Developing a “Simulation-Based” Inference Approach for Galaxy Cluster Abundance Cosmological Analysis

Yuanyuan Zhang
Assistant Astronomer @ NSF NOIRLab

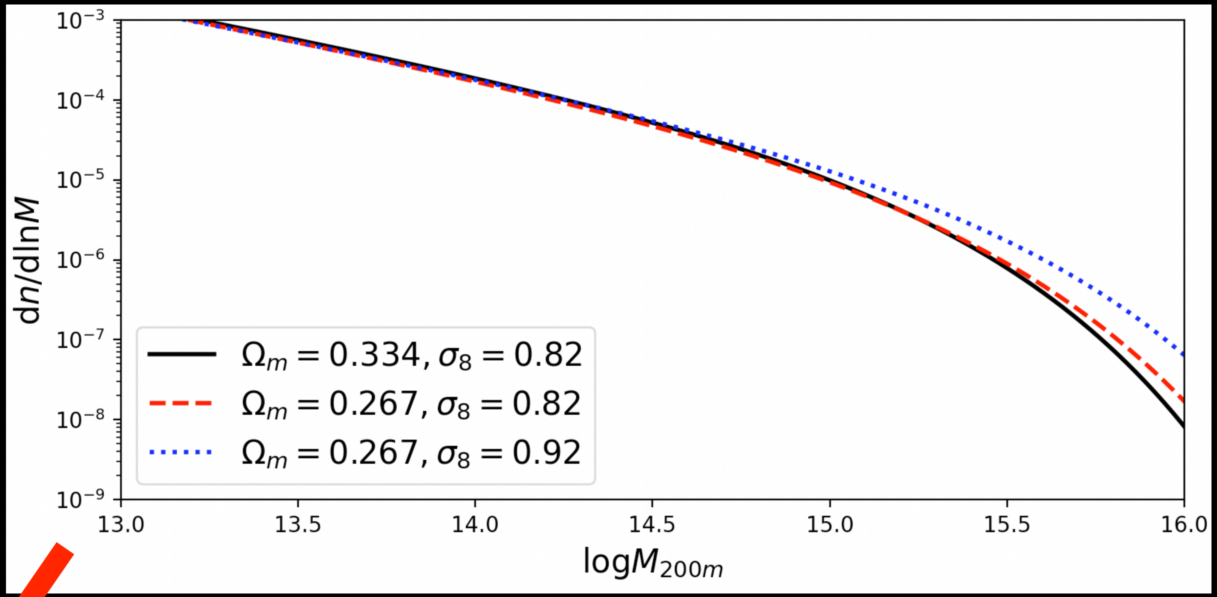
with
Moonzarin Reza, Camille Avestruz, Louis E. Strigari,
Simone Shevchuk,
Francisco Villaescusa-Navarro

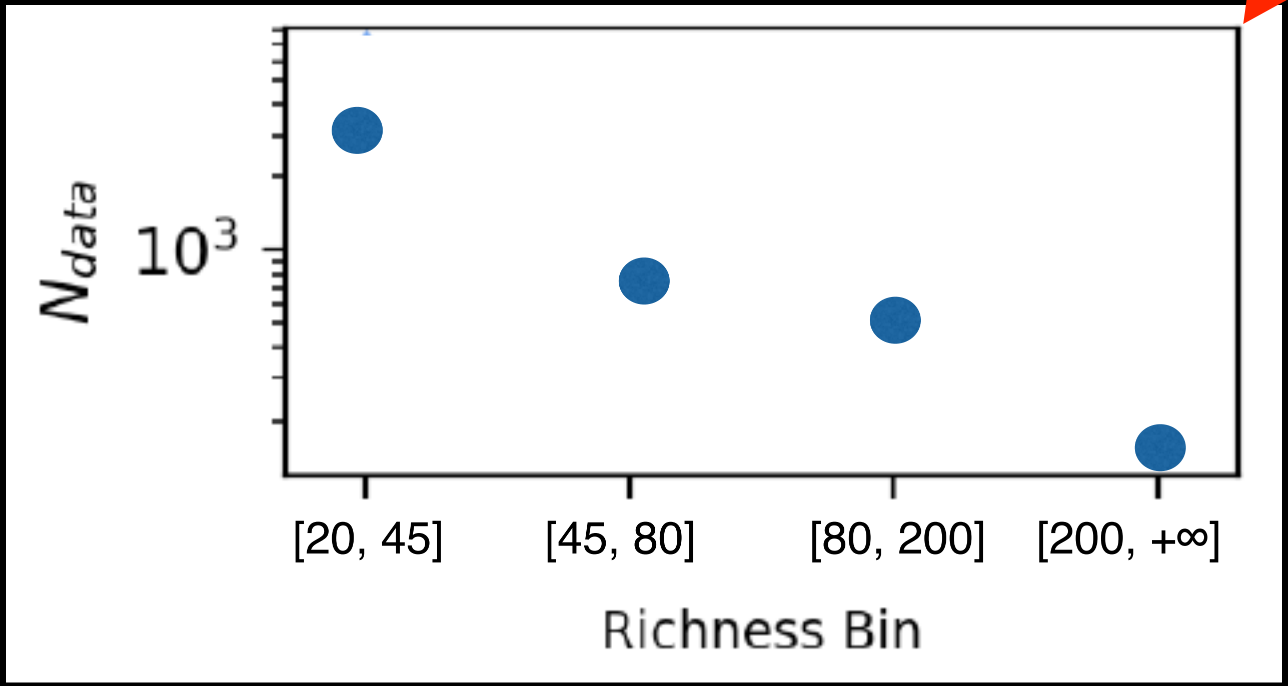
Intro: Galaxy Cluster Cosmology with Number Counts and Masses

The abundance of galaxy clusters are sensitive to cosmological parameters like Ω_m and σ_8.

In a simplified abundance analysis, we analyze
- Galaxy cluster counts in **richness** bins,
- Average masses in **richness** bins.

Theoretical dark matter halo mass function





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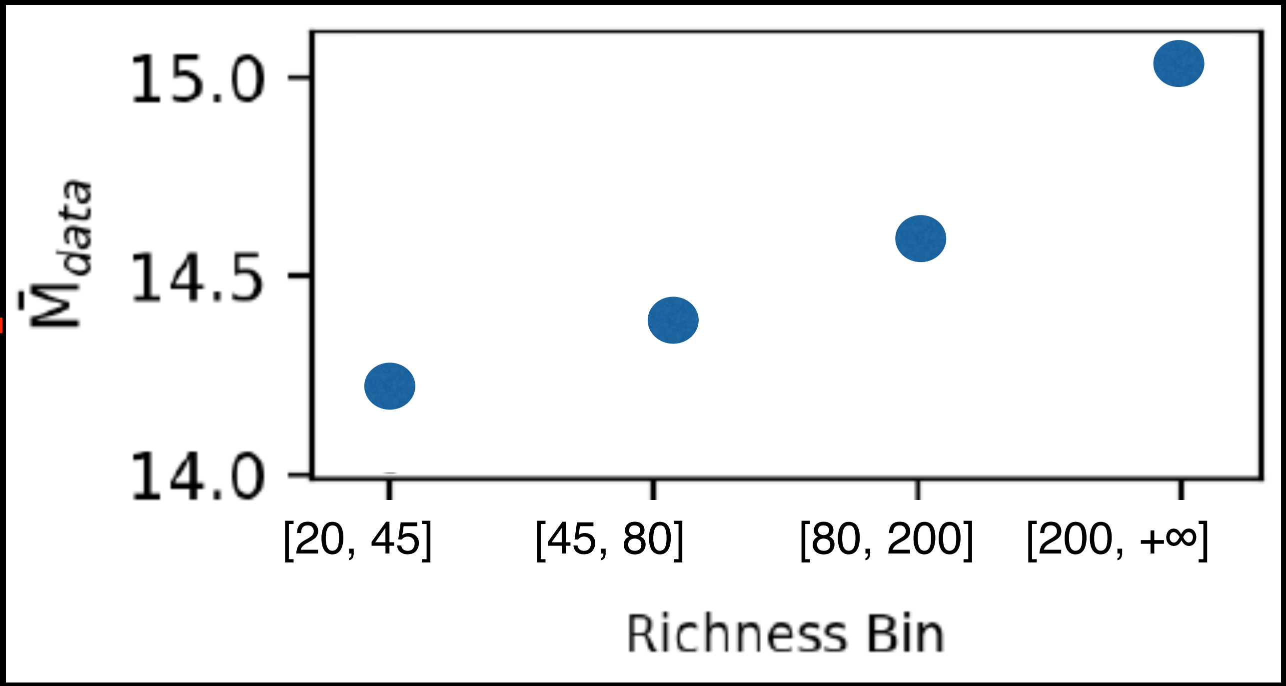
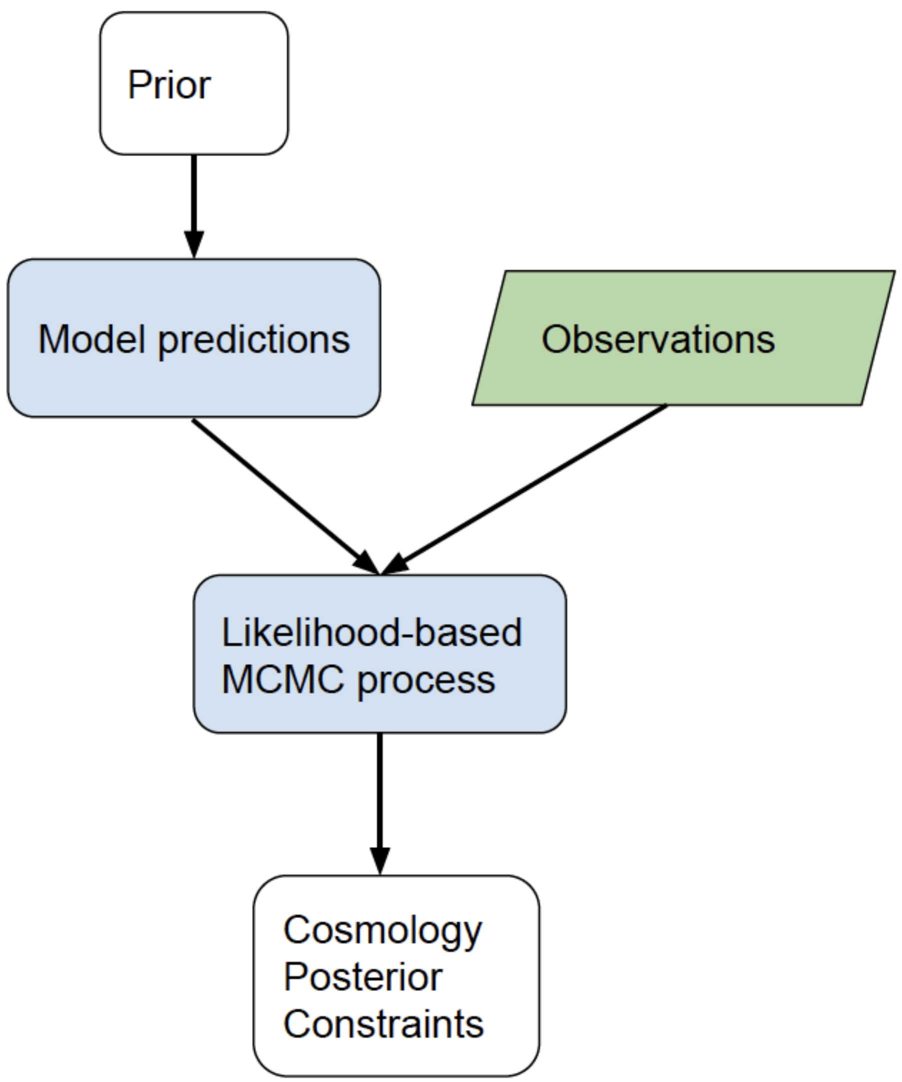


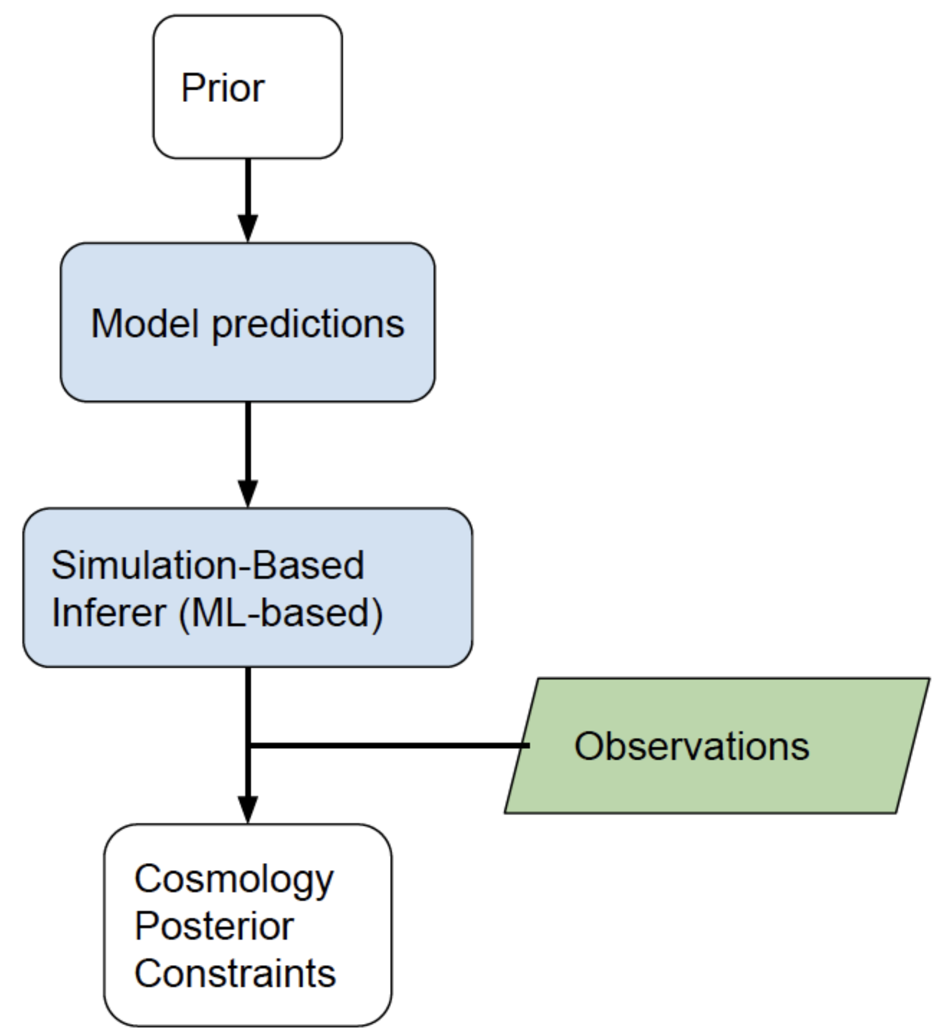
Intro: “Simulation”-Based Inference Method

The “likelihood” -based
Markov Chain Monte Carlo (MCMC)



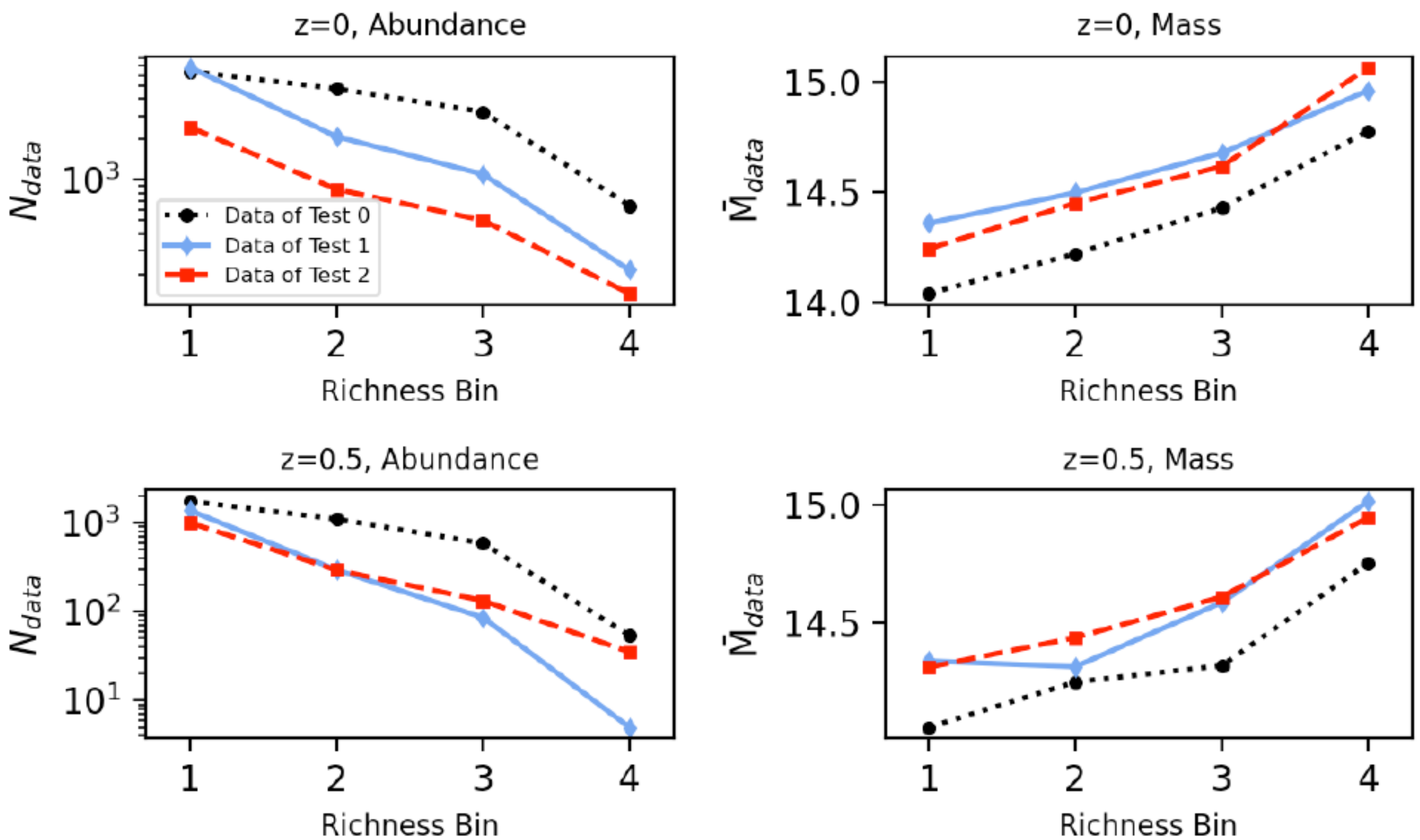
- Models/likelihood are computed “on the fly”.
- Can be quite slow to compute.

“Simulation” Based Inference (SBI)
(Cranmmer 2020)



- Training set can be pre-computed.
- The posterior steps are efficient to run.

Setting up the SBI method



SBI Procedure:

- Start with one set of parameters (cosmology, richness-mass) — $\boldsymbol{\theta}$.
- Generate a mock observable, number of clusters, averages masses in richness bins – x.
- Repeat the above process to **generate ~150,000 simulations**.
- Train an SBI inferer (Mixture Density Networks – MDN) with the simulations P($\boldsymbol{\theta}$, x).
- Use the trained MDN to derive posterior parameter distribution, for a given set of observables P($\boldsymbol{\theta}$ |x).

** We use simple analytical models without systematic effects here. Will build more complicated models for future applications!

$$\overline{N}(\theta)_{\Delta\lambda,z} = V_{\rm sim}\int_{10^{14}}^{\infty} \mathrm{d}M \int_{\Delta\lambda} \mathrm{d}\lambda\ h(M,z|\theta)P(\lambda|M,z,\theta),$$

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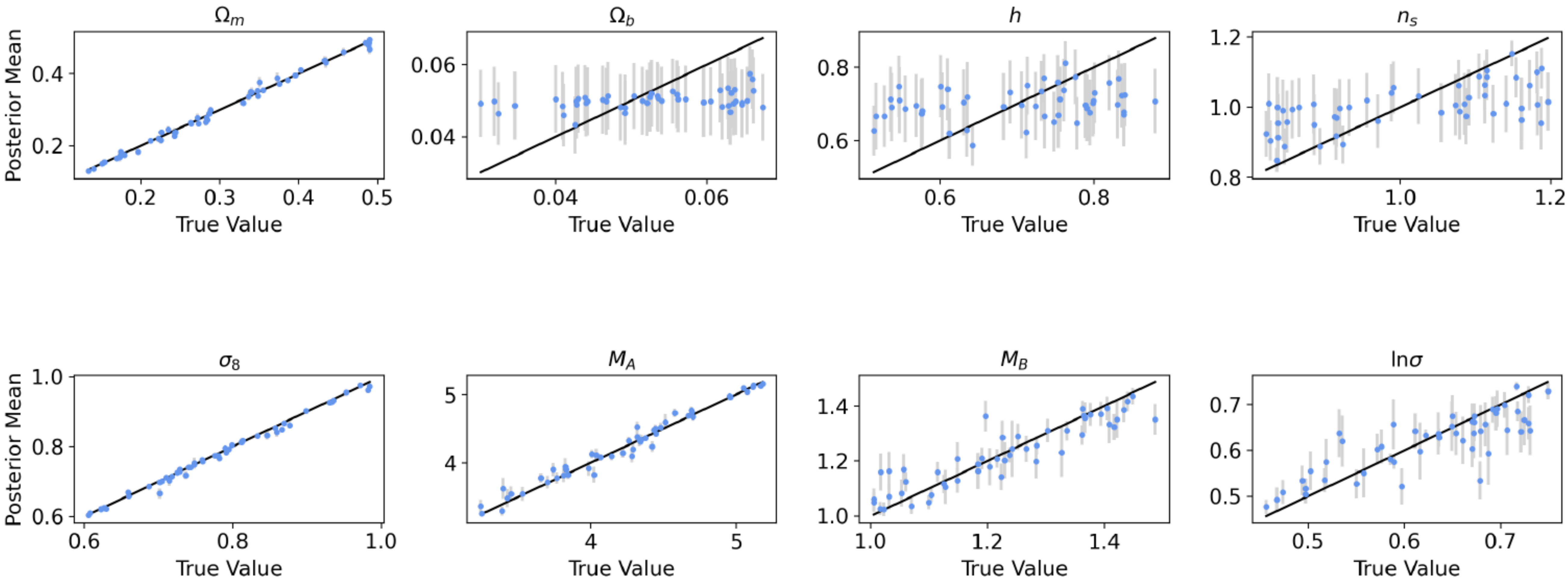
$$\overline{M}(\theta)_{\Delta\lambda,z} = \overline{NM}(\theta)_{\Delta\lambda,z}/\overline{N}(\theta)_{\Delta\lambda,z}.$$

SBI application to a "perfect" data vector (training and testing with simulations generated consistently)



- SBI constraints for **Ω**_m and σ_8 look reasonable.
- SBI constraints on parameters that clusters are not sensitive to (**Ω**_b, h and n_s) don't look great.

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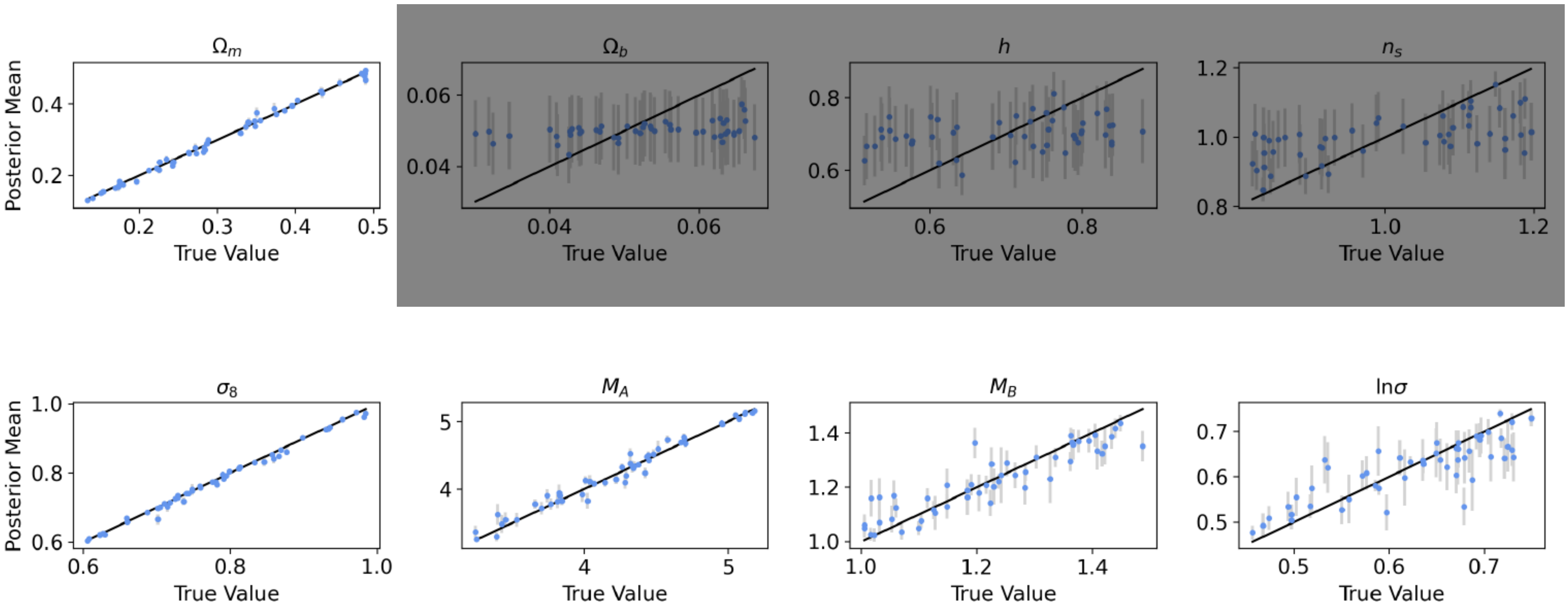

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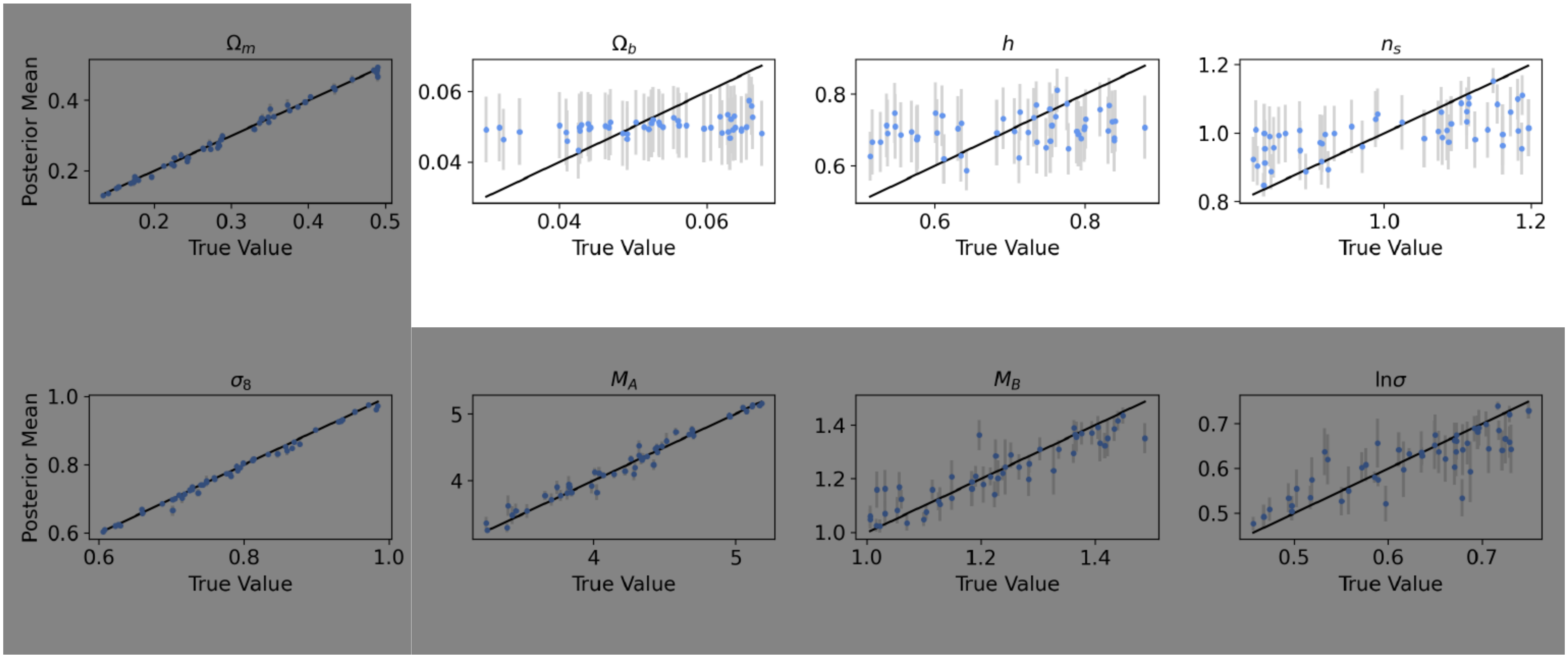

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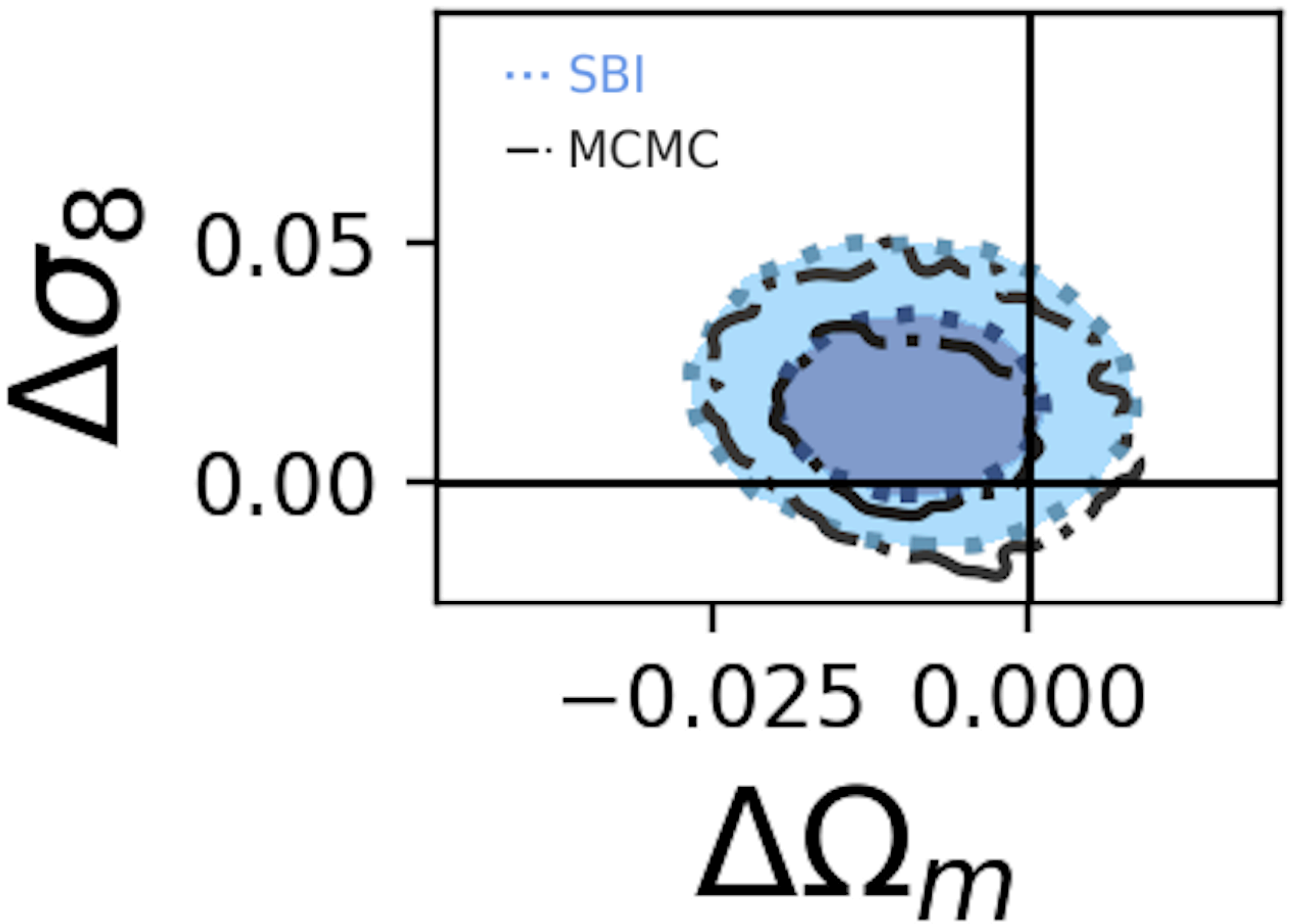


Posterior comparison between SBI vs. MCMC show that their results are consistent.

SBI application to Quijote simulations

Fiducial model (17,100 simulations)

$$\Omega_m = 0.3175, \Omega_b = 0.049, h = 0.6711, n_s = 0.9624, \sigma_8 = 0.834,$$
$$w = -1, M_\nu = 0, \delta_b = 0, f_{NL} = 0, p_{NL} = 0, f(R) = 0$$

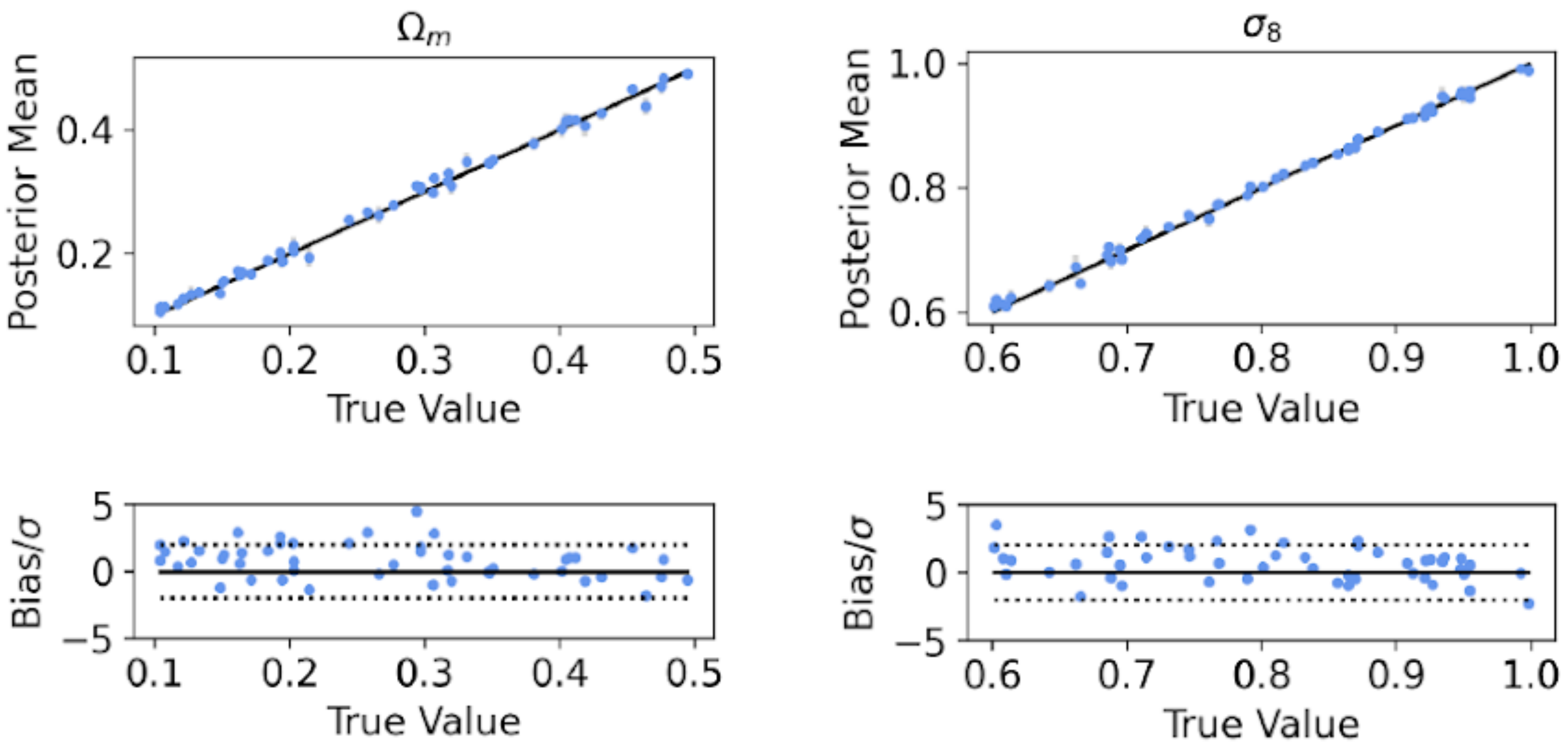
Individual parameter variations (23,000 simulations)

ΛCDM	Dark energy	Massive neutrinos	Separate Universe	Primordial non-Gaussianities	Parity violation	Modified gravity
$\Omega_m, \Omega_b, h, n_s, \sigma_8$	w	M_ν	δ_b	$f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm ortho}$	$p_{\rm NL}$	$f(R)$

Villaescusa-Navarro et al 2020

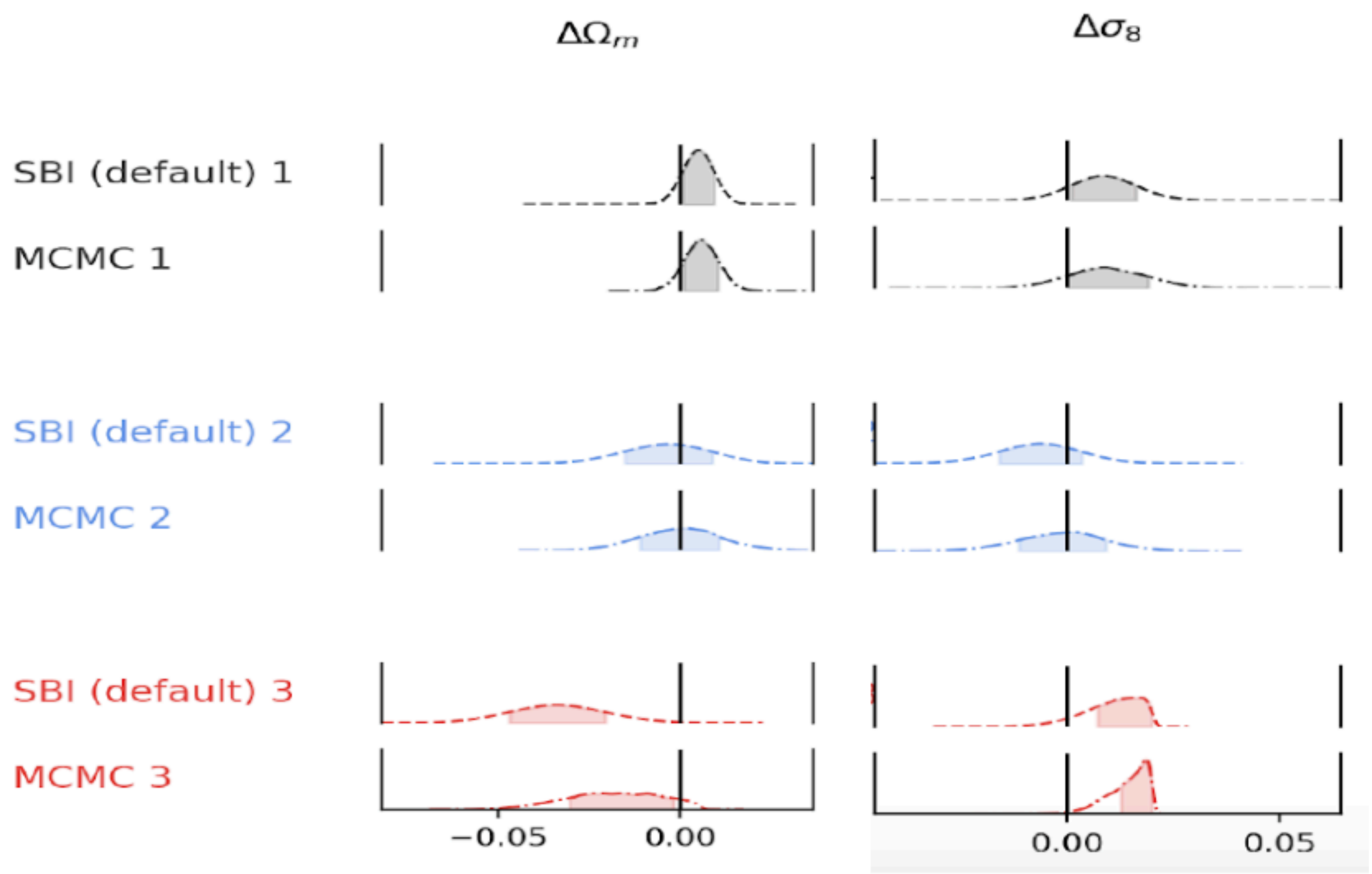
- Large simulation suite with different cosmology models/parameters.
- Particularly suitable for machine learning applications in astrophysics/ cosmology studies.

SBI application to Quijote simulations



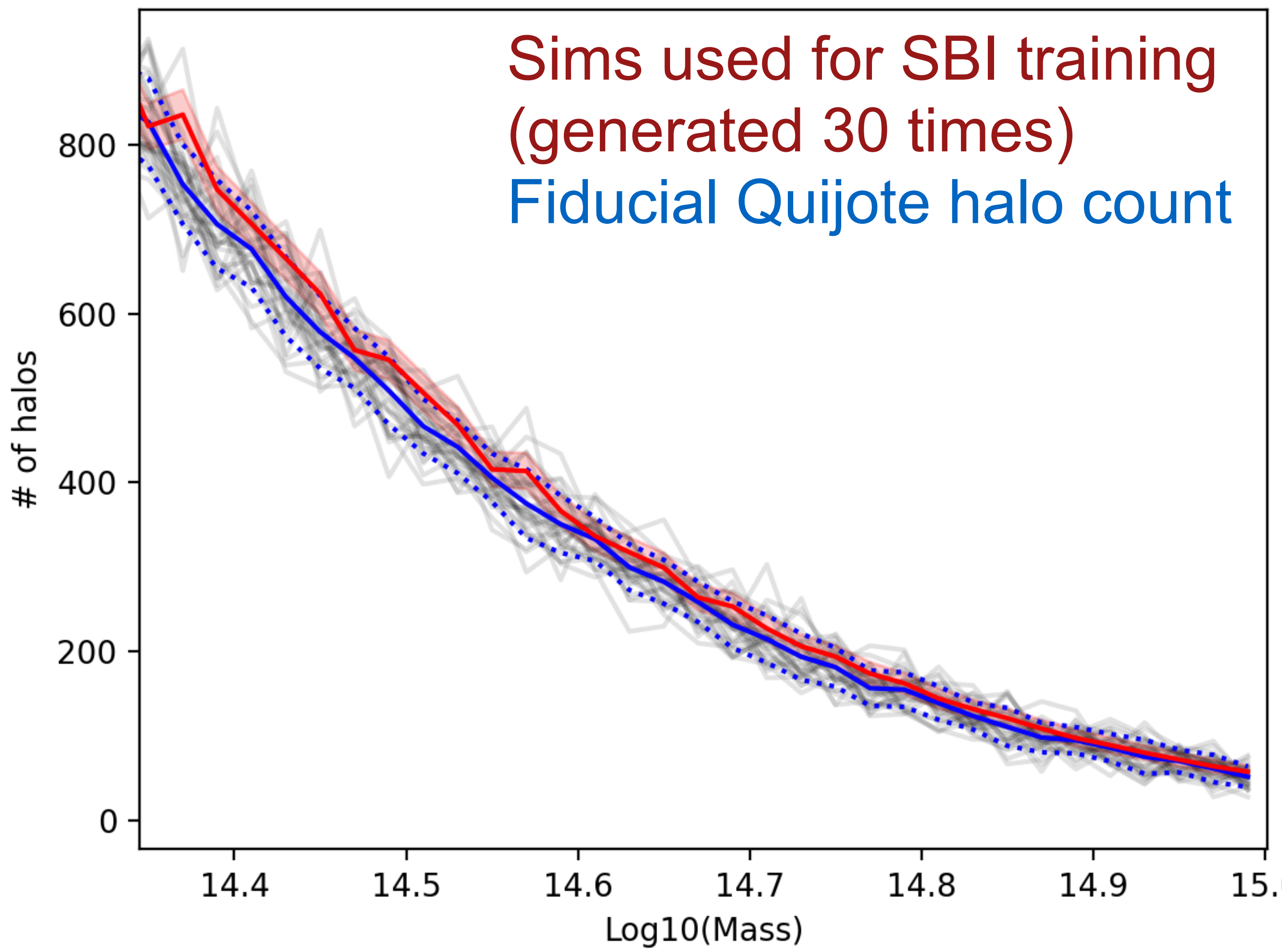
SBI posterior constraints for Ω_m and σ_8 look a bit biased.

SBI application to Quijote simulations



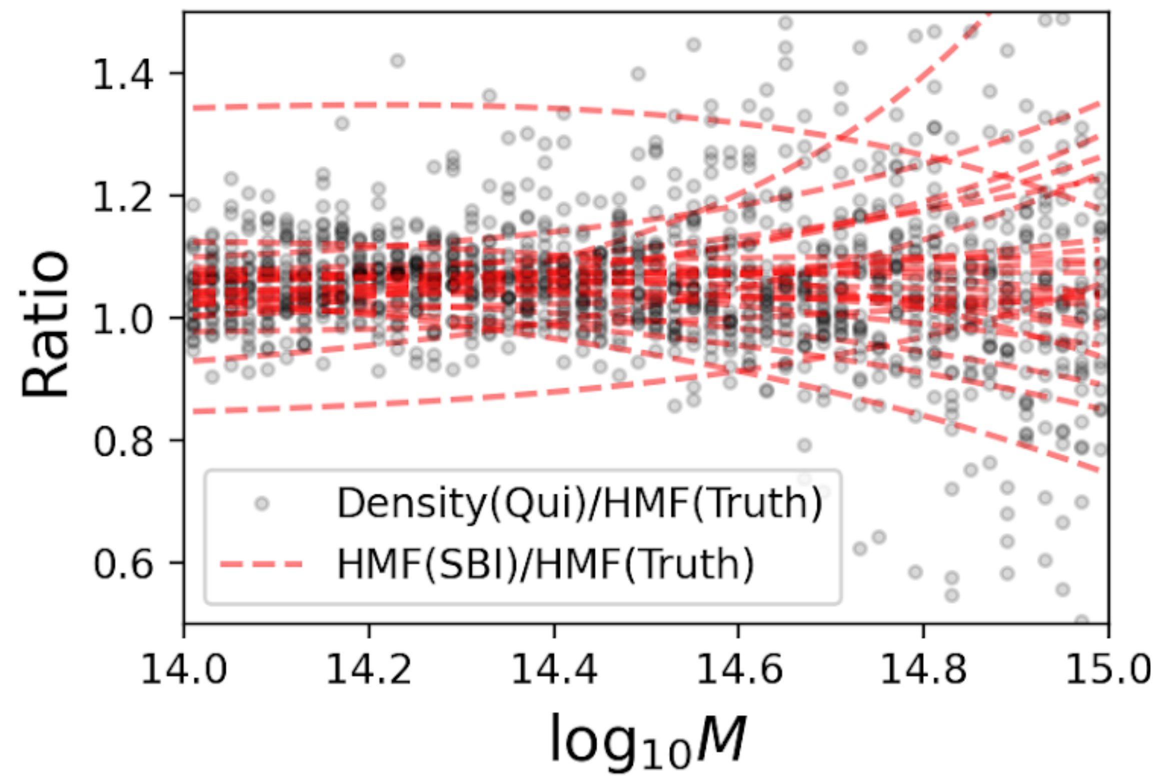
- (default) SBI posterior constraints for **Ω**_m and σ_8 look a bit biased.
- Interestingly MCMC and SBI (default) results still look comparable.

SBI application to Quijote simulations



- Halo mass function (HMF) in the Quijote simulations are different from the one we used to generate the training set.

- The SBI has to produce biased cosmology to reproduce the HMF differences between the observations and the training sets.



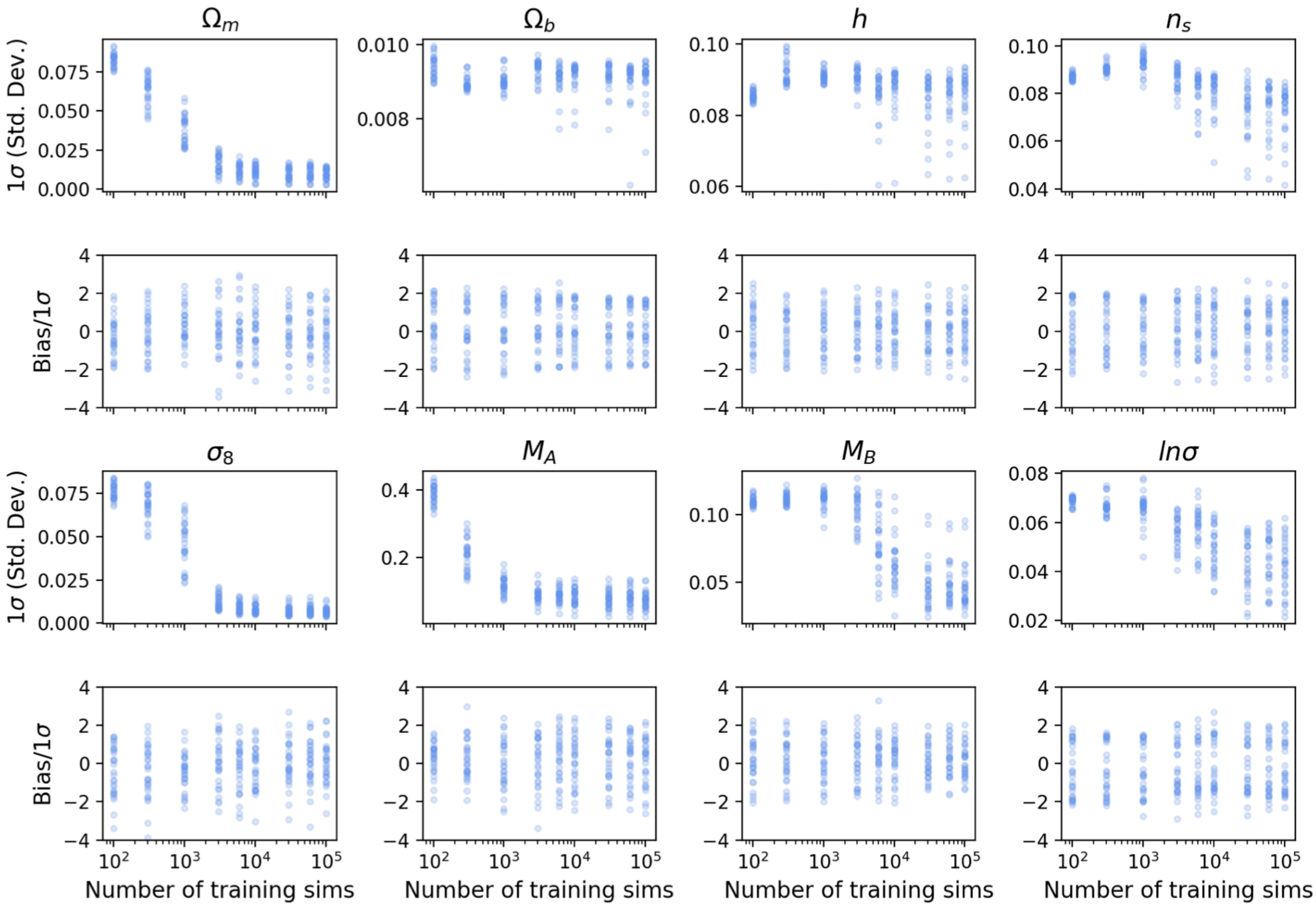
Summary

- SBI method can be accurate enough for a cluster cosmology application. Biases were interpretable and the results are comparable to MCMC.
- Compared to MCMC, SBI is fast to apply after training.
- Based on arXiv:2409.20507 (Reza, Zhang et al.)

Also, check out **Akum Gill's poster on**
SBI application to Weak Lensing Galaxy Cluster Mass Inference !

Back-up Slides

Posterior accuracies change when increasing the number of training simulations.



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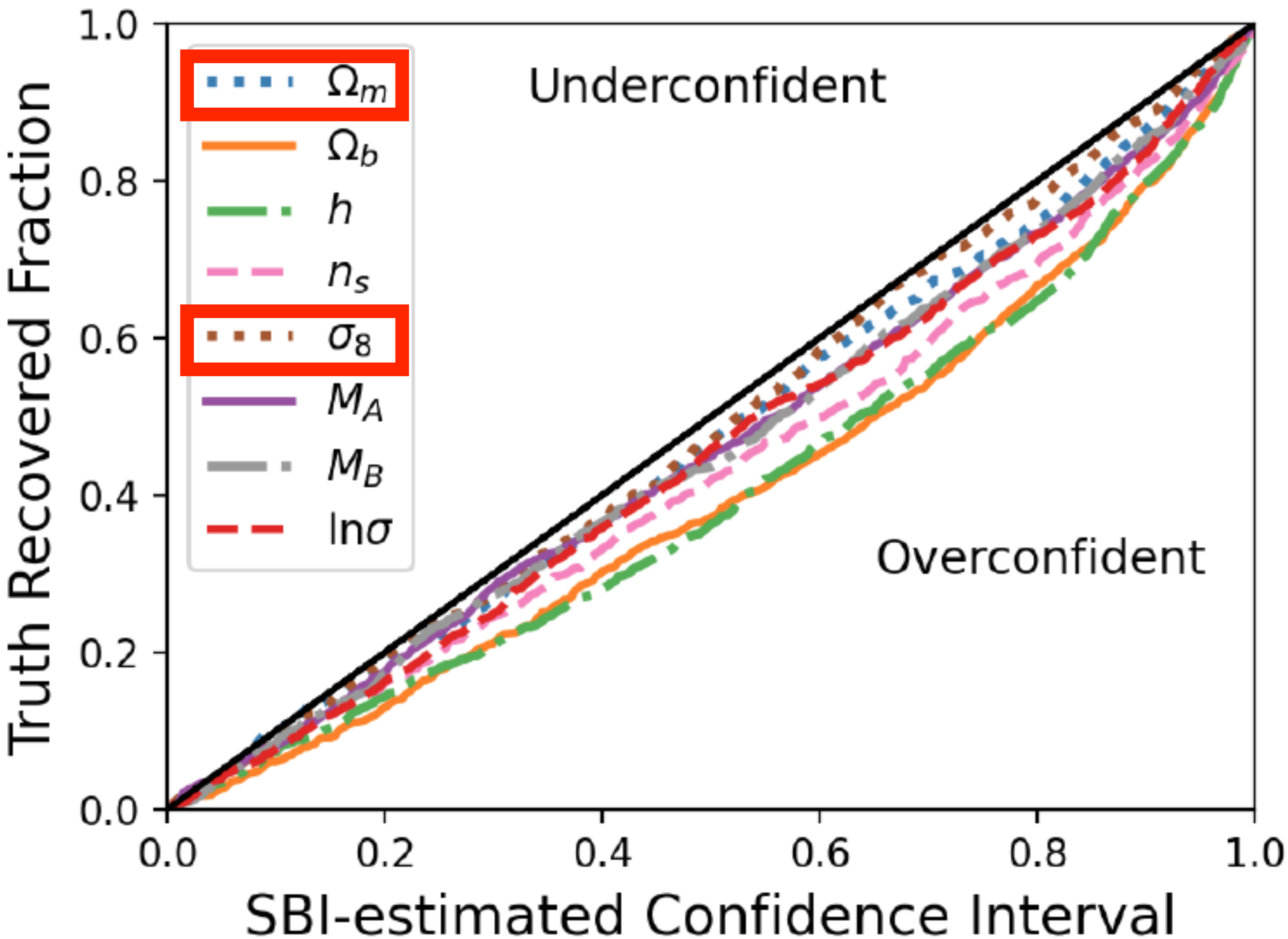

Testing whether or not the SBI uncertainties correctly
reproduce the truth occurrence rate.