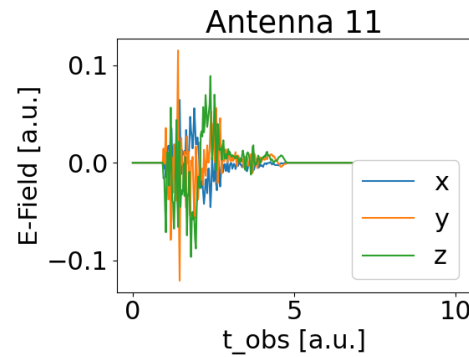
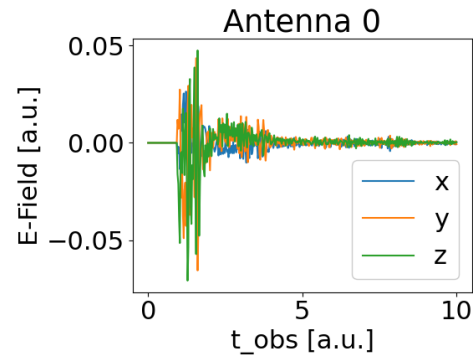
## Observer frame:

- $f(t_{\text{obs}}) = \int f(t_{\text{shower}}) \left| \frac{dt_{\text{obs}}}{dt_{\text{shower}}} \right| dt_{\text{shower}}$
- Calculate  $E$  numerically



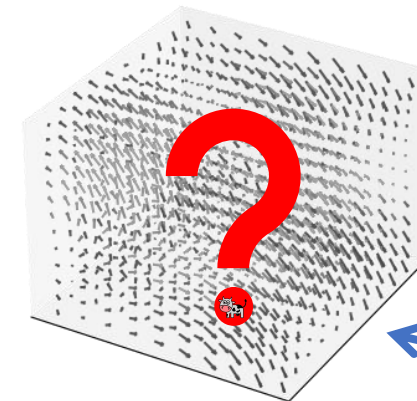
*In short:*  $\underbrace{\text{draw } j^\mu}_{\text{Generator}}$  and  $\underbrace{\text{calculate } E}_{\text{Response}}$  for the observer

# Testing the framework



**Toy Model!**

- Filled the cuboid with random structure
- Used grid of 24 antennas
- Self-consistency check:  
→do we get out what we put in?



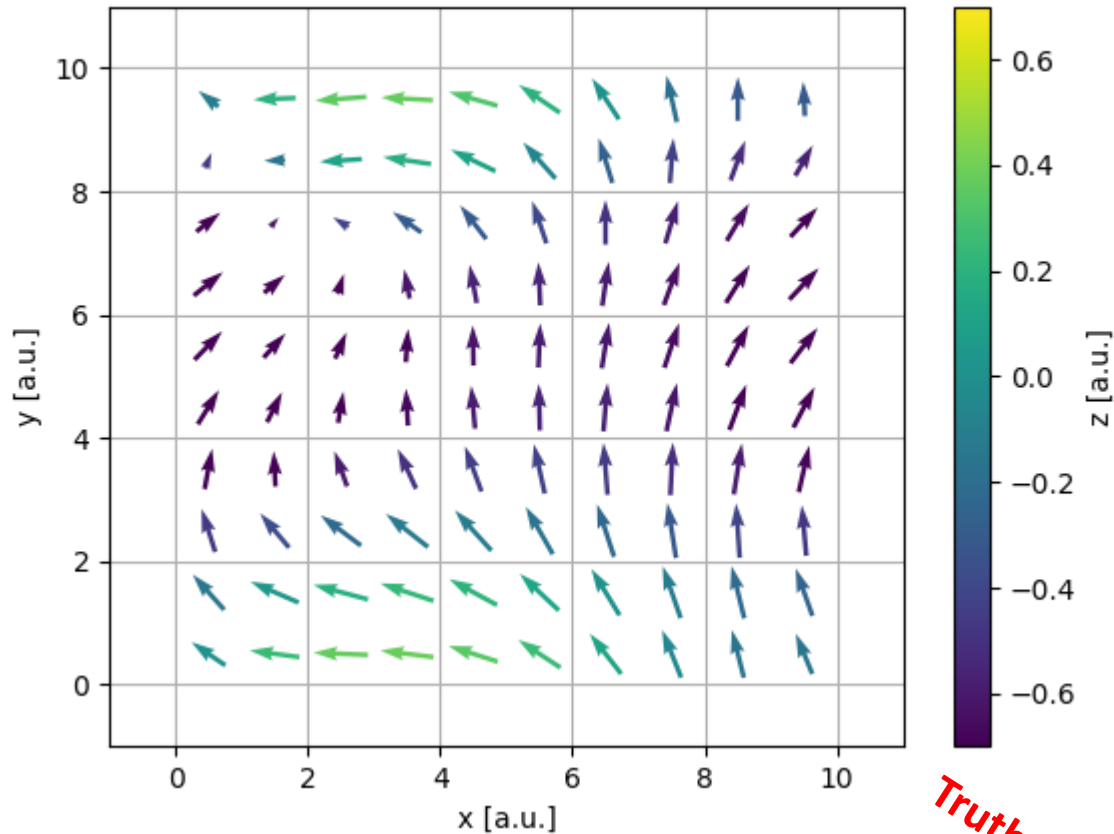
given these traces

what does the shower look like?

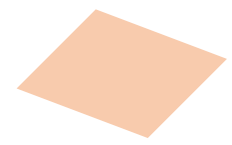
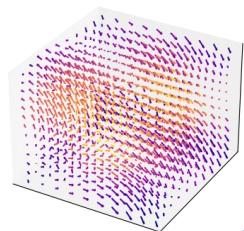
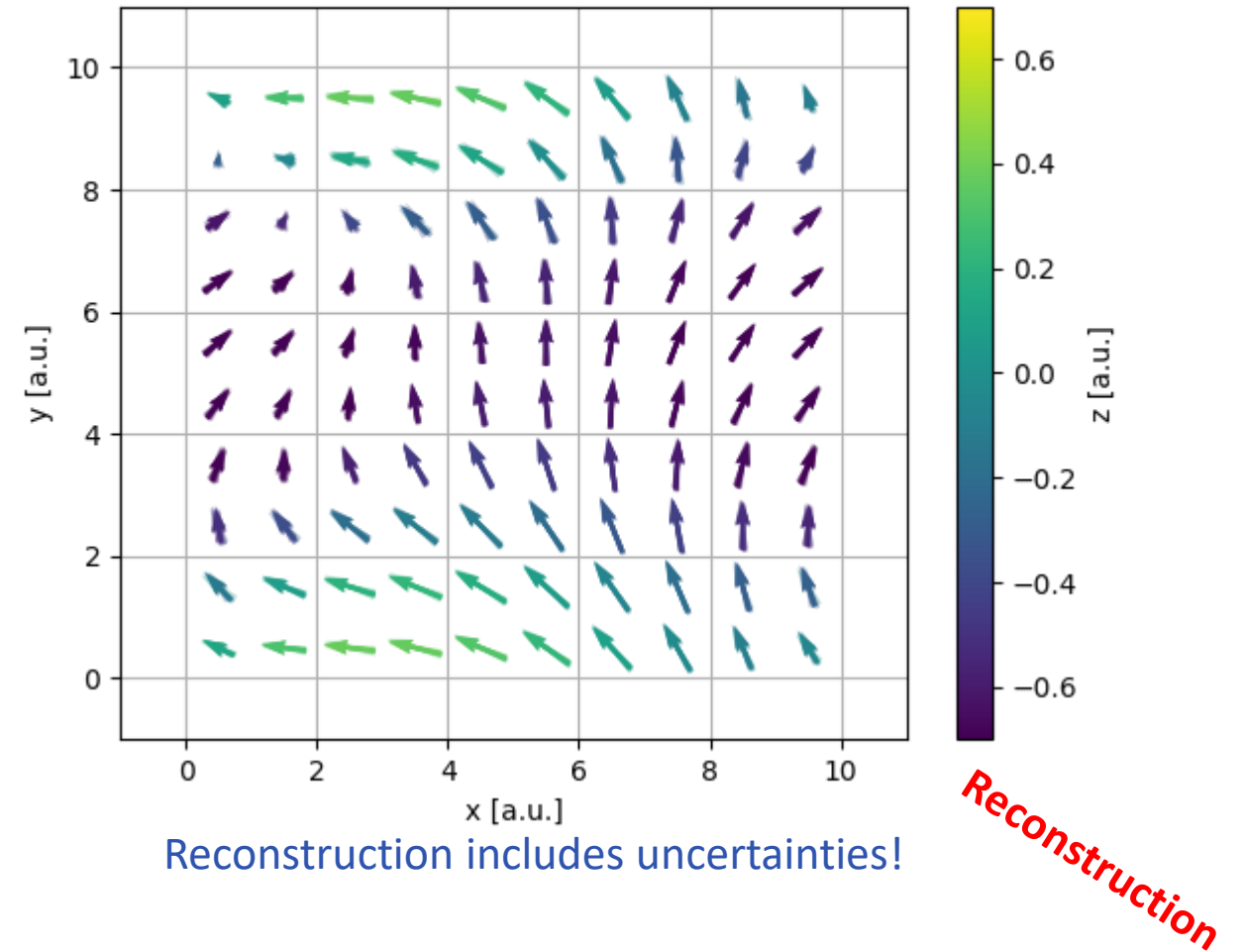


# Preliminary Results $q\beta$

Drift current for  $t_i = 42$

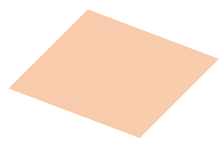
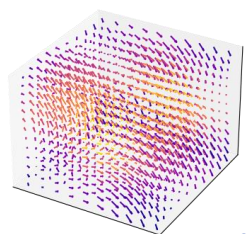
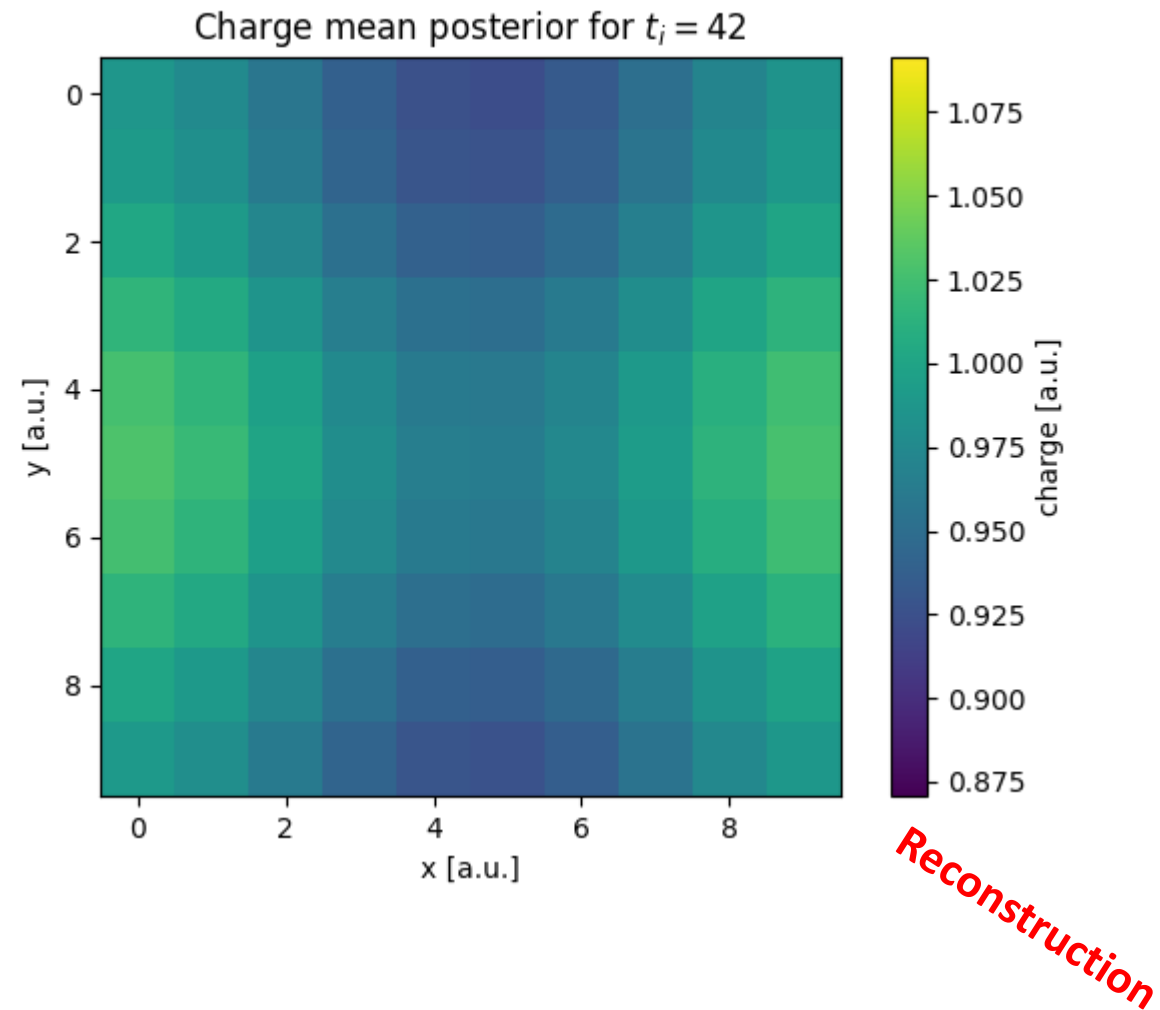
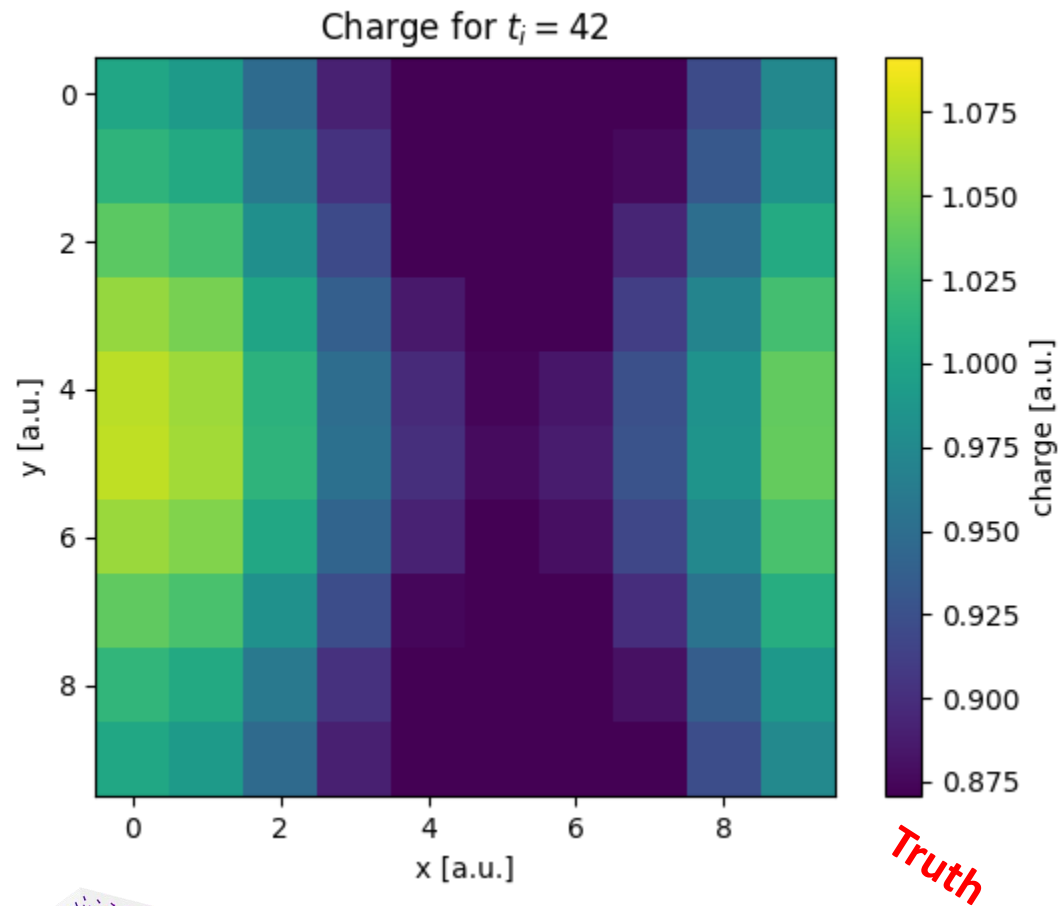


Drift current posterior samples for  $t_i = 42$



Cuboid slice at one point in time

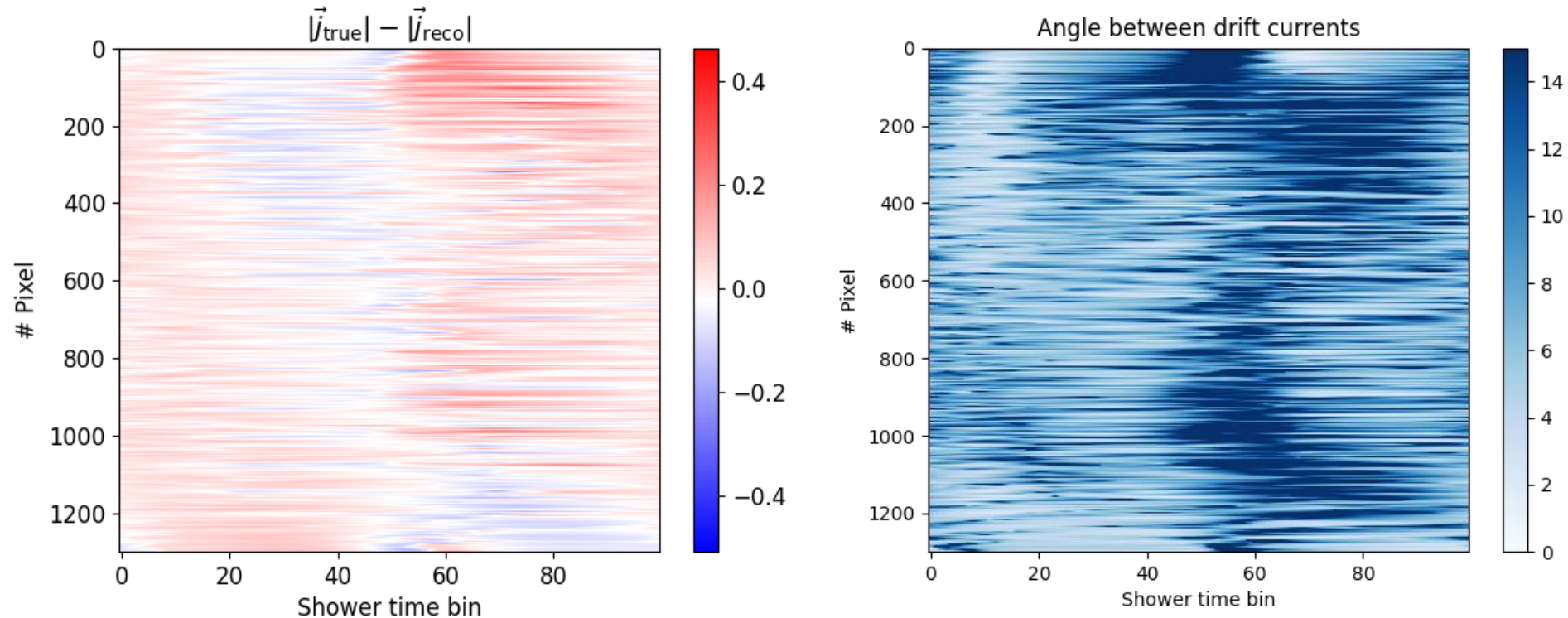
# Preliminary Results $q$



Cuboid slice at one point in time

# Preliminary Results

- Overall reconstruction similar to truth
- Expected to improve with more stringent priors



*Currently investigating!*

# Summary

- Working prototype of a macroscopic air shower imaging algorithm
- Use of Information Field Theory to fit many voxels at once
- Sample posterior for uncertainty estimation



## *Next up:*

- Expand parametrization → include physical assumptions
  - Check self-consistency: How close is the output to the truth?
  - Test physical scenarios: How flexible is the framework?
  - How degenerate is the solution space?
- 
- Application to simulated showers

