Inaugural US Muon Collider Accelerator School
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U.S. Particle Accelerator School
Education in Beam Physics and Accelerator Technology

THE UNIVERSITY OF CHICAGO

Colliders – Lecture VS2:

Collective Effects (BB, SC, etc)
Incoherent Effects (halo, cool)

Vladimir Shiltsev, Northern Illinois University

part of the "Colliders" class by V.Shiltsev, J.Eldred and B. Simmons

Muon Collider School · Aug. 04 – Aug 07, 2025 · U. Chicago

Important Beam Effects

- Beam-beam effects
- Space-charge effects
- Instabilities
- Collimation
- Cooling
- Diffusion and Intrabeam scattering
- Beamstrahlung
- Polarization
- Synchrotron radiation
- etc etc etc (20 – 40 topics at USPAS/CAS)

Beams as Moving Charges

- Beam is a collection of charges
- Represent electromagnetic potential for other
- charges
- Forces on itself (space-charge, wake-fields/impedances) and opposing beam (beam-beam effects)
  - Main limit for present and future colliders
  - Important for high density beams, i.e. high intensity and/or small beams = for high luminosity !

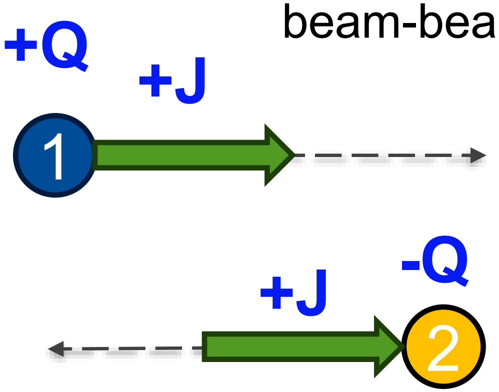
EM Forces in Beams: self fields $B_\theta = \beta E_r$

same charges repel each other (opp attract) $F = eE$

same currents attract each other (opp repel) $F = \beta e B = -\beta^2 e E$

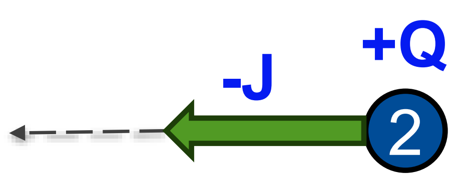
Qualitatively: Balance of These Forces

beam-beam



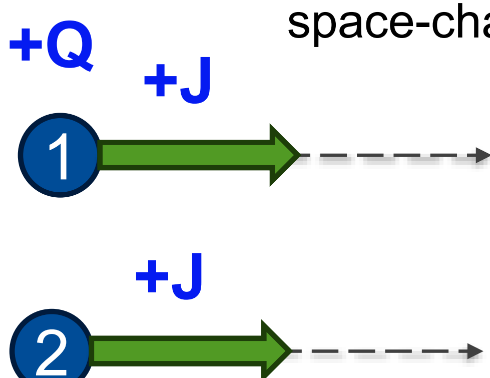
F_E=(+Q,-Q) attracts

F_B=(+J,+J) attracts



both F_E F_B repel

space-charge



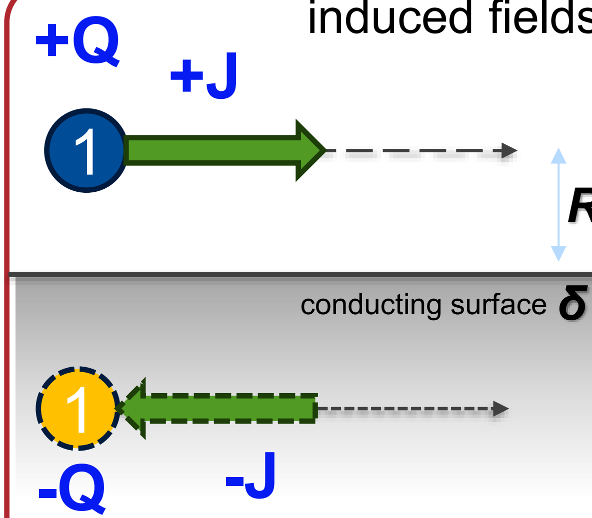
F_E=(+Q,+Q) repels

F_B=(+J,+J) attracts

Total $F_E - F_B$ repels

$= F_E - \beta^2 F_E = F_E / \gamma^2$

induced fields



F_E=(+Q,-Q) attracts

F_B=(+J,-J) repels

Diff $F_E - F_B$ attracts,

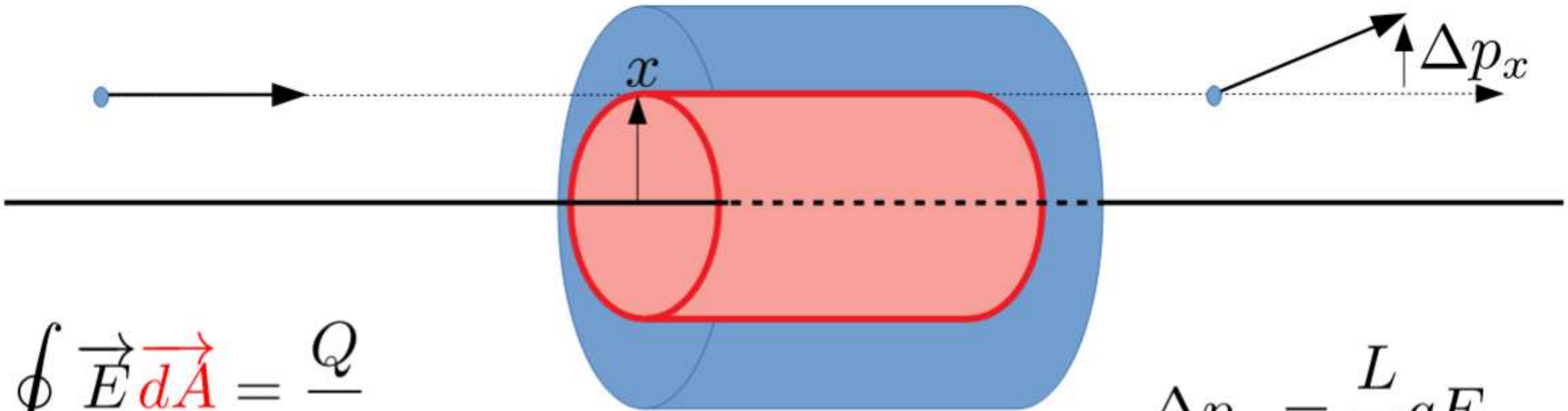
eg skin-eff $\Delta F \sim F_E \frac{\delta}{R}$

BEAM-BEAM (1)

Beam-Beam Effects

- A beam acts on particles like an electromagnetic lens, but:
  - Does not represent simple form, i.e. well-defined multipoles
  - Very non-linear form of the forces, depending on distribution
  - Can change distribution as result of interaction (time dependent forces ..)
- Results in many different intensity dependent effects and problems:
  - unstable betatron oscillations
  - growth of beam sizes
  - particle losses

Beam-Beam Fields: Start with a Cylinder



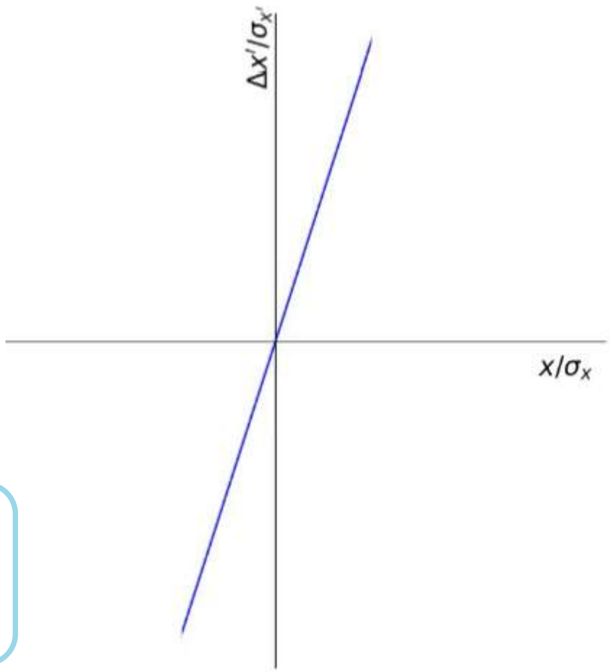
$$\oint \vec{E}\,\vec{dA} = \frac{Q}{\epsilon_0}$$

long cylinder

$$\Delta p_x = \frac{L}{c} q E_x$$

Inside: $$E_x 2\pi x L = \frac{\pi x^2 L \rho}{\epsilon_0} \qquad E_x = \frac{\rho}{2\epsilon_0} x$$

Outside: $$E_x 2\pi x L = \frac{Q_{tot}}{\epsilon_0} \qquad E_x = \frac{Q_{tot}}{2\pi L \epsilon_0} \frac{1}{x}$$



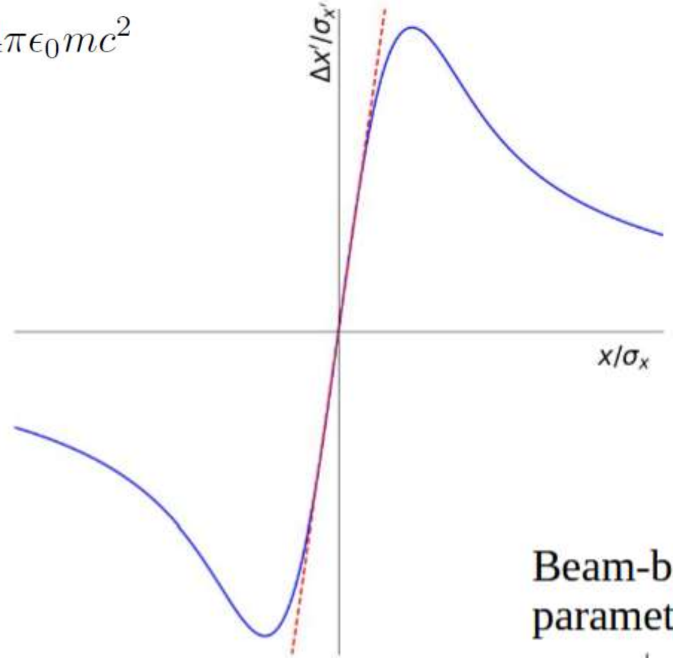
i.e. force is linear inside, nonlinear outside

Kick from a Round Gaussian Beam

- (i.e. focusing) for opposite charged beams
+ otherwise

$r_0 = e^2/4\pi\epsilon_0 mc^2$

$$\Delta x' = \pm \frac{2Nr_0}{\gamma} \frac{x}{r^2} \left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)$$



- The strength of the beam-beam force is usually characterized by the slope at the center → focal length f_{bb}

$$\cos(2\pi(Q_0 + \Delta Q_{BB})) = \cos(2\pi Q_0) - \frac{\beta_0^*}{2 f_{BB}} \sin(2\pi Q_0)$$

- For small shifts and away from integer and half-integer resonances we have:

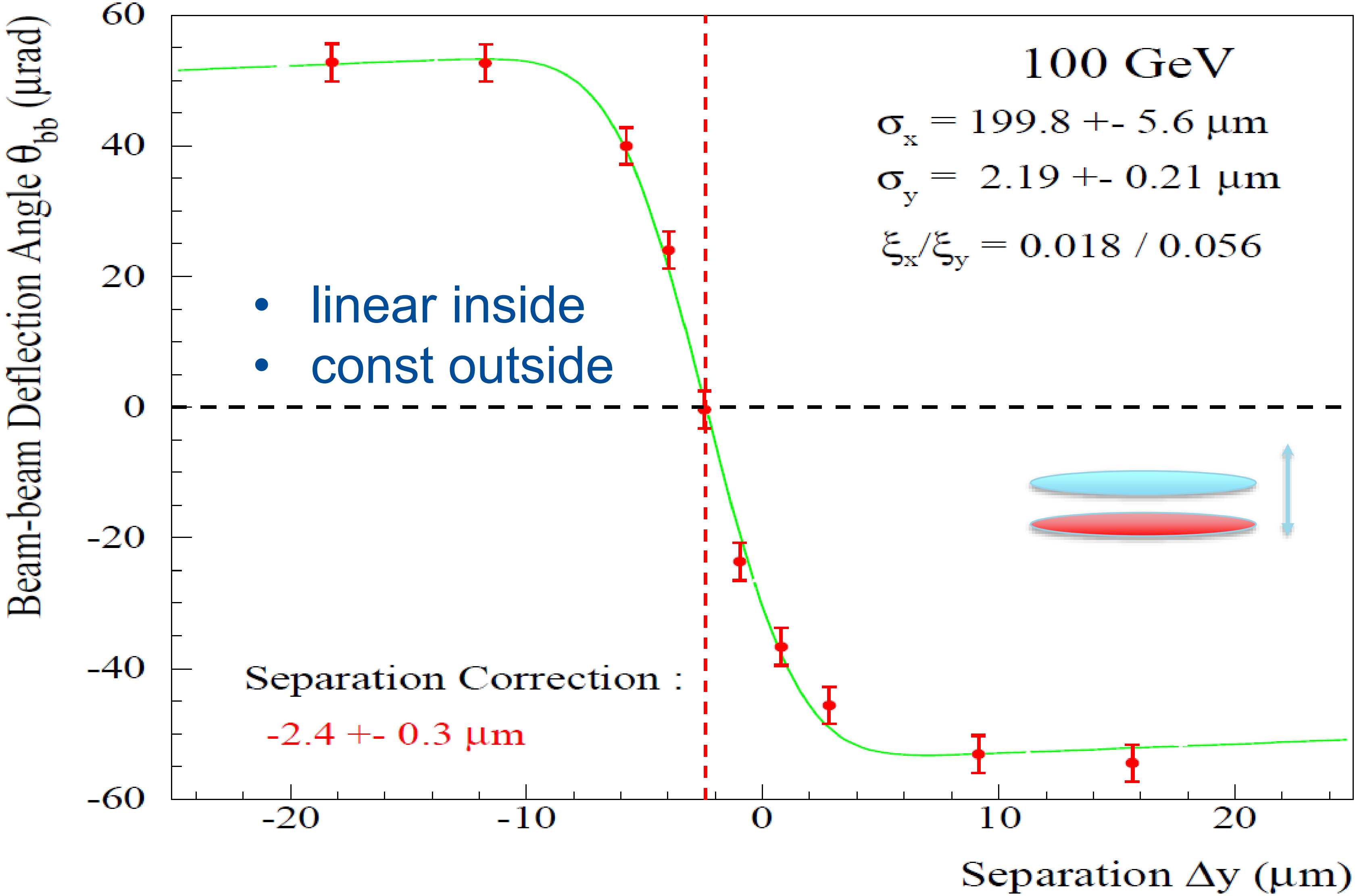
Beam-beam parameter

$$\Delta Q_{BB} \approx \mp \frac{N r_0}{4\pi\epsilon_n} \equiv \xi$$

- In these conditions the beam-beam tune shift is **independent of the beam energy** and of β^*

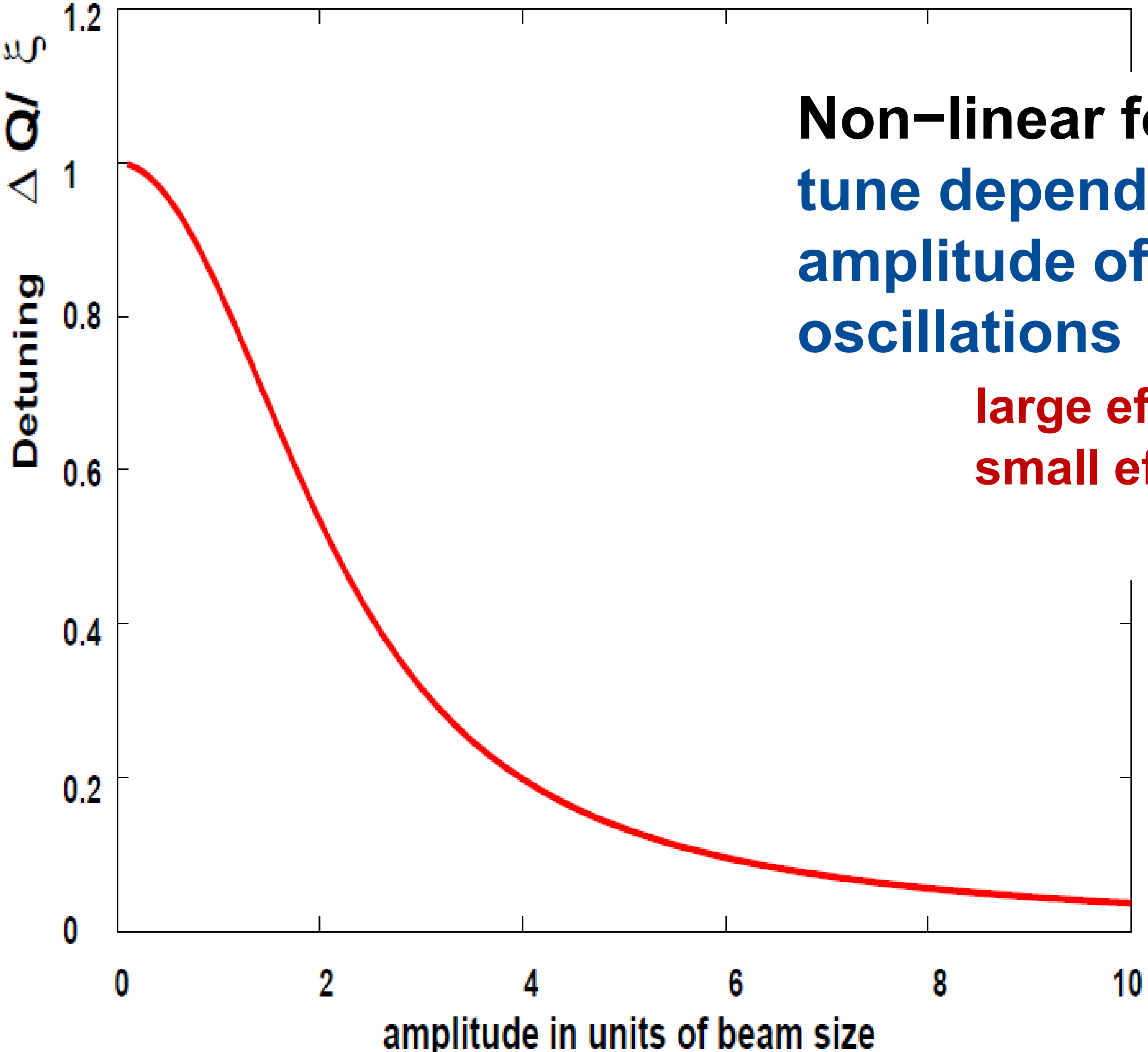
Beam-beam kick in reality

Deflection scan (LEP measurement)



- linear inside
- const outside

Beam-beam Detuning with Amplitude



Non−linear force →

tune depends on the amplitude of betatron oscillations

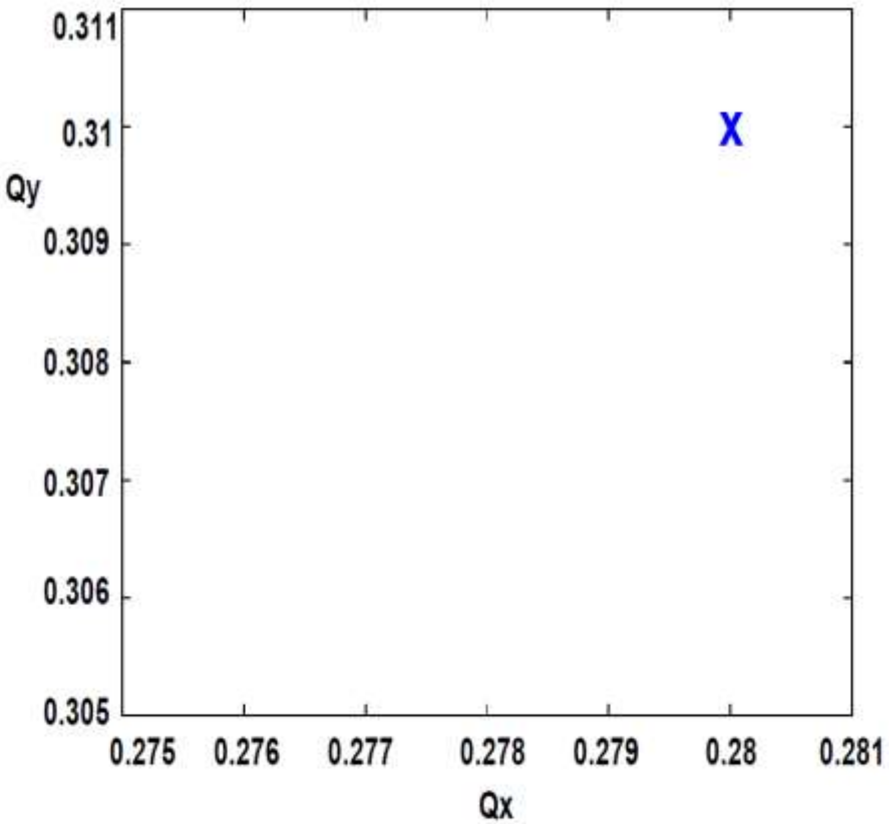
large effect for A<sigma

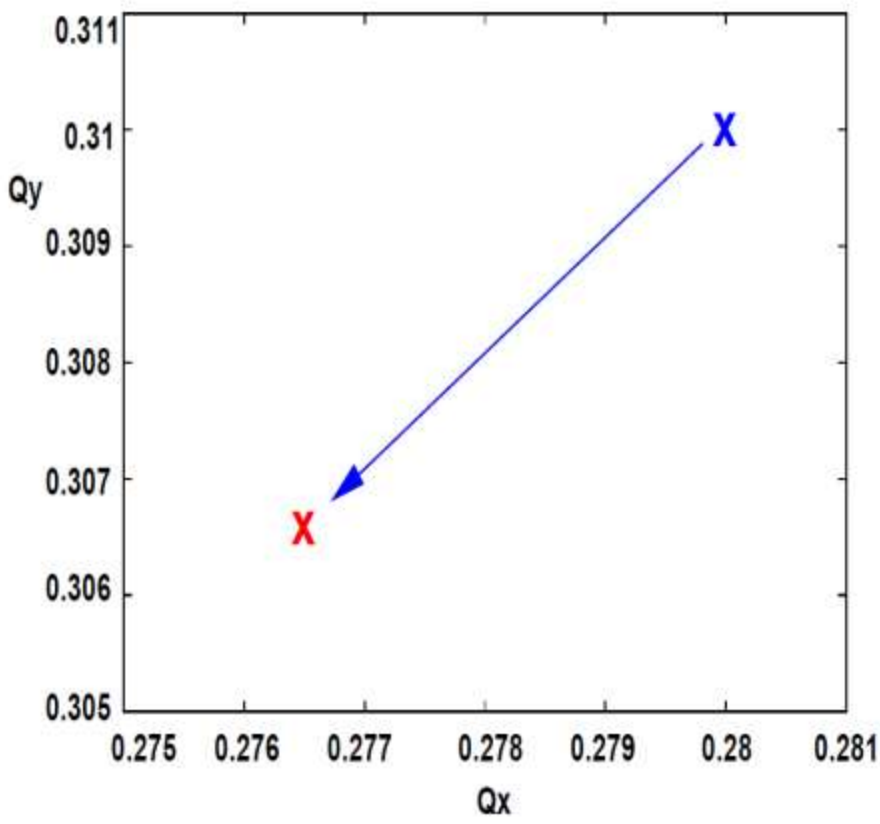
small effect for A>>sigma

Linear tune shift - two dimensions

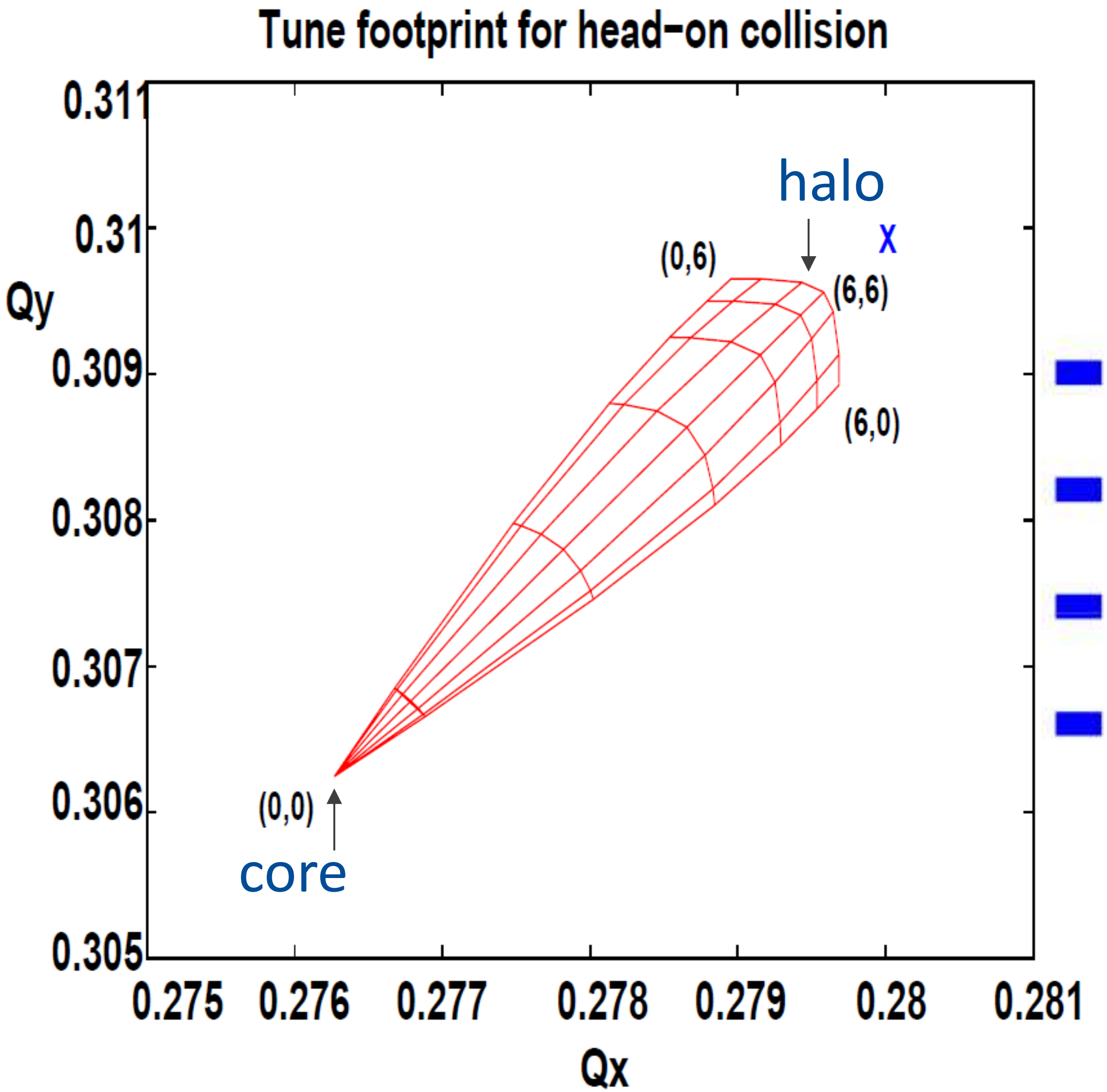
“bare lattice” tune

“bare lattice” tune + linear shift due to beam-beam (=core particles)





Non-linear tune shift in two dimensions



- Tunes depend on x **and** y amplitudes
- No single tune in the beam
- Compute and plot for every amplitude (pair) the tunes in both planes
- In 2 dimensions: plotted as **footprint**

e+e- LEP vs p-pbar collider Tevatron

	LEP	Tevatron
Beam sizes	160 - 200μm · 2 - 4μm	30 μm · 30 μm
Intensity N	4.0 · 10^{11}/bunch	3 · 10^{11}/bunch
Energy	100 GeV	980 GeV
β_x^* · β_y^*	1.25 m · 0.05 m	0.28 · 0.28 m
Beam-beam parameter(ξ)	0.0700	0.012 x2 IPs

Observations (Reality of Beam-Beam)

- Remember:

$$\mathcal{L} = \frac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y}$$

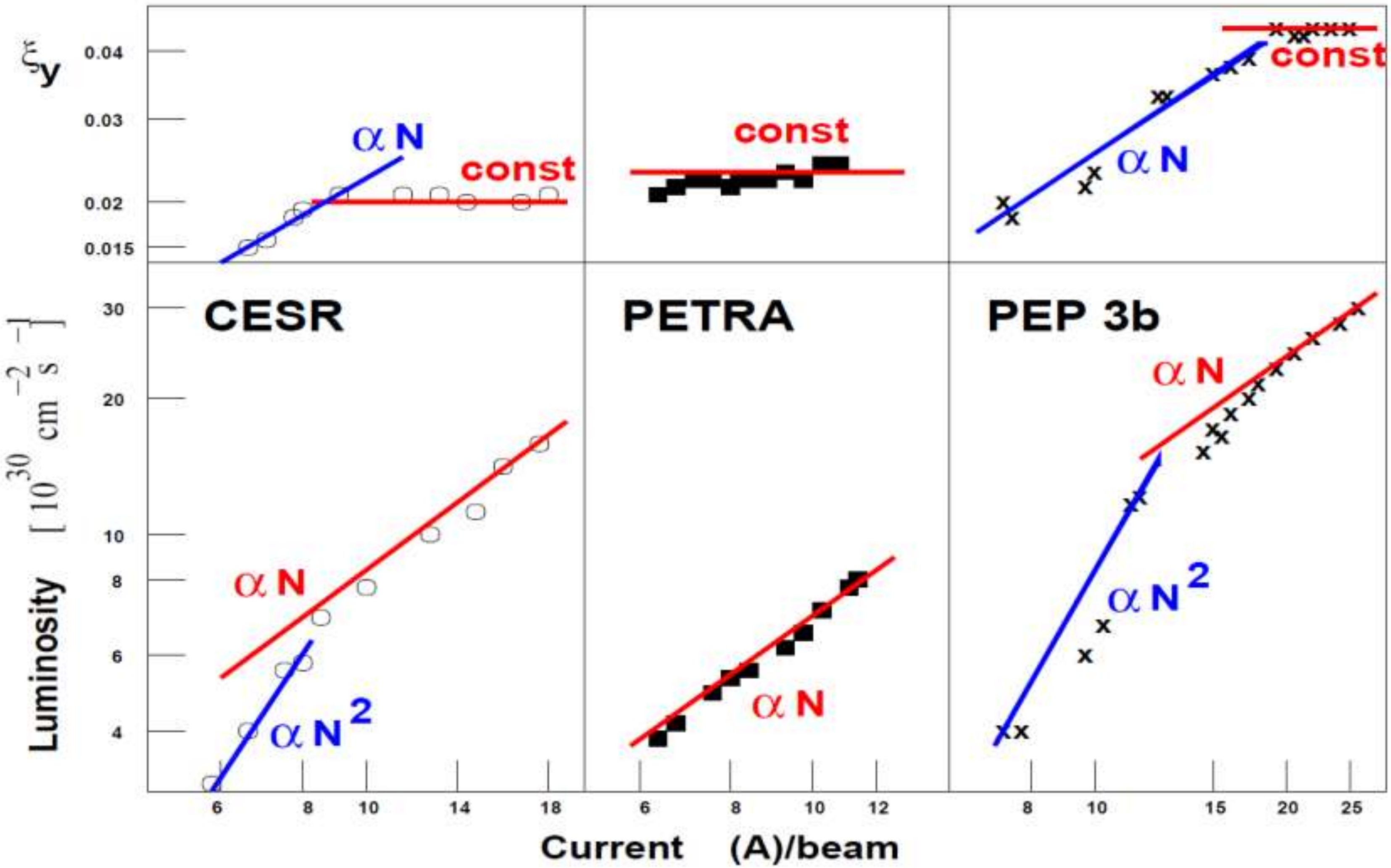
- Luminosity should increase $\propto \; N_1 N_2$

  for:

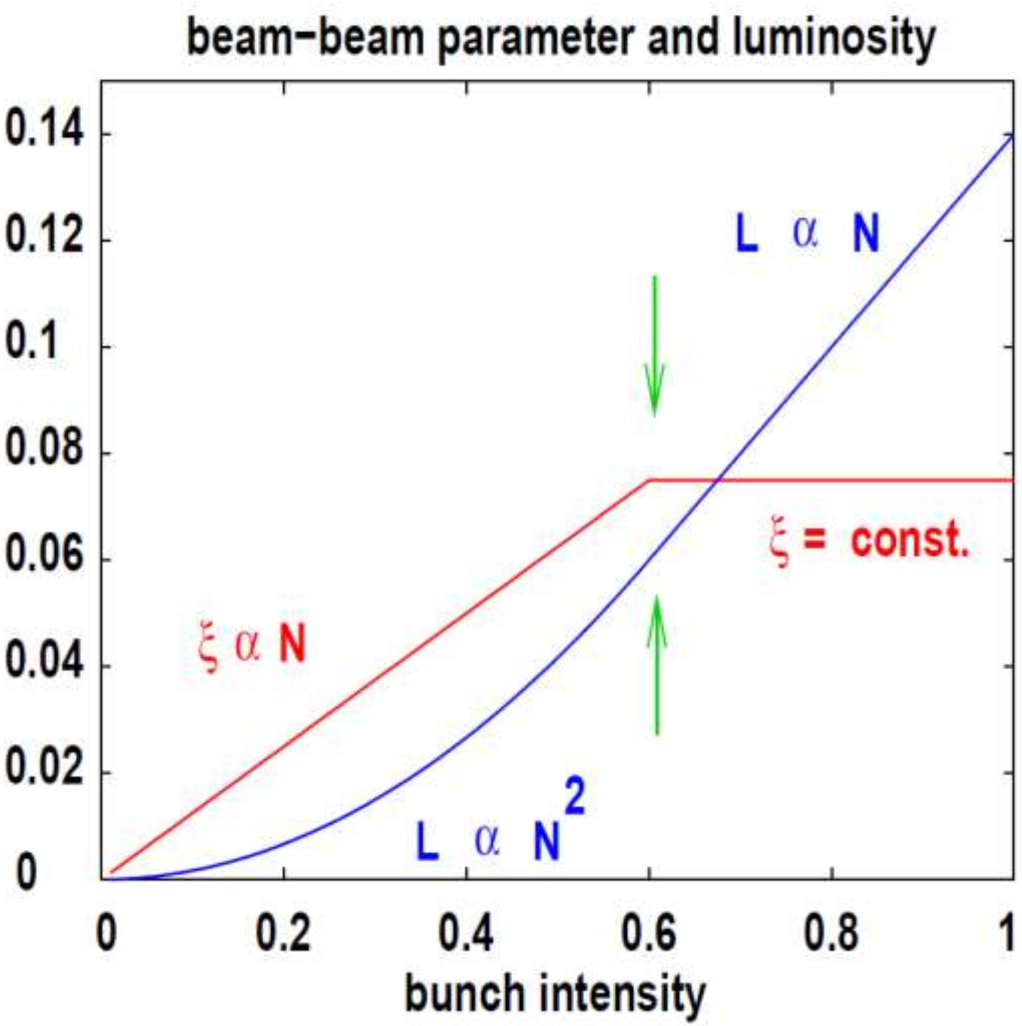
$$N_1 = N_2 = N \quad \rightarrow \quad \propto \; N^2$$

- Beam-beam parameter should increase $\propto \; N$
- But:

Beam-Beam Limits : e+e- Colliders



Beam-beam Limit on Luminosity



First - Beam−beam parameter increases linearly with intensity
Saturation above some intensity

Then – luminosity increases only linearly with N above the so−called
beam−beam limit

What's happening?

$$\xi_y = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \overset{(\sigma_x \gg \sigma_y)}{\approx} \frac{r_0 \beta_y}{2\pi\gamma(\sigma_x)} \cdot \frac{N}{\sigma_y}$$

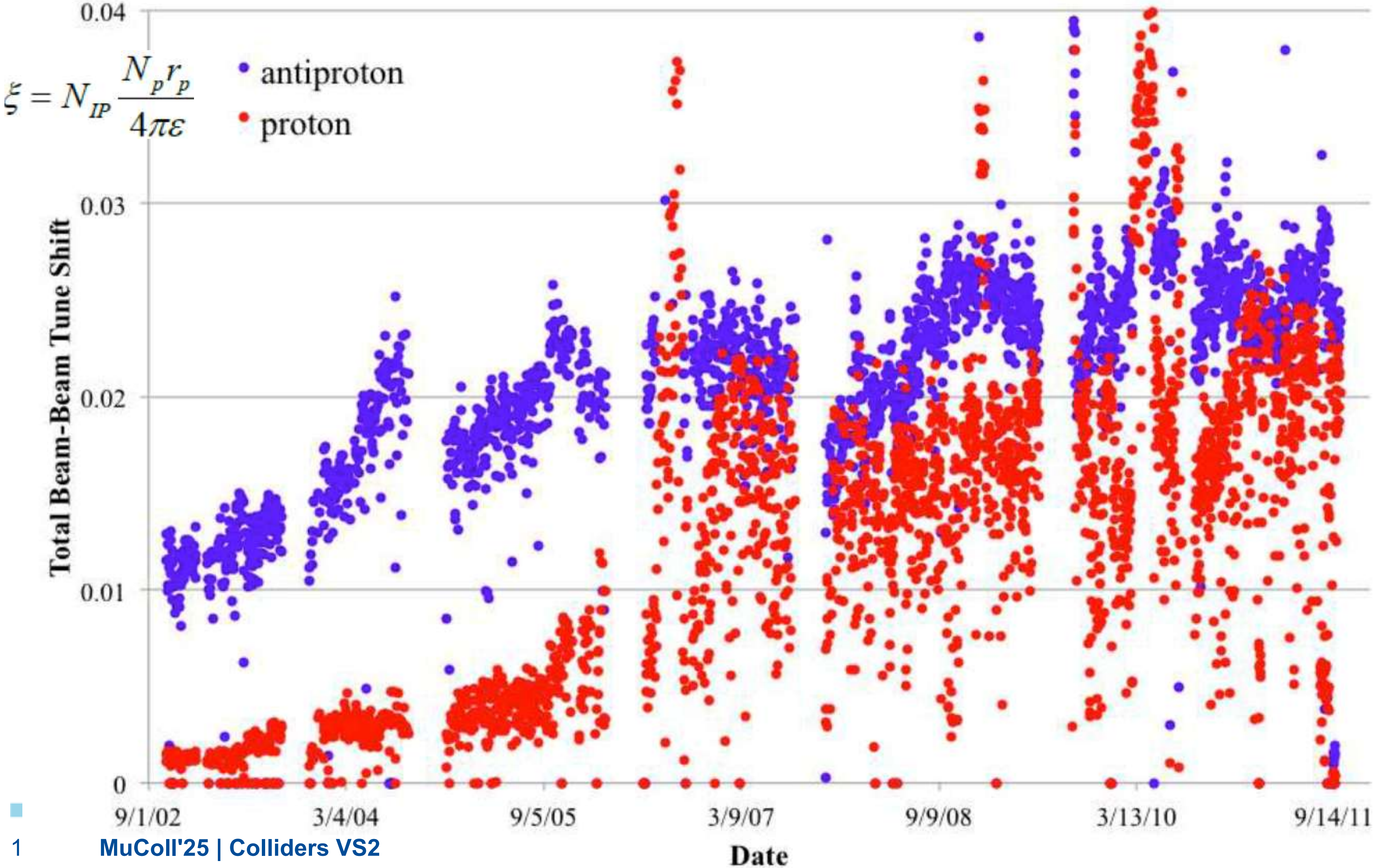
and

$$\mathcal{L} = \frac{N^2 f B}{4\pi\sigma_x\sigma_y} = \frac{N f B}{4\pi\sigma_x} \cdot \frac{N}{\sigma_y}$$

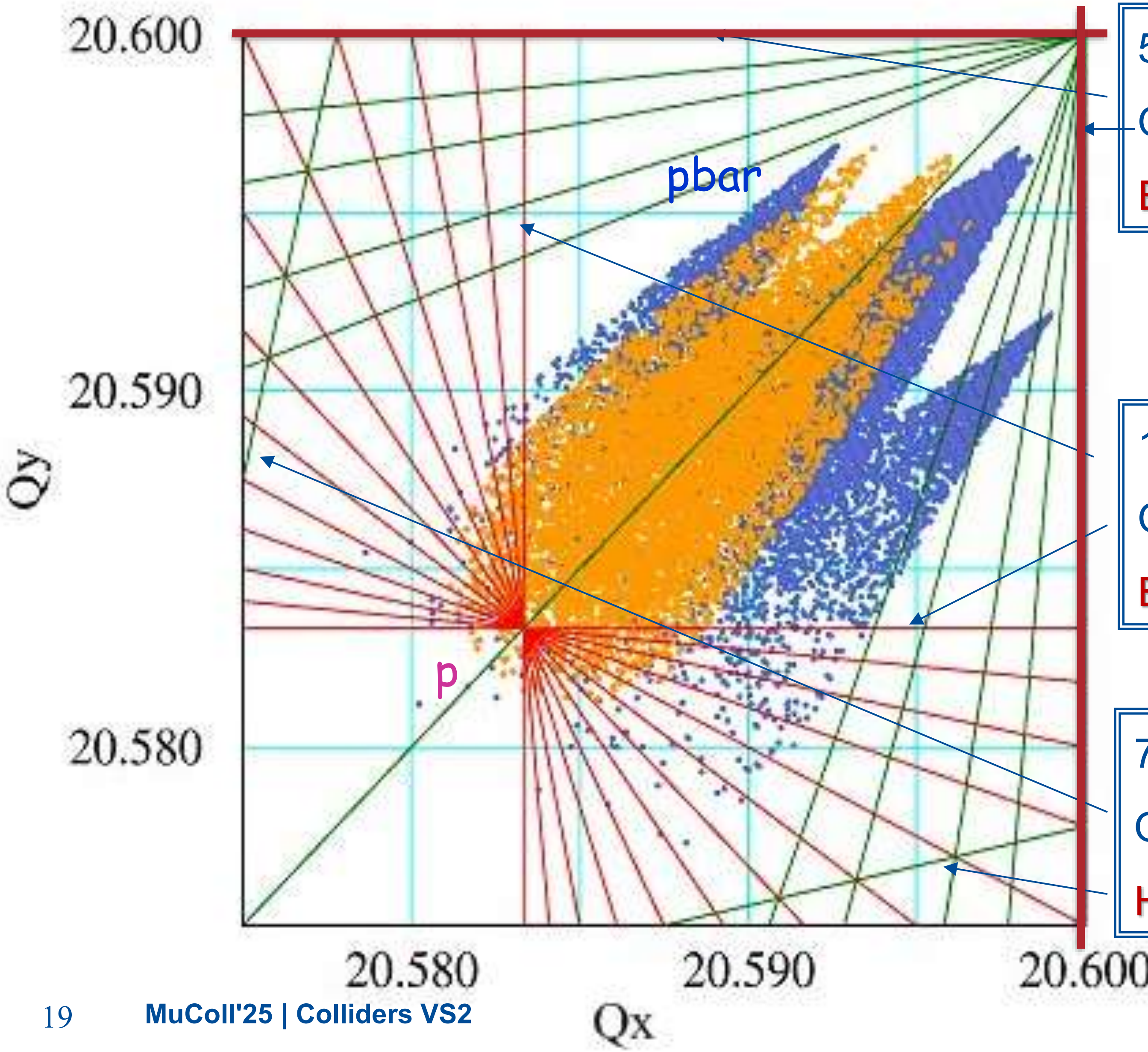
- Above beam-beam limit: σ_y increases when *N* increase
  - to keep constant → equilibrium emittance !
- Therefore:
- is NOT a universal constant !
  - depends on tunes/WPs, damping rates, etc
  - difficult to predict exactly for hadron machines

Beam-Beam Limits: pp/pbar Colliders

Tevatron Collider Run II



Tevatron Tune Footprint "Confinement"



5th order resonances:

Q=3/5=0.600 –

EMITTANCE BLOWUP

protons
antiprotons

12th order resonances:

Q=7/12=0.583 -

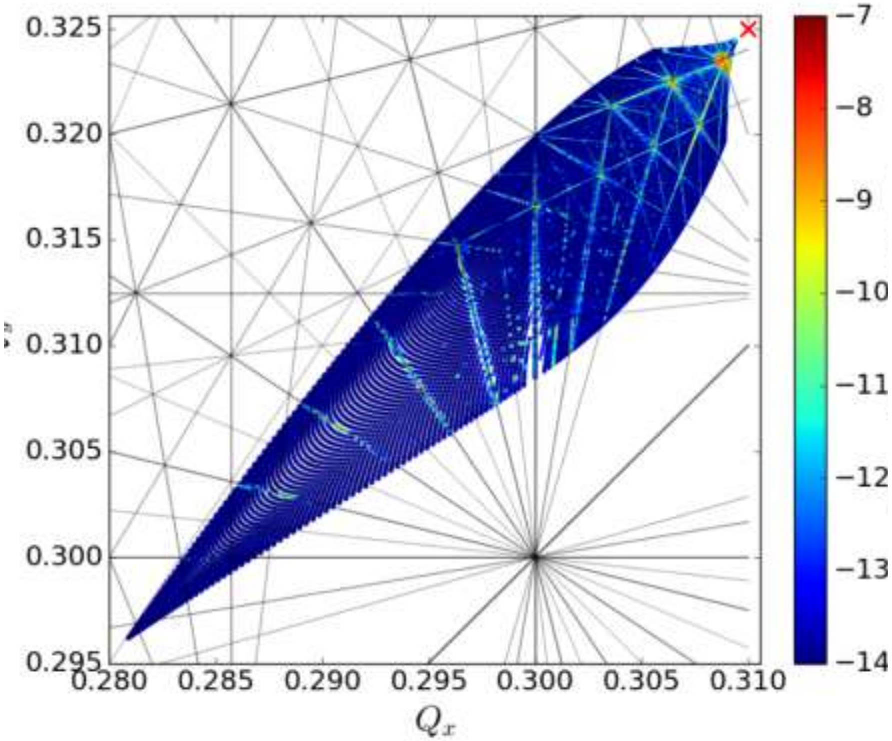
BAD BEAM LIFETIME

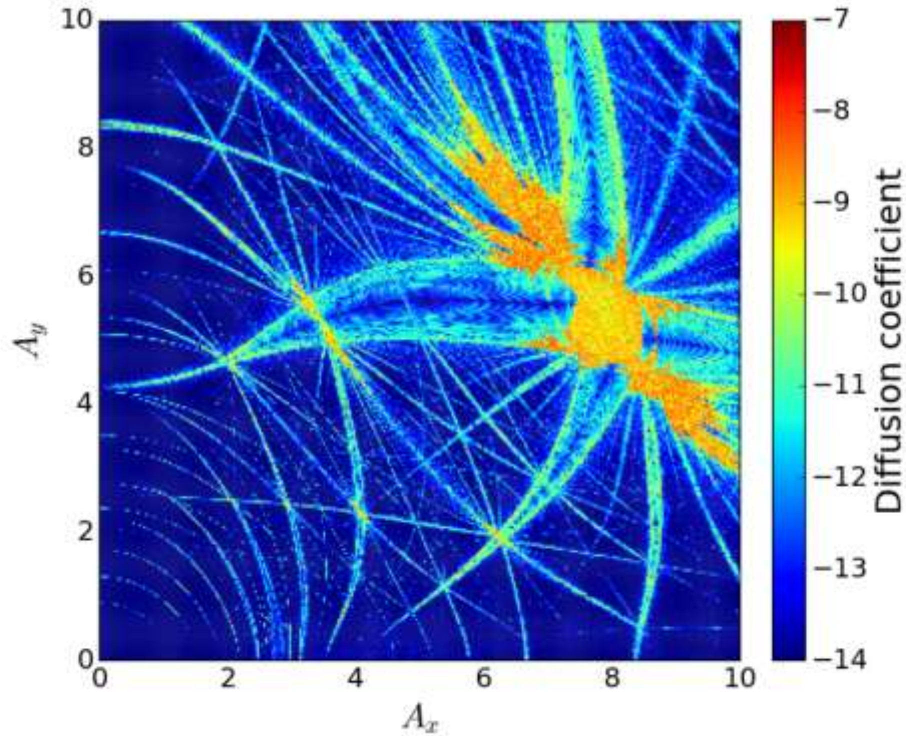
7th order resonances:

Q=4/7=0.571 -

HIGH LOSSES

Resonances matter! … Diffusion





Tune map: LHC (simul)

Shown resonances up to order 20

– $\xi_{tot} = 0.03$, $Q_x = 0.31$, $Q_y = 0.325$
$N_{mp} = 1e5$, 4D BB , $Q' = 0$

– IC: Grid in 2D (x,y)

Amplitude map: LHC (simul)

Shown diffusion rates vs Ax/Ay

$$D_i = \log_{10} \sqrt{\frac{dQ_{x,i}}{d\text{turn}}^2 + \frac{dQ_{y,i}}{d\text{turn}}^2}$$

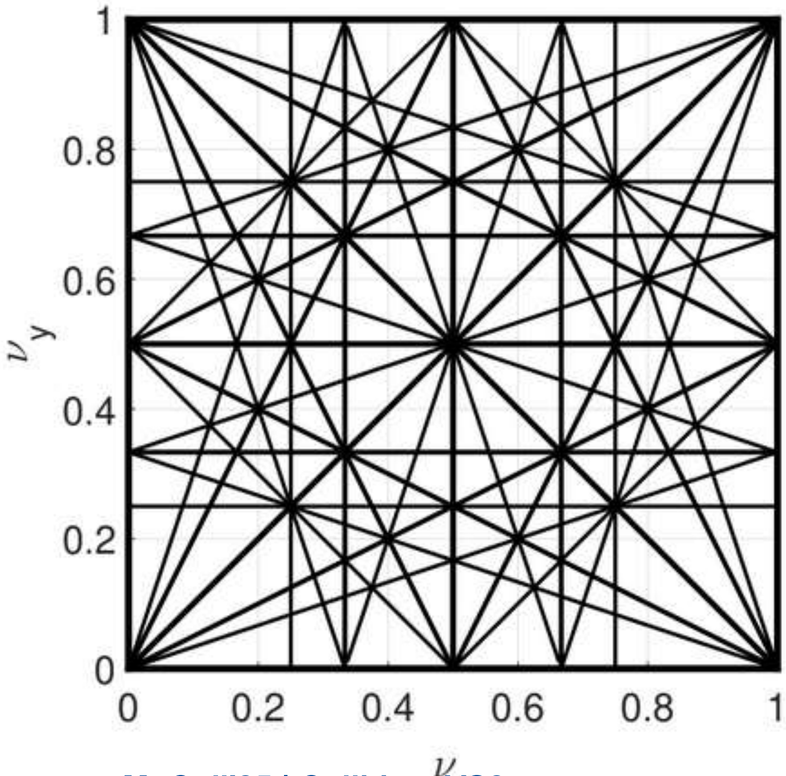
Measure tune of a particle based on (here) 4096 turns -Calculate linear change over 10 measurements, separated by 10k turns

Non-linear Resonances

harmonics of

ω_x ω_y ω_0

- Nonlinear terms in the force $F(x,y,t) \sim x^l \, y^p \, \delta(t-kT)$ lead to appearance of driving terms oscillating with frequencies mQ_x+nQ_y, and therefore open opportunities for nonlinear resonances if



$$mQ_x+nQ_y=p$$

$|m|+|n|$ *is order of the resonance*

i.e. resonance diagram up to fourth order; importance of the resonance depends on the force shape and order (low order = more serious; often longitudinal deviations matter if $mQ_x+nQ_y+lQ_s=p$

How to control beam-beam effects?

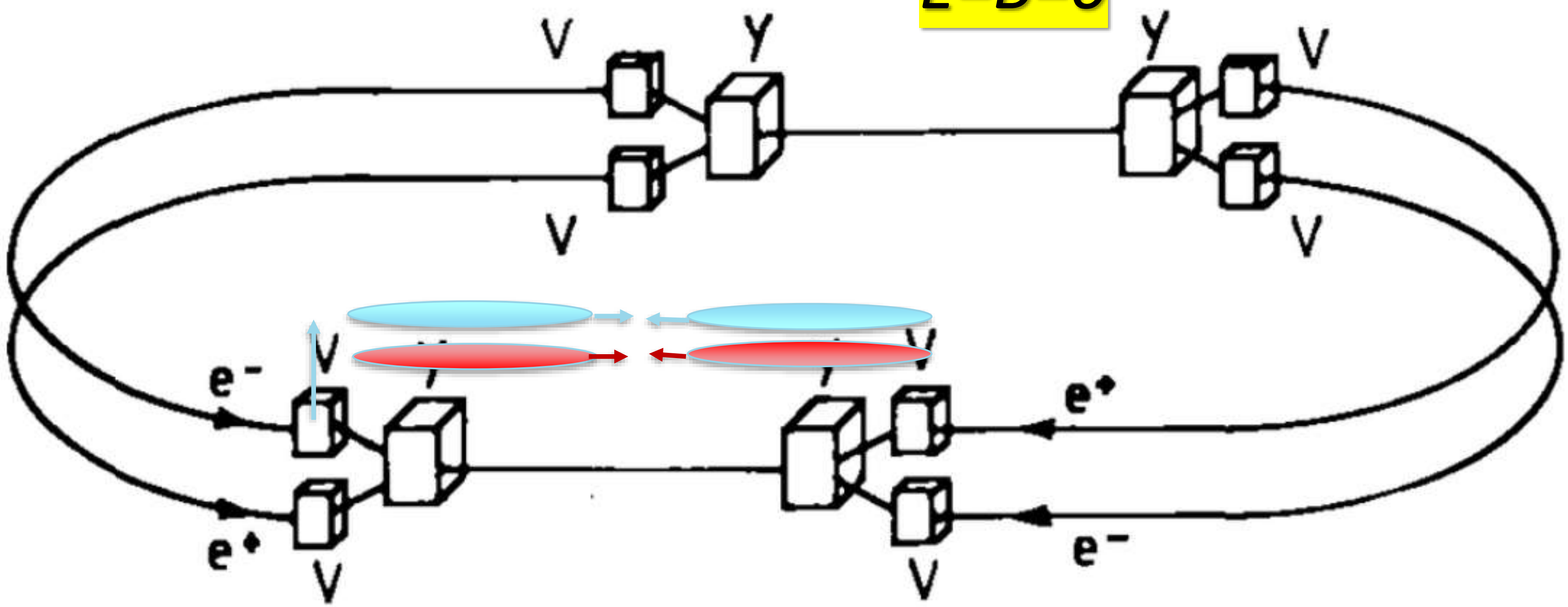
- Find 'lenses' to correct beam-beam effects
- Head on effects: force is *~r*
  - Linear "electron lens" to shift tunes
  - Non-linear "electron lens" to reduce spread
  - Successful e-lenses at FNAL and RHIC
- Long range effects:
  - At very large distance: force is ~*1/r*
  - Same force as a wire !
- Overall - success with active compensation

Attempt #1: Four beams e-e+ e-e+

four-beam collider *Dispositif de Collisions dans l'Igloo* (DCI, 1970s) at Orsay with two 0.8 GeV electron beams and two positron beams of the same energy, all meeting at the same interaction point *(J.LeDuff et al)*

Q1+Q2+Q3+Q4=0 *J1+J2+J3+J4=0*

E=B=0



Attempt #1: Four beams compensation

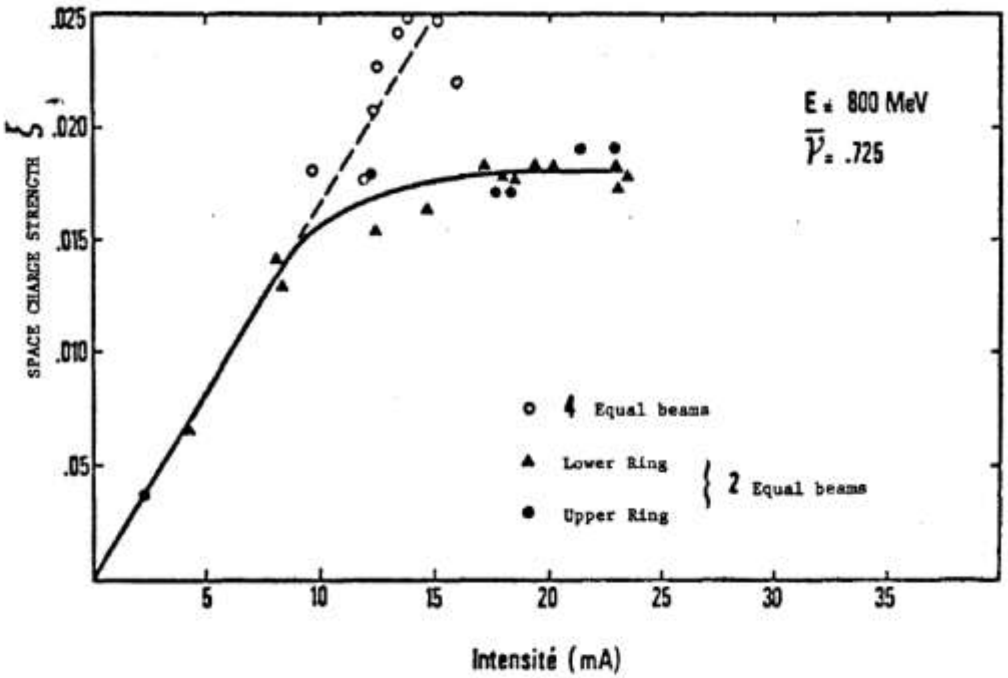


Fig. 12. Comparison between four beams and two beams: ξ versus current at E = 800 MeV and $\bar{\nu}$ = .725

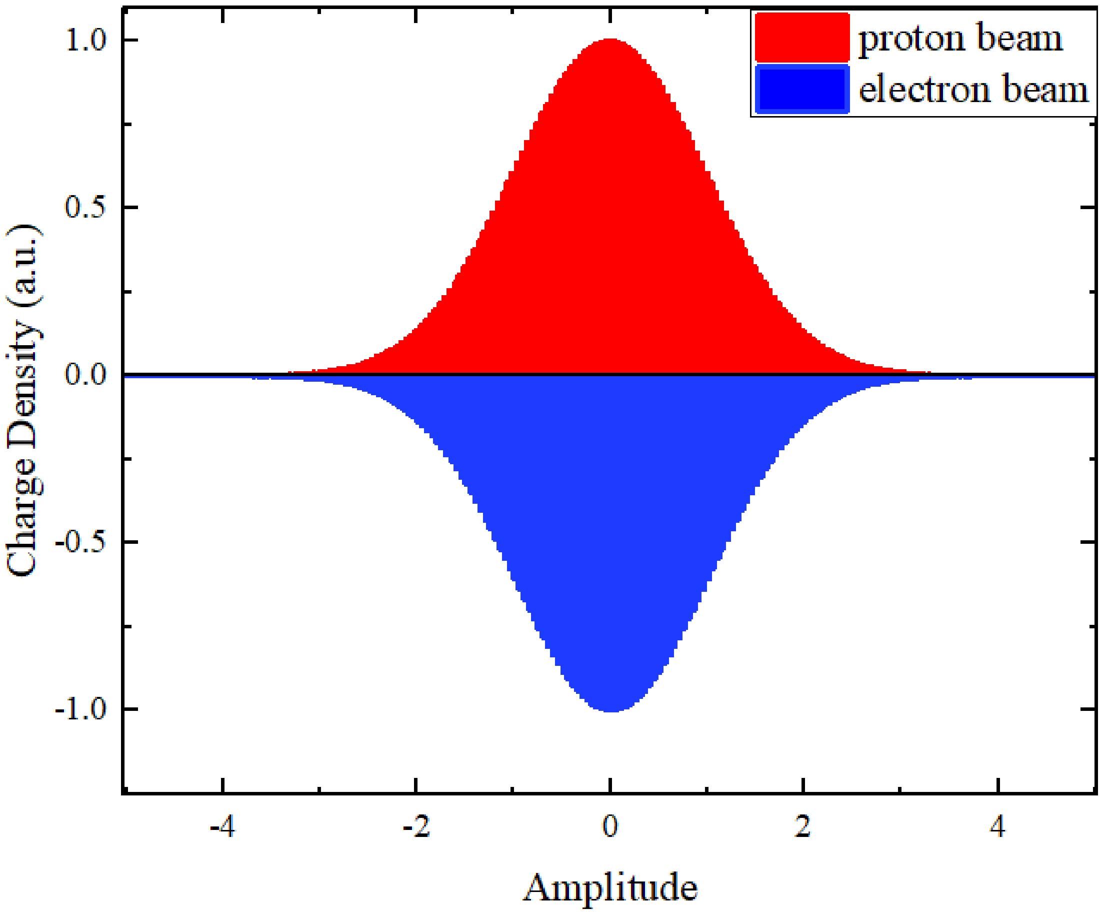
No improvement of performance was obtained in the four-beam configuration compared to collisions of just two beams of electrons and positrons.

A transverse dipole feedback as well as a detuning of the two rings did not help.

The compensation is believed to be unsuccessful due to the loss of beam stability, both for dipole and higher order modes of coherent motion.

Approach #2: Electron lens

e- profile same as ***p+*** $N_e = N_{IP} N_p / (1 + \beta_e)$.



Protons focus pbars +
Electrons defocus

Net effect = zero

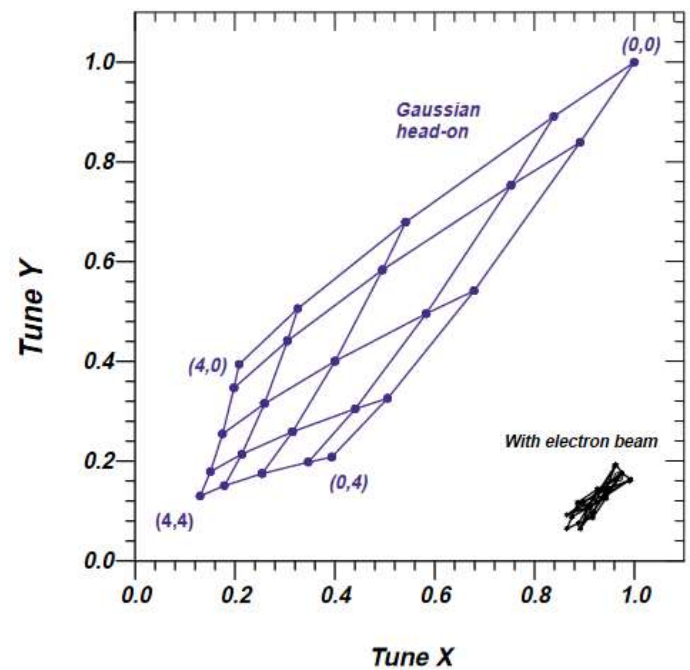
Footprint compressed

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 2, 071001 (1999)

Considerations on compensation of beam-beam effects in the Tevatron with electron beams

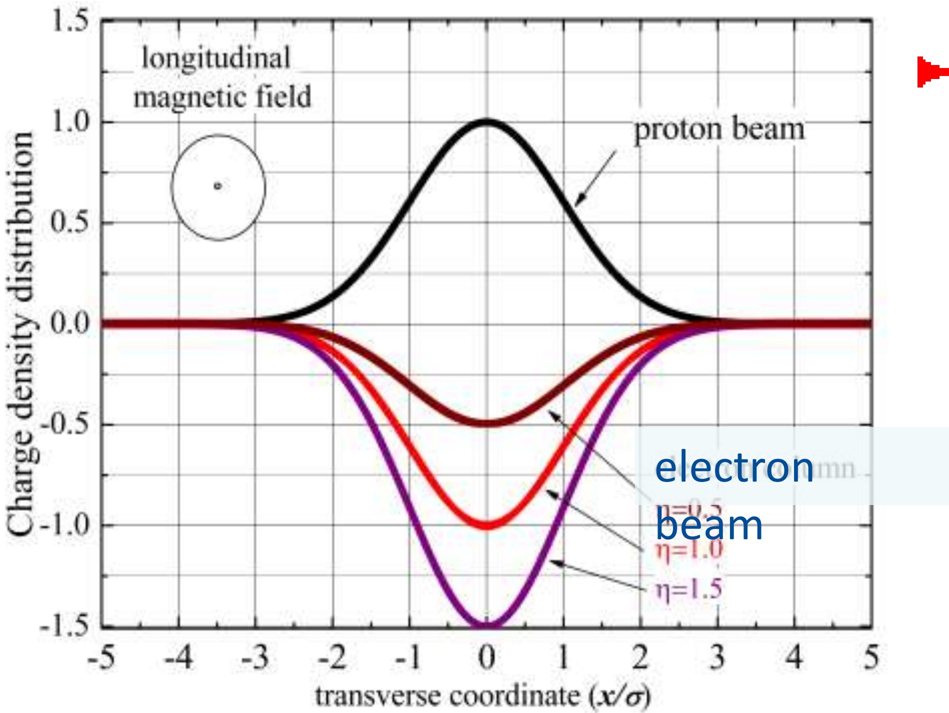
V. Shiltsev, V. Danilov,* D. Finley, and A. Sery†
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 8 October 1998; published 28 July 1999)

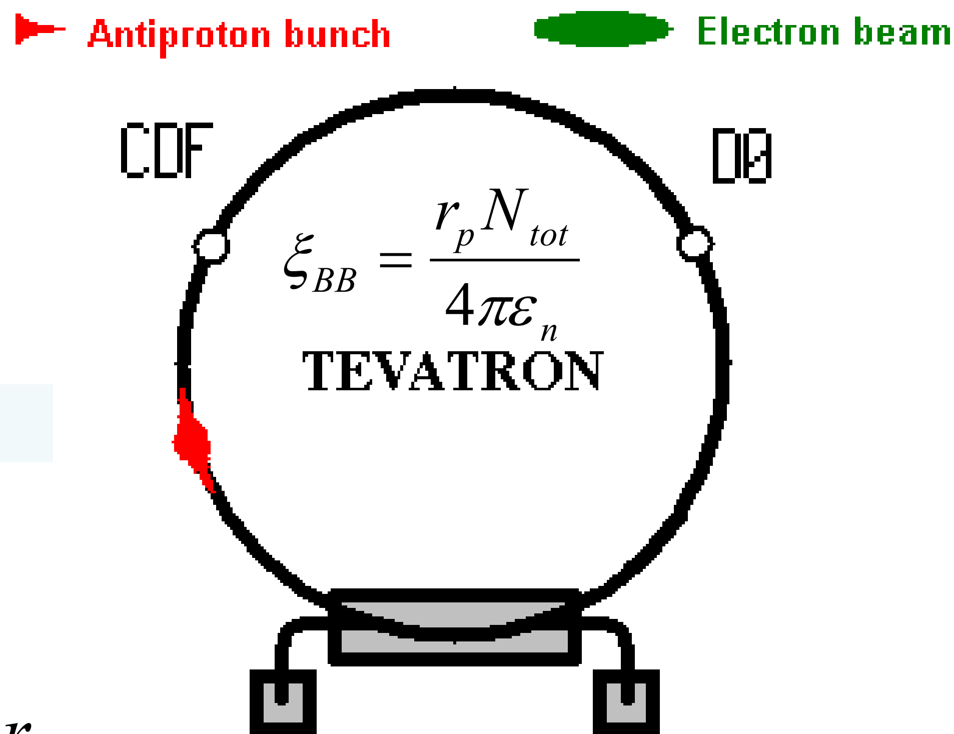
The beam-beam interaction in the Tevatron collider sets limits on bunch intensity and luminosity. These limits are caused by a tune spread in each bunch which is mostly due to head-on collisions, but there is also a bunch-to-bunch tune spread due to parasitic collisions in multibunch operation. We propose to compensate these effects with the use of a countertraveling electron beam, and we present general considerations and physics limitations of this technique.



(V.Shiltsev et al)

Electron Lens Compensation





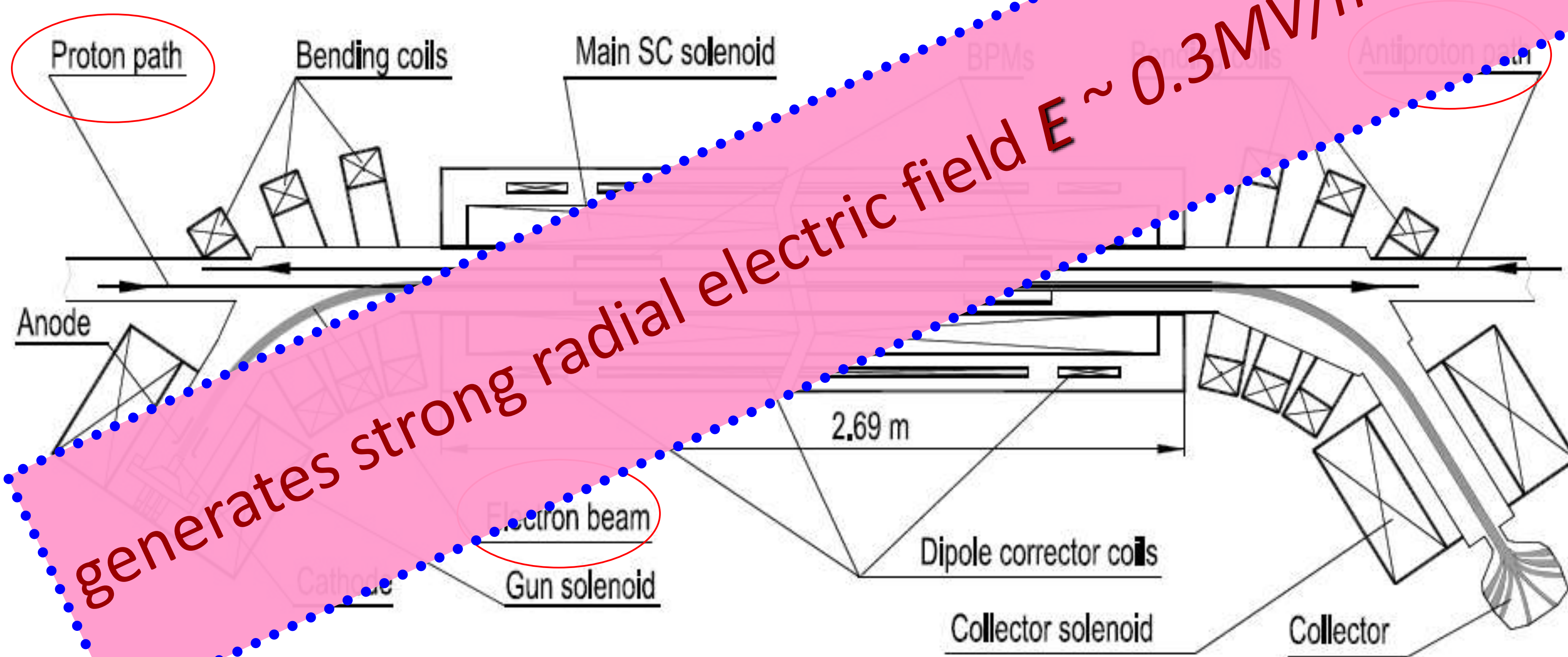
$$dQ_{x,y} = \mp \frac{\beta_{x,y}}{2\pi} \cdot \frac{1 \pm \beta_e}{\beta_e} \cdot \frac{J_e \cdot L_e \cdot r_p}{e \cdot c \cdot a_e^2 \cdot \gamma_p}$$

"…to compensate (in average) space charge forces of positively charged protons acting on antiprotons in the Tevatron by interaction with a negative charge of a low energy high-current electron beam "
(V.Shiltsev, 1997)

Some Facts on Electron Lenses

~4 mm dia 2 m long very straight beam of ~10kV

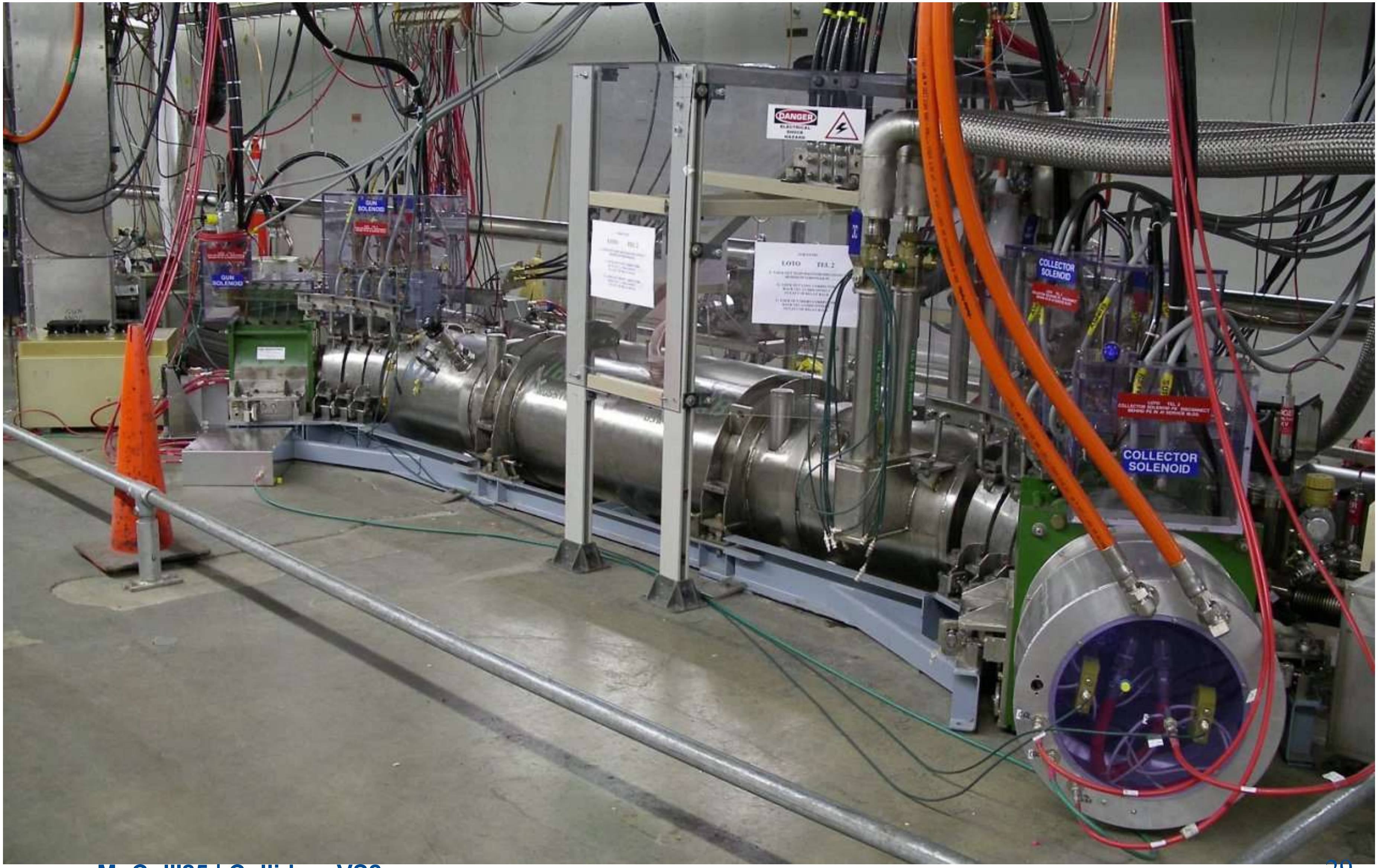
~1A electrons (~10^{12}) immersed in 3T solenoid



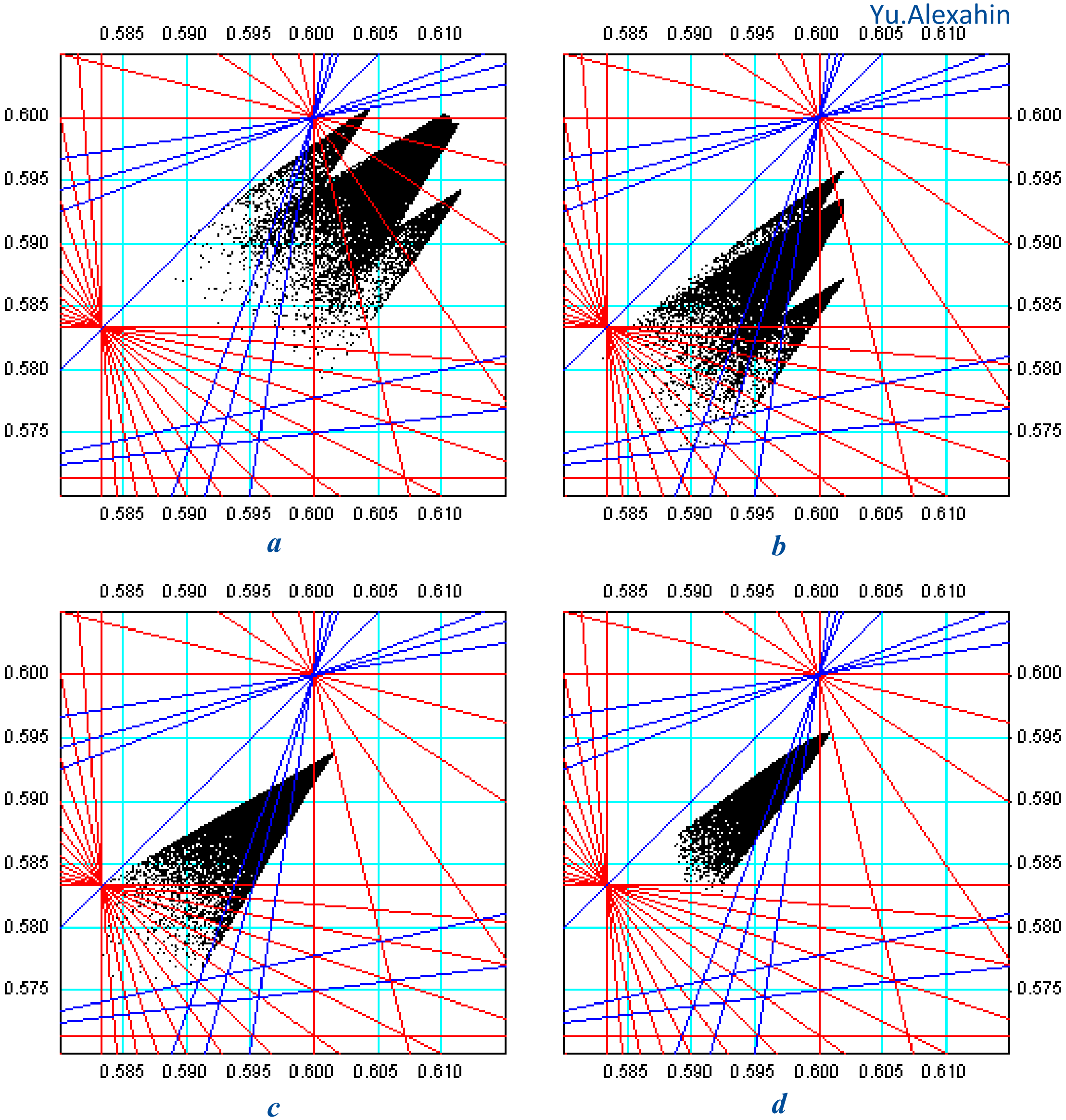
generates strong radial electric field $E \sim 0.3$MV/m

Tevatron Electron Lens #1 (F48)

TEL2 in the Tevatron Tunnel (A11)



Compensation with Two TELs



- Tev Run II: 36x36 bunches in 3 trains
- compensate beam-beam tune shifts
  - a) Run II Goal
  - b) one TEL
  - c) two TELs
  - d) 2 nonlinear TELs
- requires
  - 1-3A electron current
  - stability dJ/J<0.1%
  - e-pbar centering
  - e-beam shaping

Electron Charge Distribution

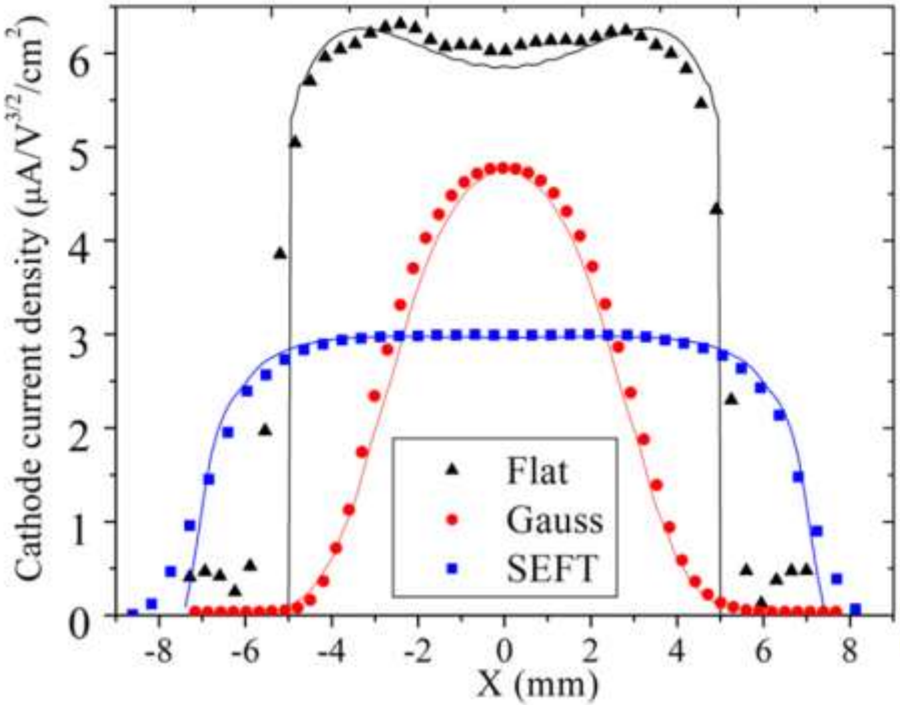


Figure 2. Three profiles of the electron current density at the electron gun cathode: black, flattop profile; red, Gaussian profile; blue, SEFT profile. Symbols represent the measured data and the solid lines are simulation results. All data refer to an anode–cathode voltage of 10 kV.

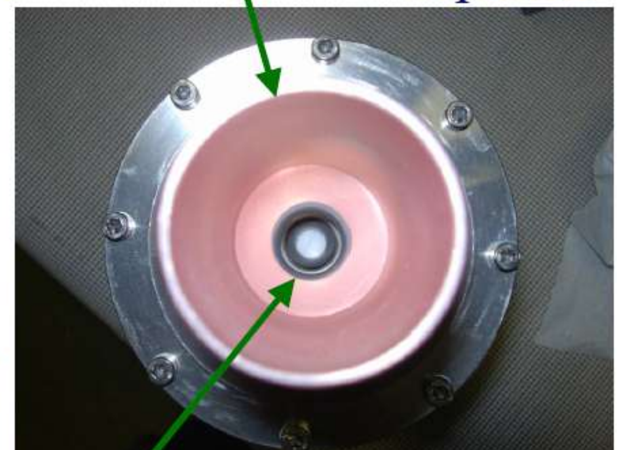
Shiltsev et al., *PRL* 99, 244801 (2007).
Shiltsev et al., NJP 10, 043042 (2008).

Electron gun

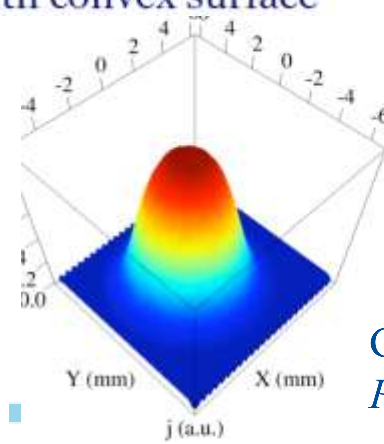
Copper anode

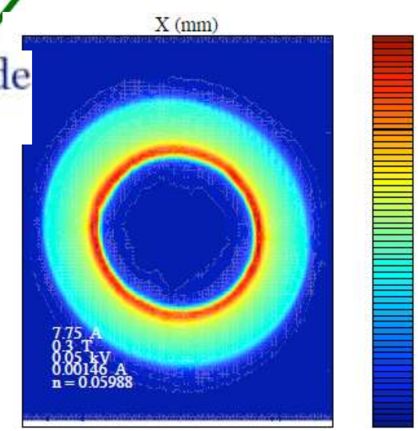
side view

top view



Tungsten dispenser cathode with convex surface

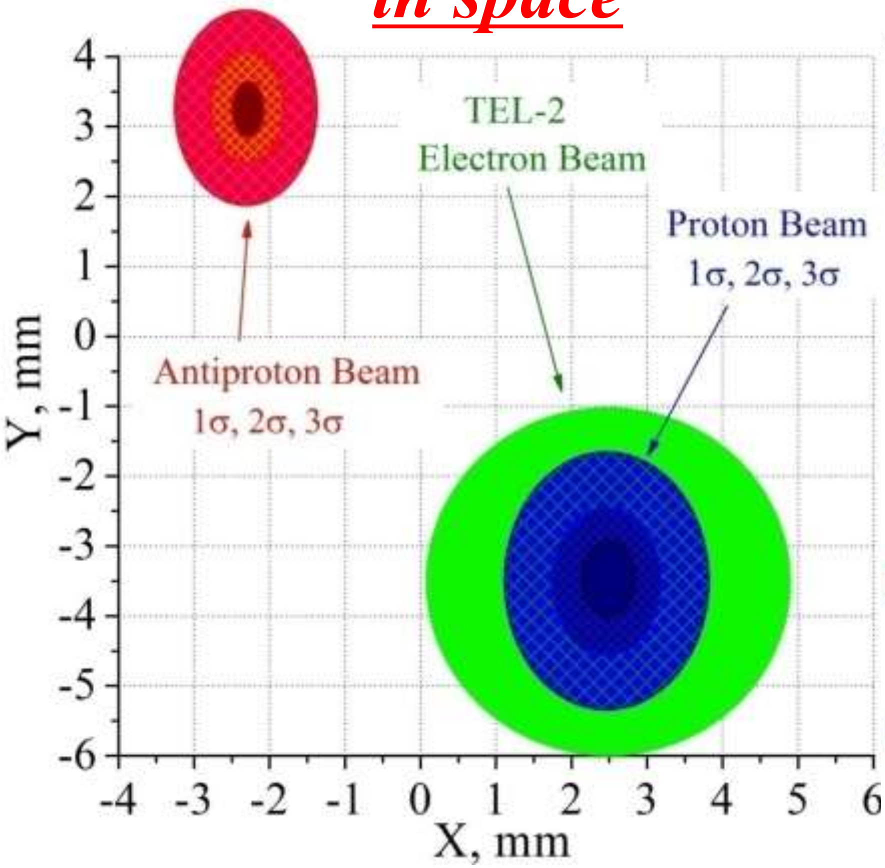


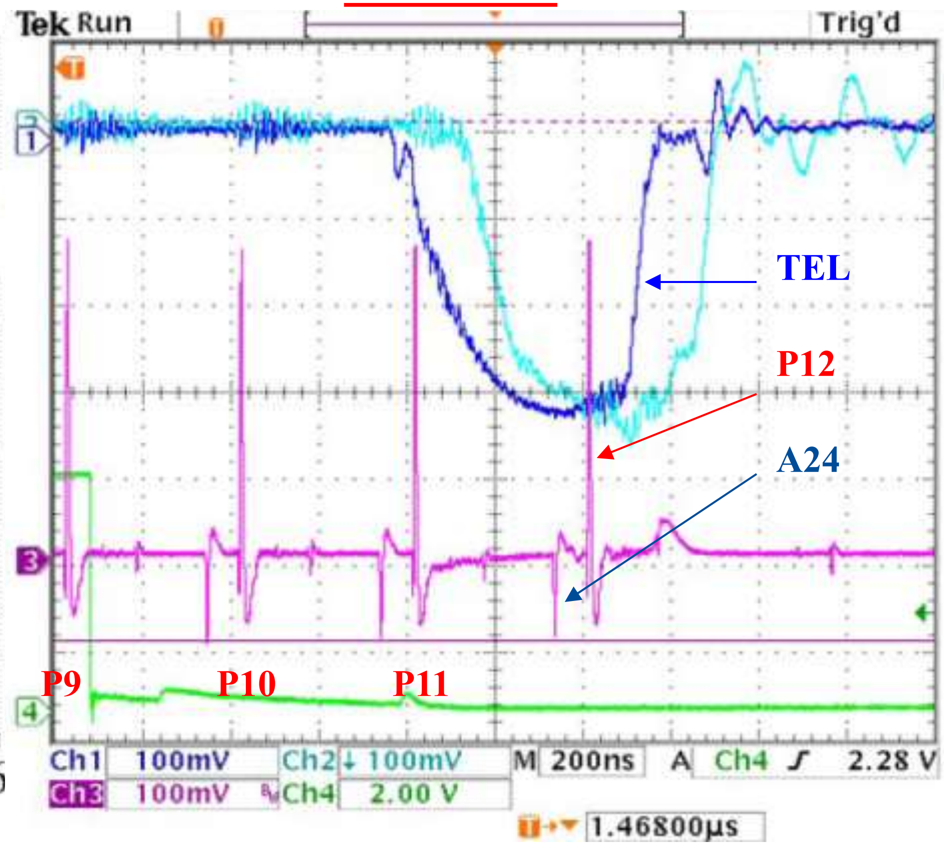


G. *Stancari*, et al., (2011)
Phys. Rev. Lett. 107, 084802

TEL e-beam aligned and timed on protons

in space / ***in time***



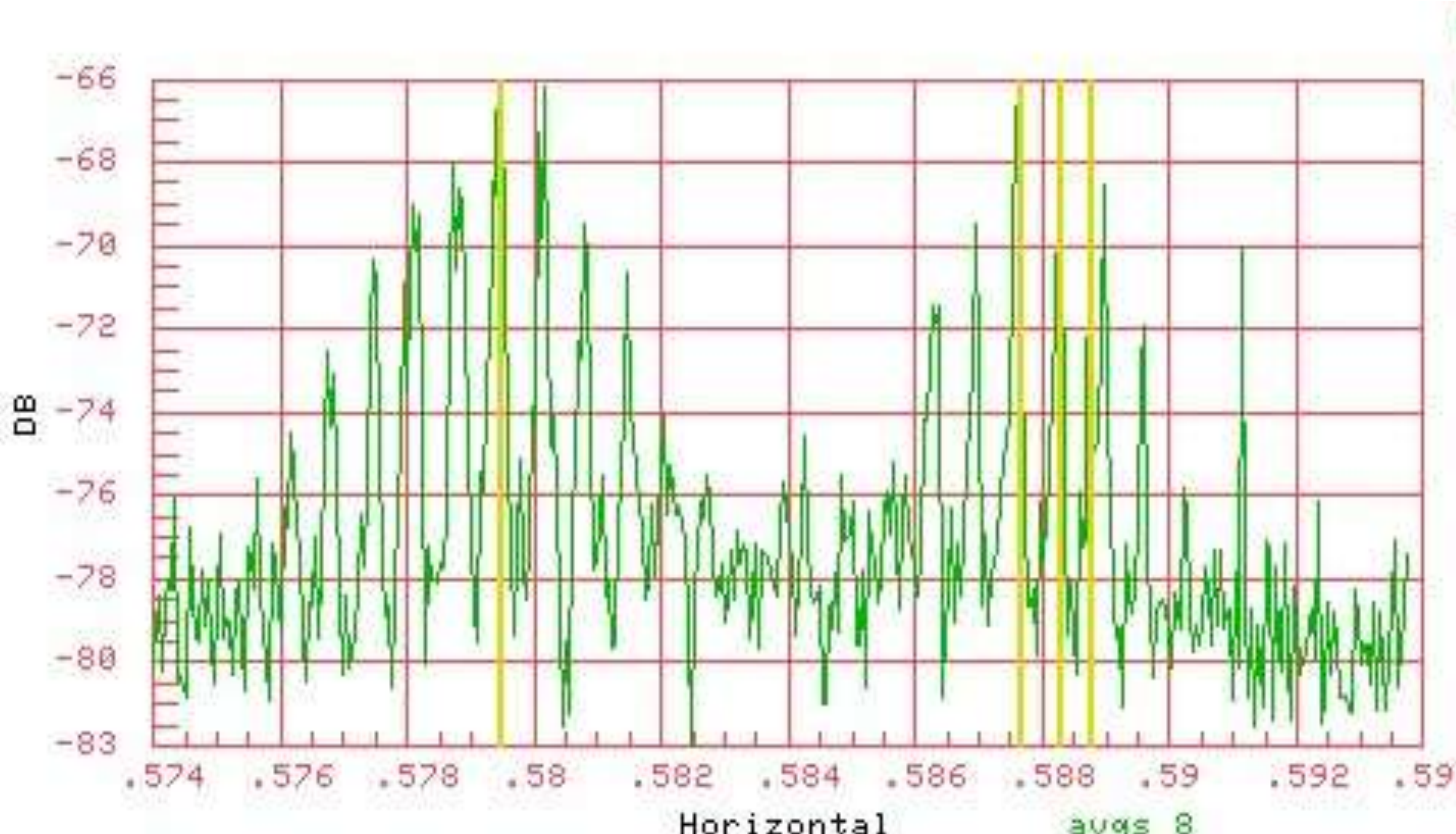


Transverse e-p alignment **is very important for minimization of noise effects and optimization of positive effects due to e-beam.** ***Timing*** **is important to keep protons on flat top of e-pulse – to minimize noise and maximize tune shift.**

Tevatron Electron Lenses (2001-2011)

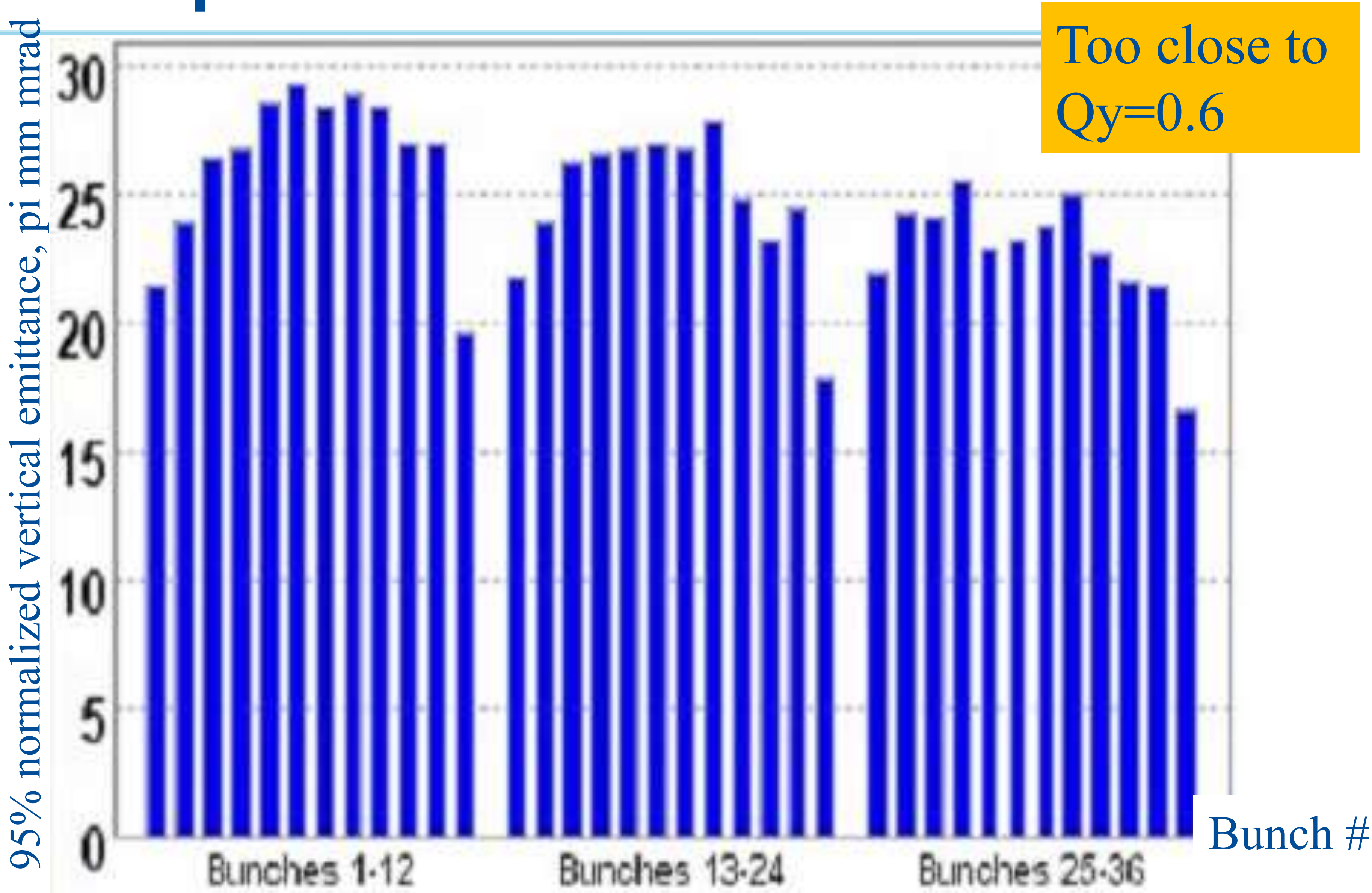
- Technology proven, tune shift ~0.01 demo'd
- First successful active compensation
- *Head on effects compensation:*
  - Reduced emittance growth of a PACMAN antiproton bunch ("scallops" effect)
- *Long range effects compensation:*
  - Significant (x2) improvement of the lifetime of most affected proton bunches
    - By shifting tunes of otherwise unfavorable bunch away from resonances

Tuneshift dQ_{hor}=+0.009 by TEL

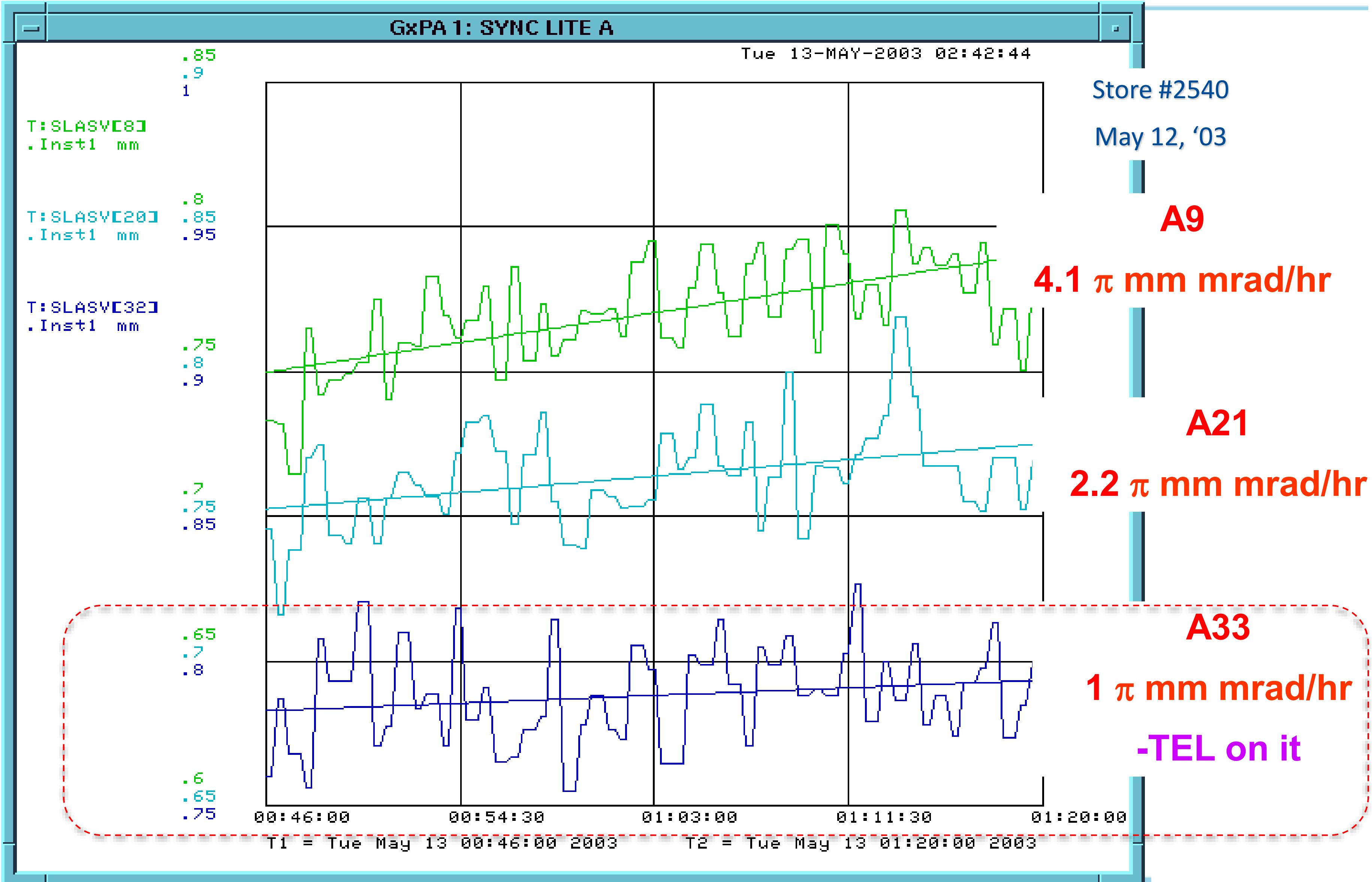


Three bunches in the Tevatron, the TEL acts on one of them

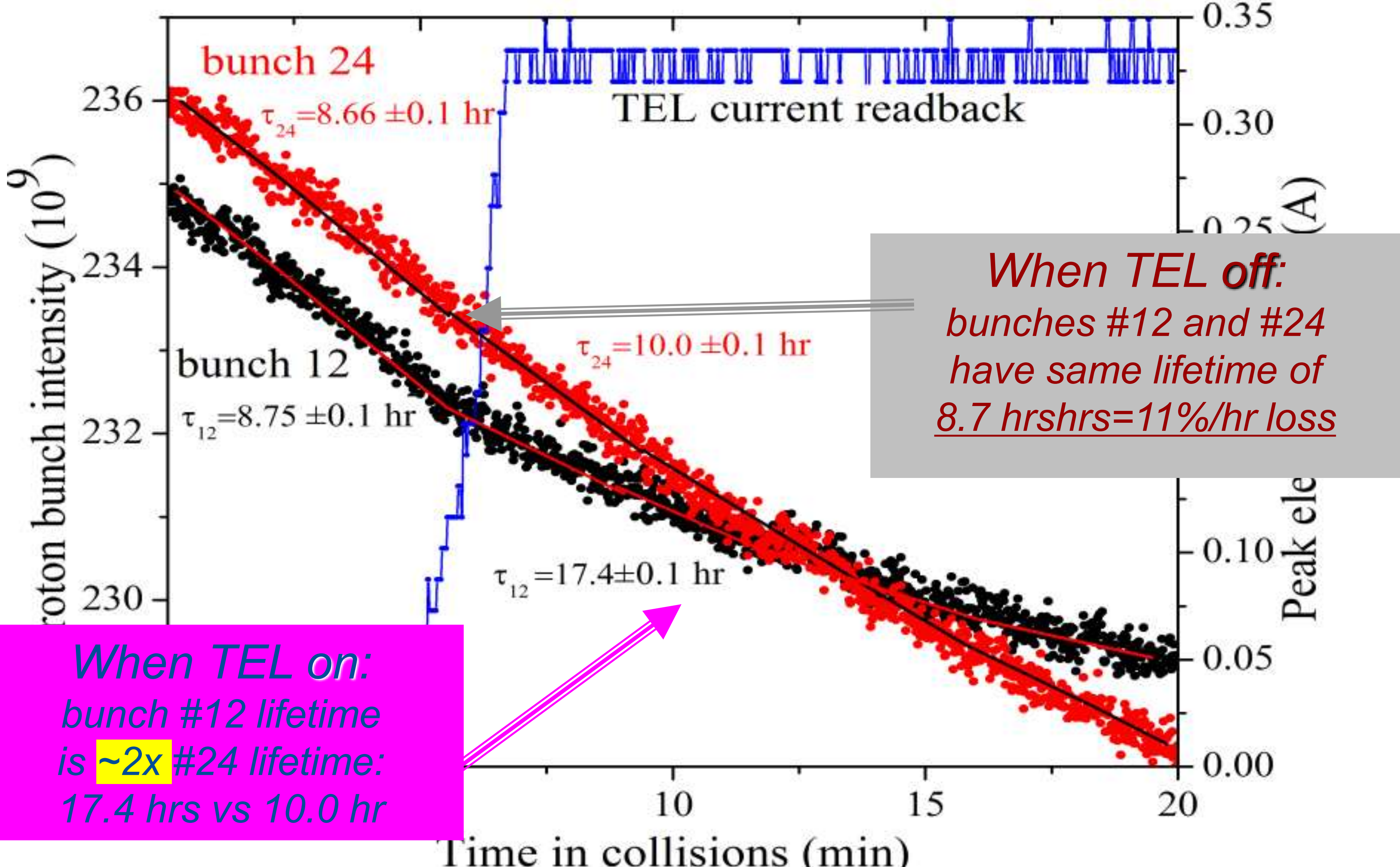
“Scallops” in Pbar Bunch Emittances



ittance Growth of A33 Suppresed by TEL



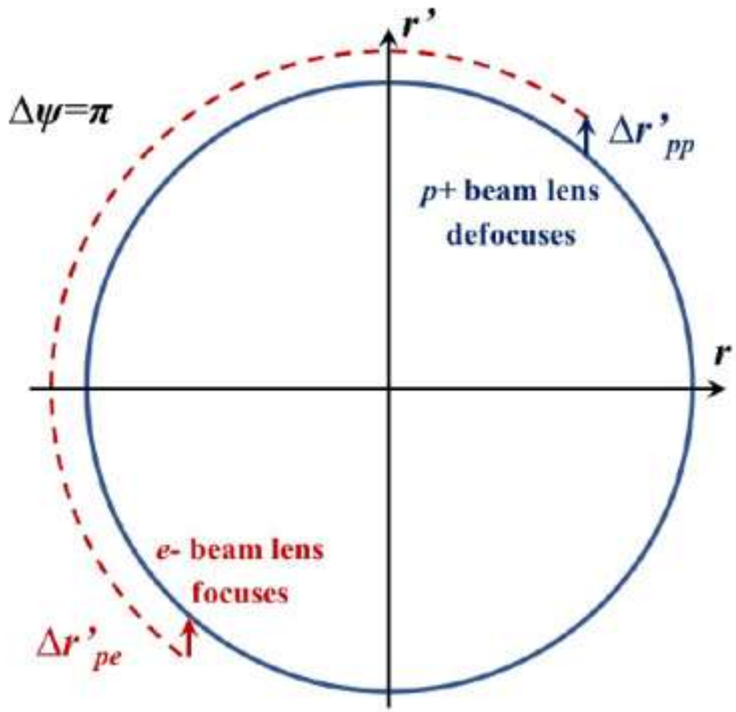
TEL2 on One Proton Bunch P12



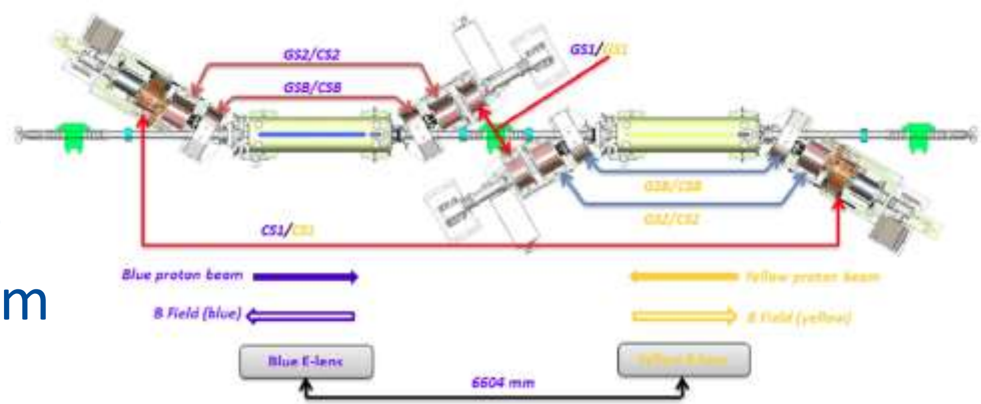
When TEL off:
bunches #12 and #24
have same lifetime of
8.7 hrshrs=11%/hr loss

When TEL on:
bunch #12 lifetime
is ~2x #24 lifetime:
17.4 hrs vs 10.0 hr

Approach #3: Head-On Comp'n in RHIC



(W.Fischer et al)



With e-lens, one can compensate Head-On effect: not only the tune footprint, but also the *resonant driving terms* if elens is placed 180 degrees (betatron phase) away from the main IP (one IP compensation)

RHIC pp 2015 elens Success

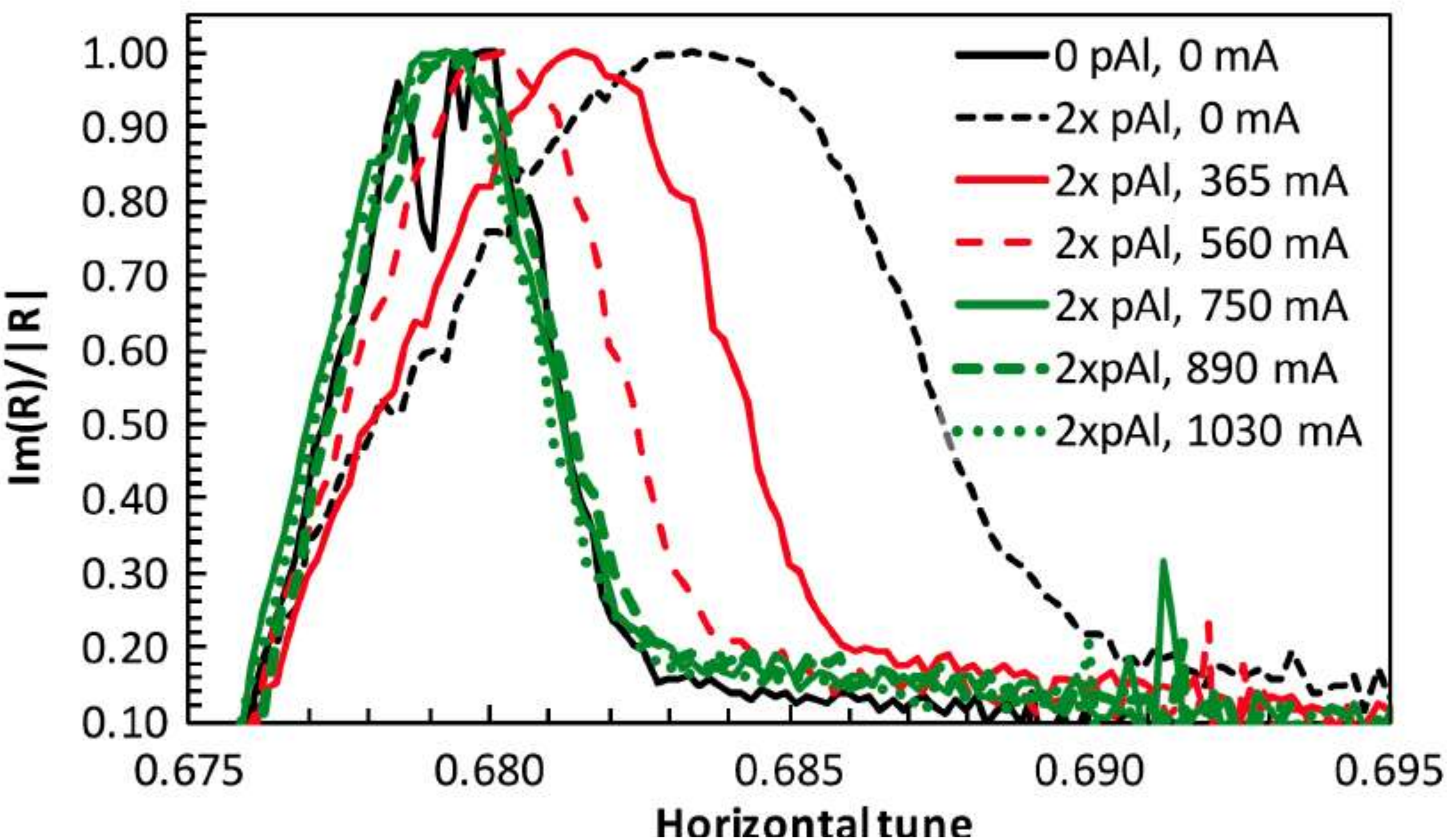


Figure 7: Tune distribution width reduction with the RHIC electron lens, measured in the proton beam with $p{+}Al$ collisions. The distribution widens due to two beam-beam interactions, and narrows again with increase of the electron lens current to 1.03 A [9].

RHIC pp 2015 elens in Ops

Electron lens parameters

Distance of center from IP	m	1.5	1.5
Effective length L_e	m	2.1	2.1
Kinetic energy E_e	kV	5	5
Relativistic factor β_e		0.14	0.14
Relativistic factor γ_e		1.0002	1.0002
Current I_e	A	1.0	0.43/0.60
Electron beam size at interaction	μm	350	650
Linear tune shift		0.0147	0.01

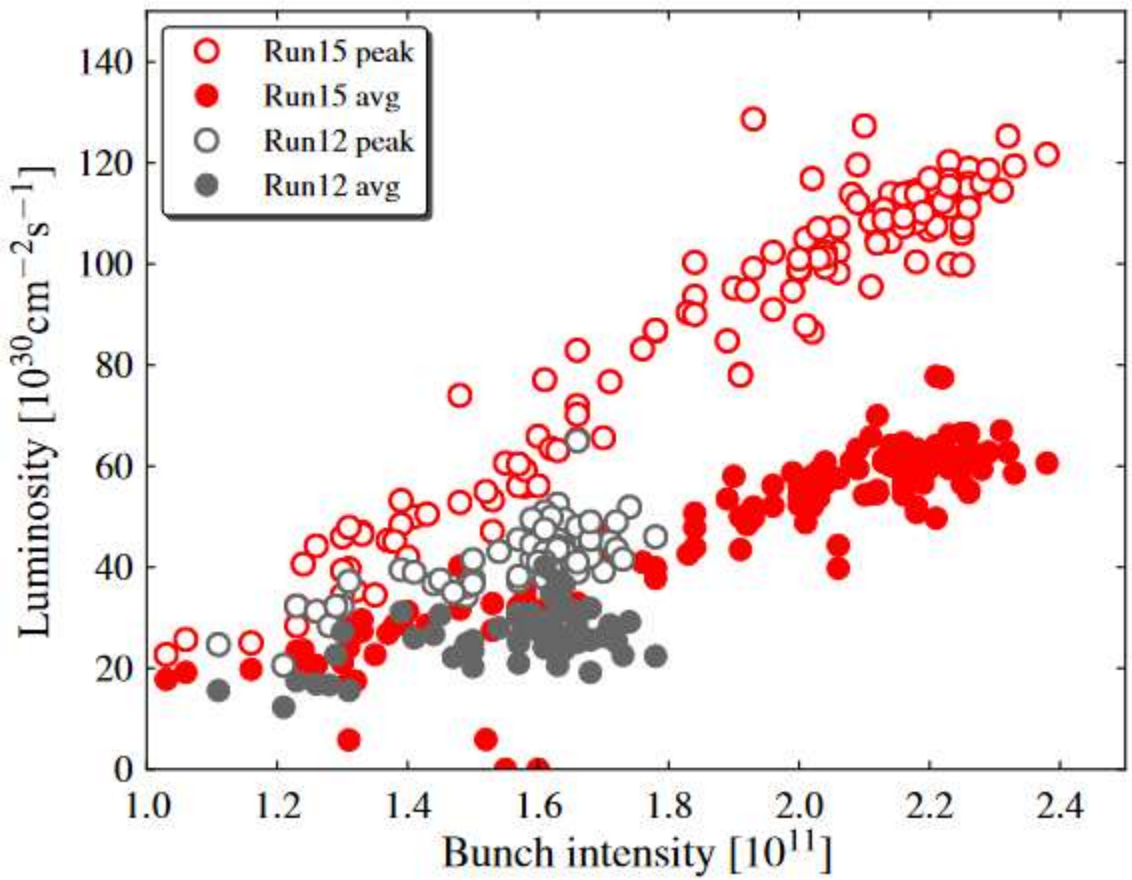


FIG. 3. Peak and average store luminosity in polarized proton operation at 100 GeV beam energy in 2012 and 2015.

With 0.6A, 2.1m long, 5 kV e-beam, essentially:

- one out of 2 IP head-on effect cancelled,
  - max allowed beam intensity increased by ~40%,
  - peak average lumi ~tripled, averaged lumi ~ doubled

Approach #4 : Wire Compensation of Long Range Beam-Beam Interactions

Fields of separated p+ beam:

$E \sim N_{IPs}\, N_p / d$

$B = E$

Field of separated conductor (wire):

$E = 0$

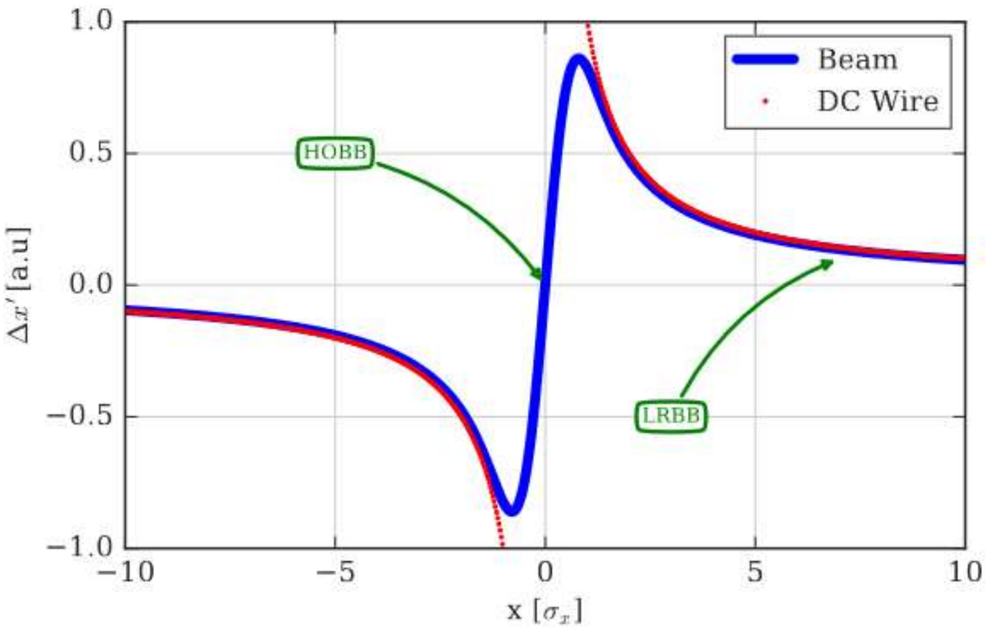
$B \sim 2J_e / d$

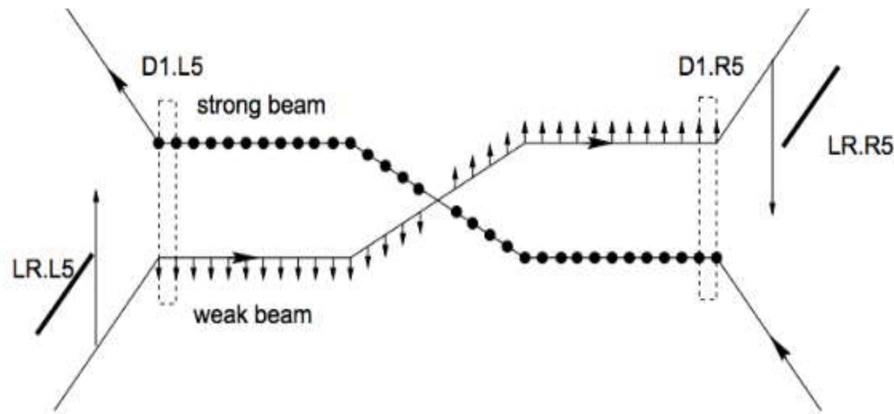
Combined effects of p+ beam + e- beam will cancel out if

wire is placed at the same d

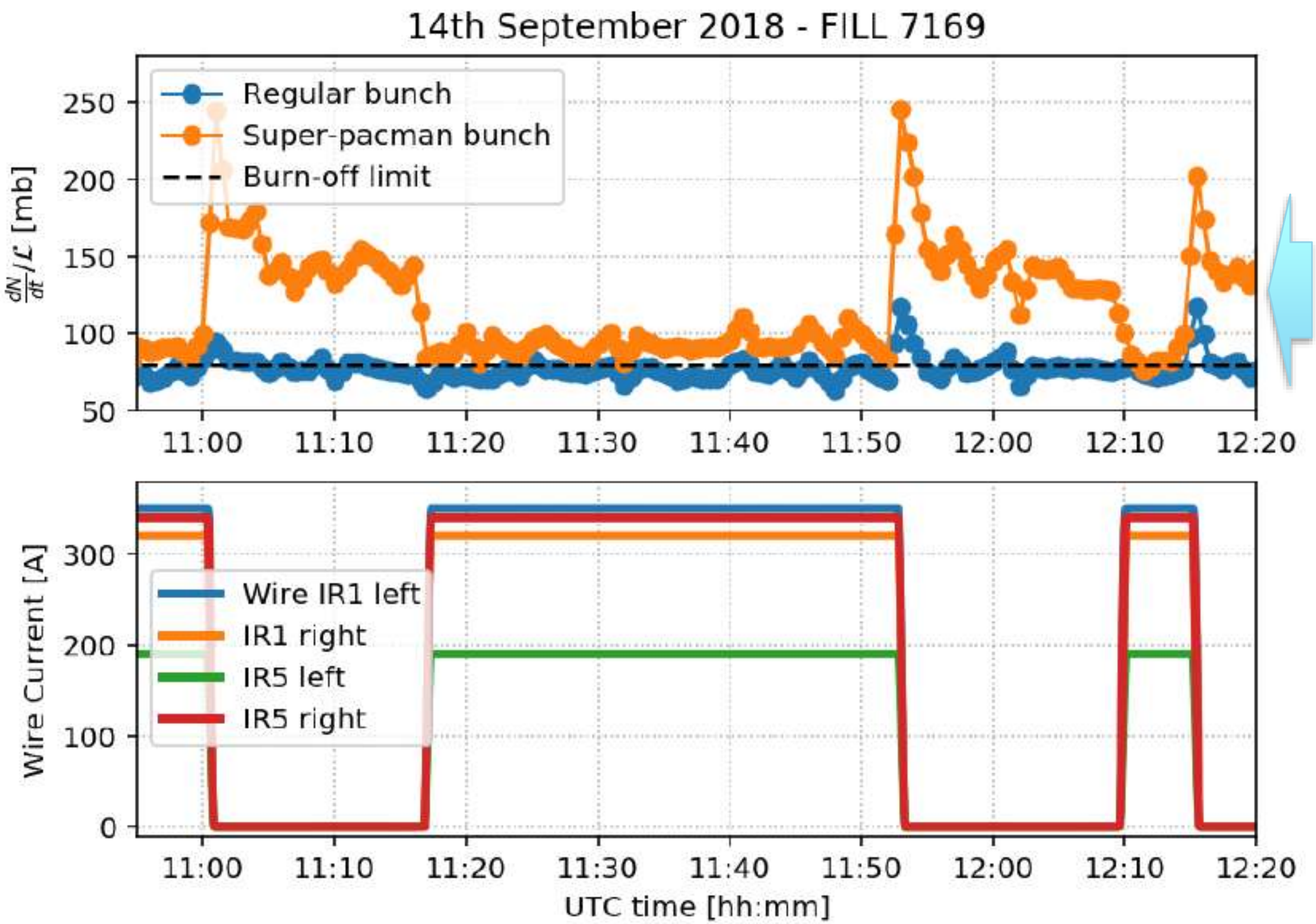
wire kick J×length matches $N_{IPs} N_p$

(J.P.Koutchouk, G.Sterbini et al)





Wire Compensation in the LHC (2018)



Proton losses in collisons are due to: Luminosity burn up *dN/dt=-L*x80 mbarn

and beam-beam effects - different for regular and PACMAN bunches

So, plotted is *dN/dt/Lumi* for regular and PACMAN bunches

Beam-beam topics also include…

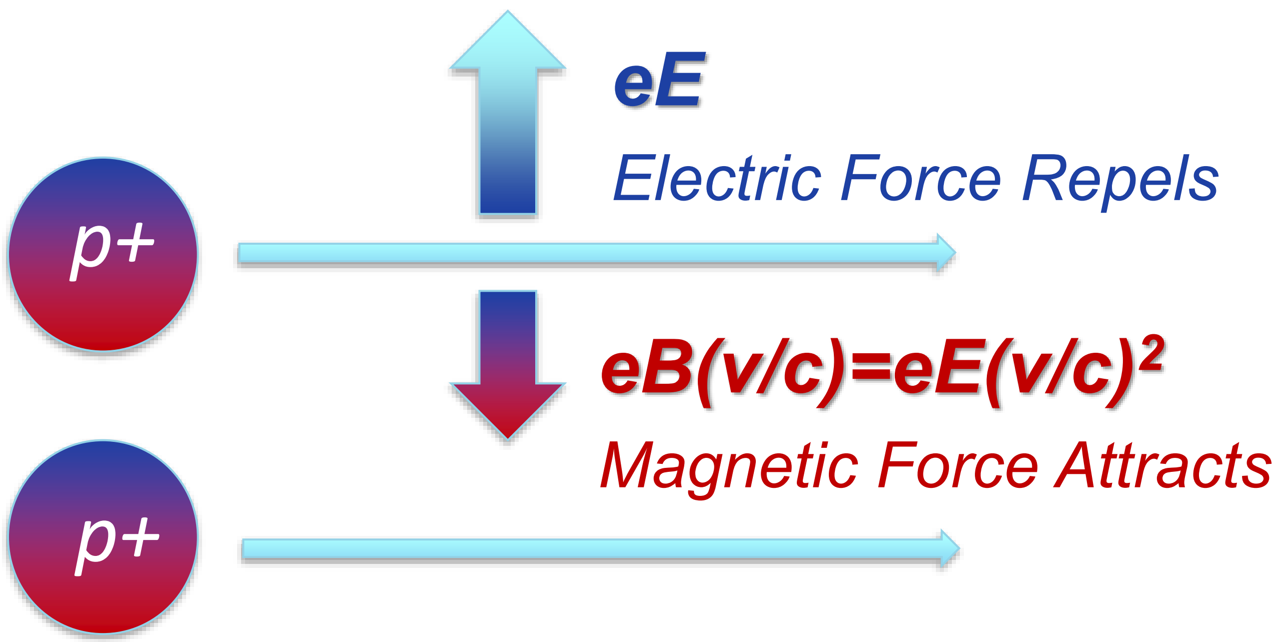
- Beam-beam effects in linear colliders
- Beamstrahlung
- Asymmetric beams
- Synchrobetatron coupling
- Crabbed and crab-waist schemes
- Monochromatization
- Beam-beam simulation codes

... etc.

BREAK (!...?)

SPACE-CHARGE EFFECTS

Intense Beams : Forces and Losses (1)

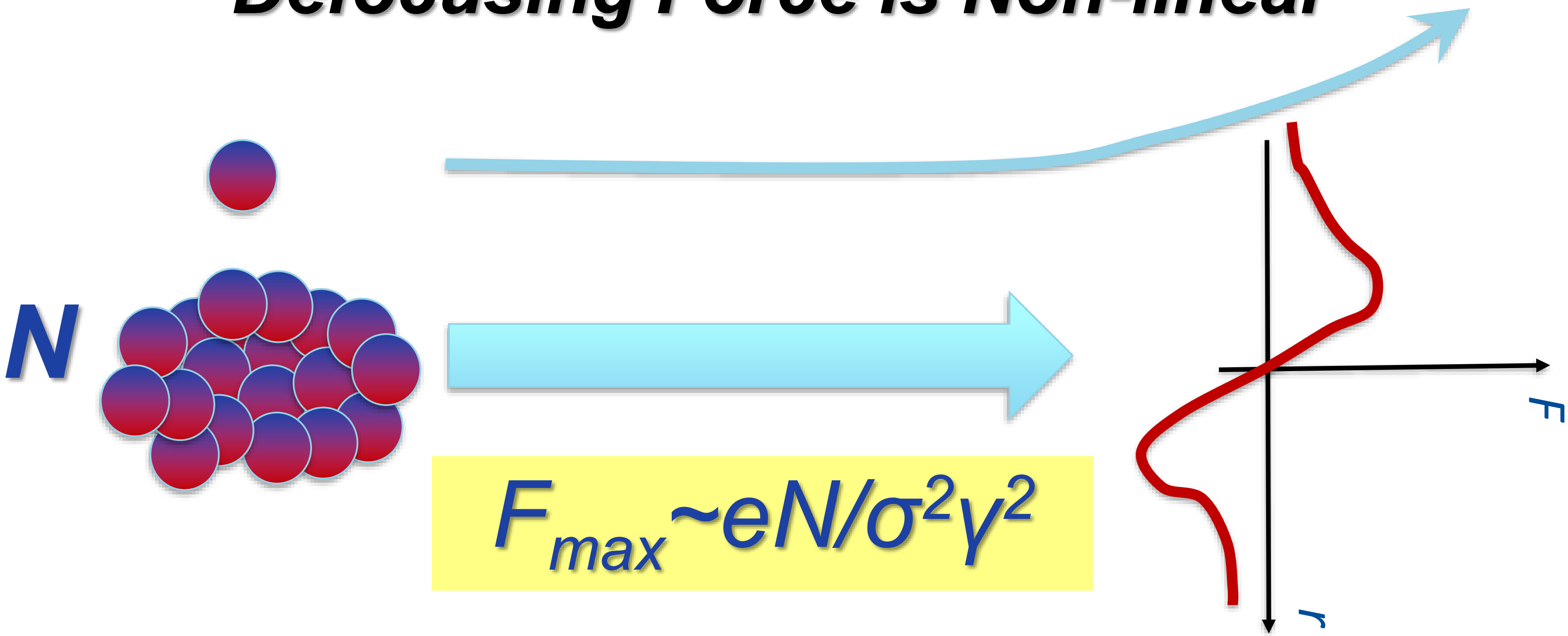


Net Force: Repels

$$eE - eE(v/c)^2 = eE\,(1-\beta^2) = eE/\gamma^2$$

Intense Beams : Forces and Losses (2)

Defocusing Force is Non-linear



$$F_{max} \sim eN/\sigma^2\gamma^2$$

Space-charge effects (emittance growth, losses):

a) proportional to current (N)

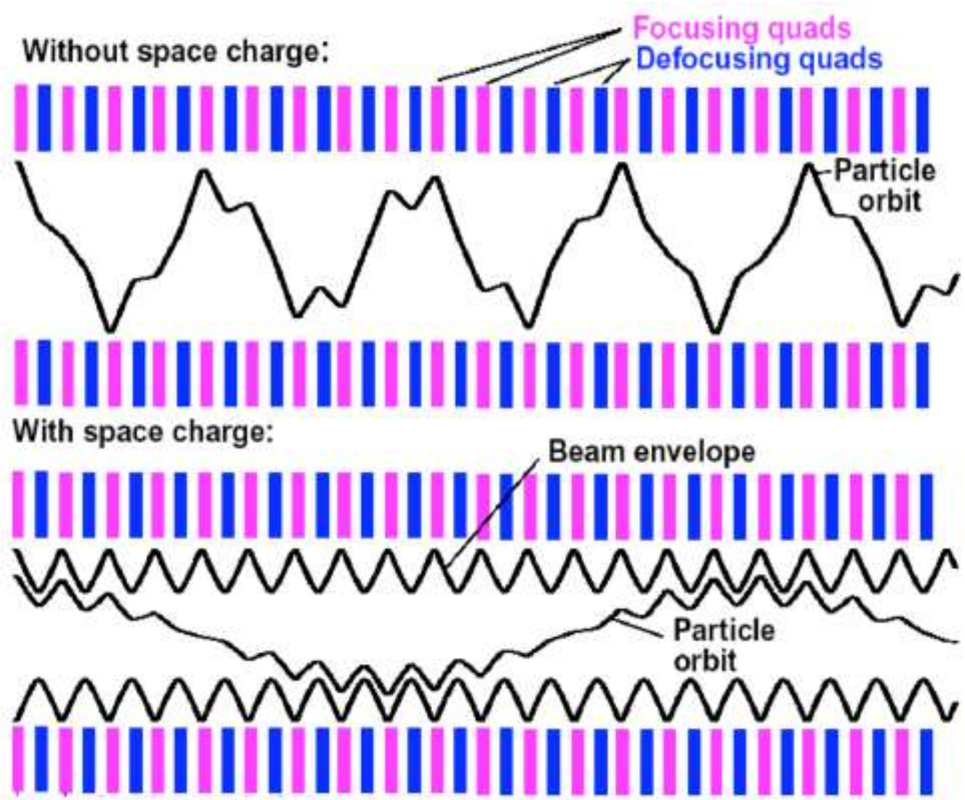
b) scale inversely with beam size (σ)

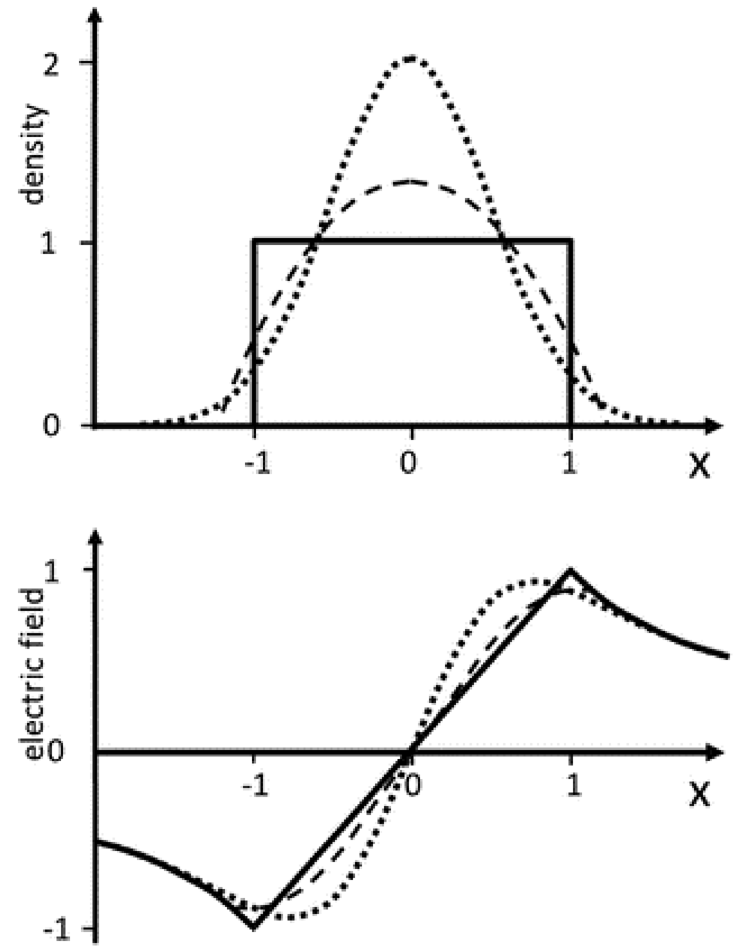
c) scale with time at low energies (γ)

Linacs 5-20 MeV/m

Rings 0.002-0.01 MeV/m

Space-charge: Core vs Tail



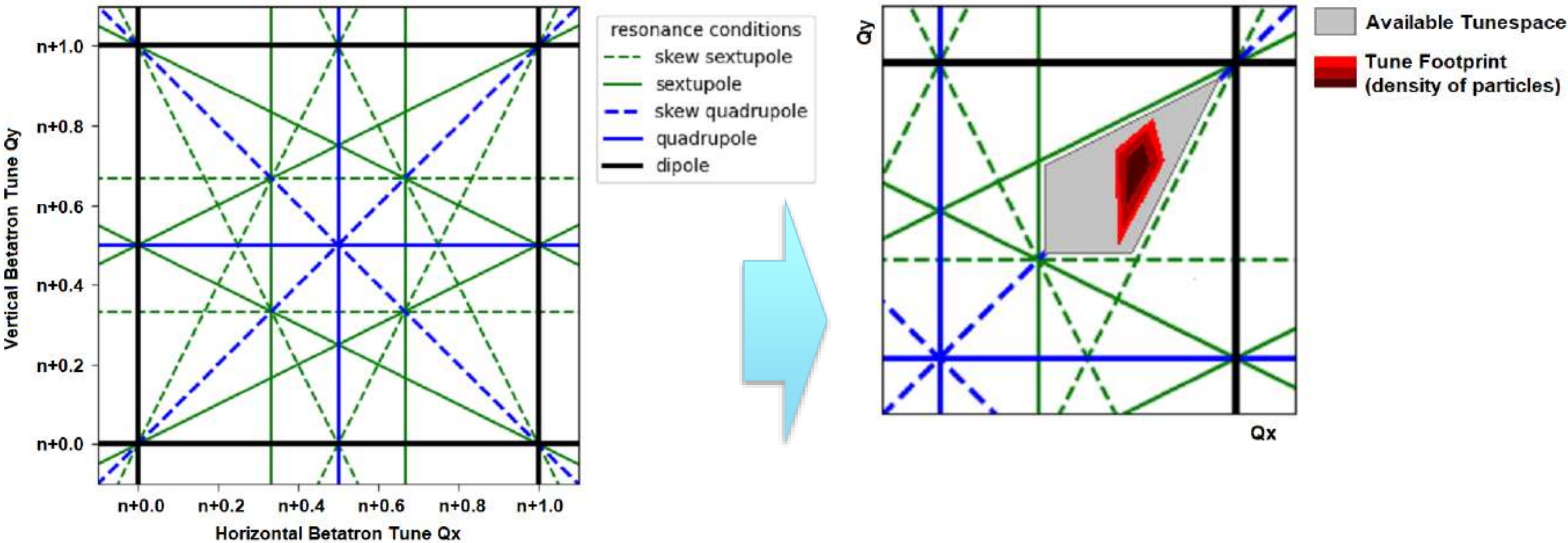


S. Lund

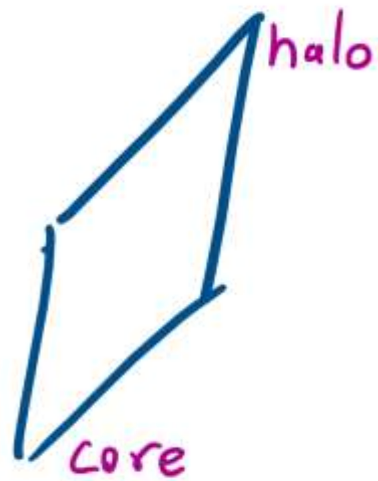
Space-charge effects: Proton Rings

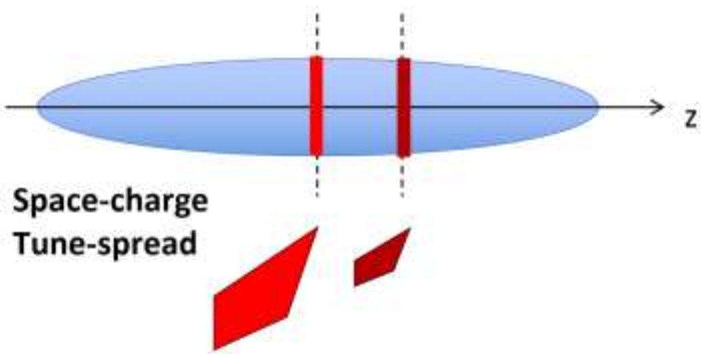
- SC tune shift

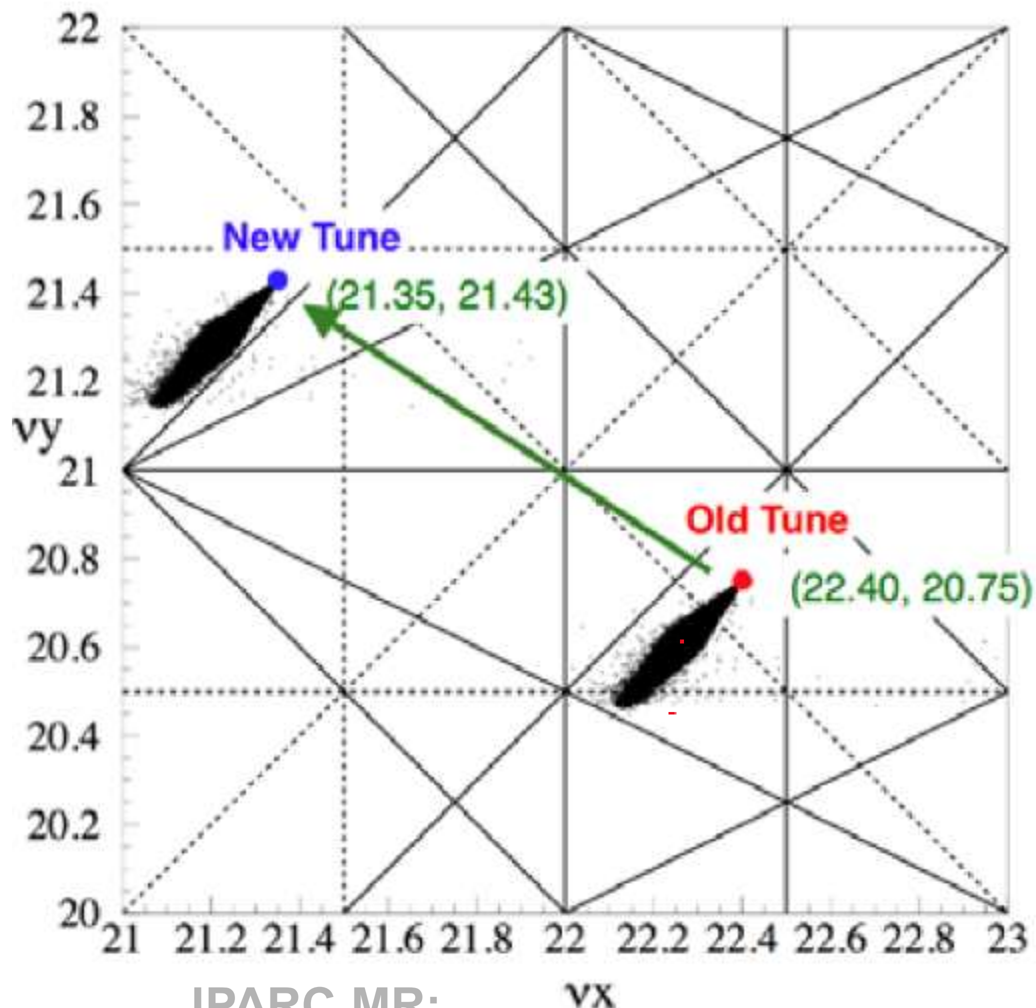
$$\Delta Q_{SC} = -\frac{N_p r_p B_f}{4\pi\varepsilon\beta_p\gamma_p^2}$$



Space-charge Tune-spread & Betatron Resonances







G. Franchetti et al.
PRSTAB 2017

JPARC MR:
M. Friend

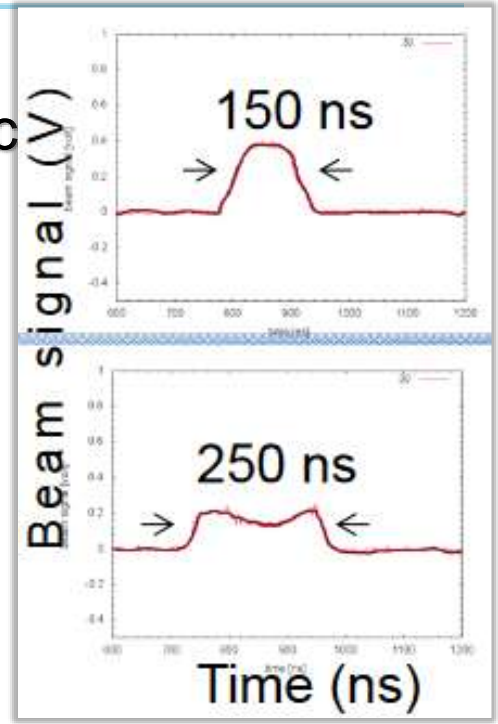
Max SC tuneshift Achieved: -0.2…-0.5

	E_i/E_p	N_p	T	P	ΔQ_{sc}	$\%_{N_p}$	$\%_\epsilon$	C	S	$Q_{h,v}$
ISIS	0.07/0.80	3.1	0.01 s	200	0.4	2		163	10	4.31/3.83
PS-B	0.05/1.4	0.25	1.2	n/a*	0.50	5	20	157	16	4.3/4.45
CSNS	0.08/1.6	1.6	0.02	100	0.28	1	20	228	4	4.86/4.78
J-RCS	0.4/3	4.2	0.02	500	0.35	0.3	10	348	3	6.45/6.32
FNAL-B	0.4/8	0.45	0.03	84	0.60	5	20	474	24	6.78/6.88
CERN-PS	1.4/28	1.5	3.6	n/a*	0.24	3	5	628	50	6.12/6.24
JPARC-MR	3/30	27	1.5	515	0.4	1.5	10	1568	3	21.35/21.43
FNAL-MI	8/120	5.1	0.62	803	0.09	2.5	5	3319	1	26.46/25.38
CERN-SPS	28/450	0.9	19	n/a*	0.21	5	10	6911	6	20.13/20.18
PSR	0.8	3.1	6e-4	80	0.29	0.3		90	10	3.18/2.19
SNS-R	1	14	0.001	1400	0.15	0.01		248	4	6.23/6.20
FNAL-RR	8	5.2	0.84	54	0.09	2.5	10	3319	1	25.44/24.43

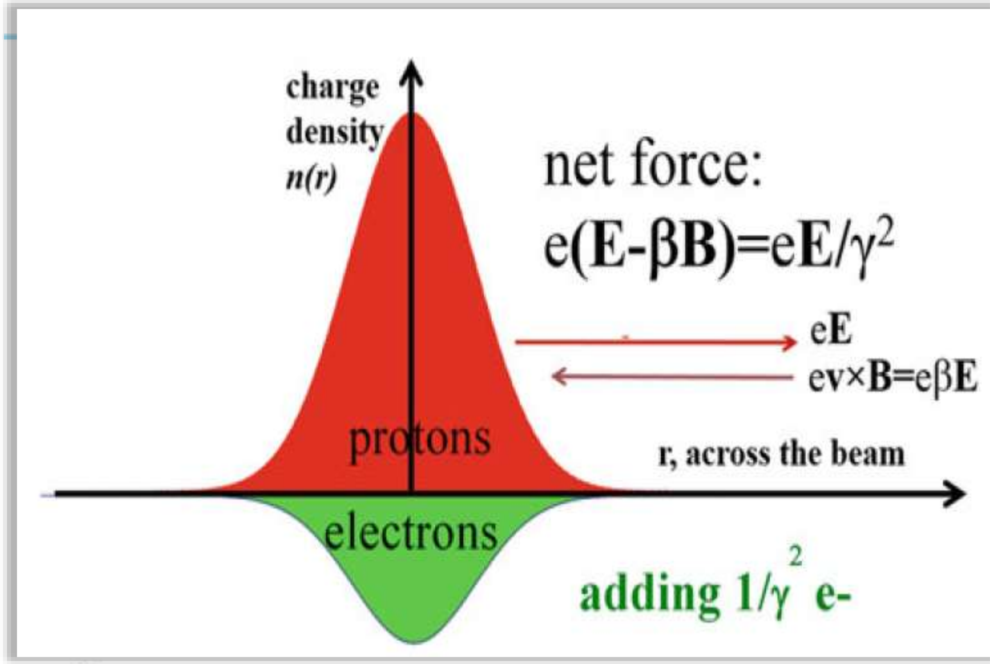
Figure 3: Operational high intensity RCSs and accumulator rings: injection/extraction kinetic energies E_i/E_p in GeV, number of protons per pulse N_p in 10^{13}, beam acceleration/storage time T in s, average beam power P in kW, maximum SC tune shift ΔQ_{sc}, fractional intensity loss $\%_{N_p} = \Delta N_p/N_p$ and emittance growth $\%_\epsilon = \Delta\epsilon/\epsilon$ in %, circumference C in m, lattice periodicity S and tunes $Q_{h,v}$. (* For CNGS operation in 2005-2012, the SPS delivered 510 kW average power at 400 GeV). Figure and caption from [10].

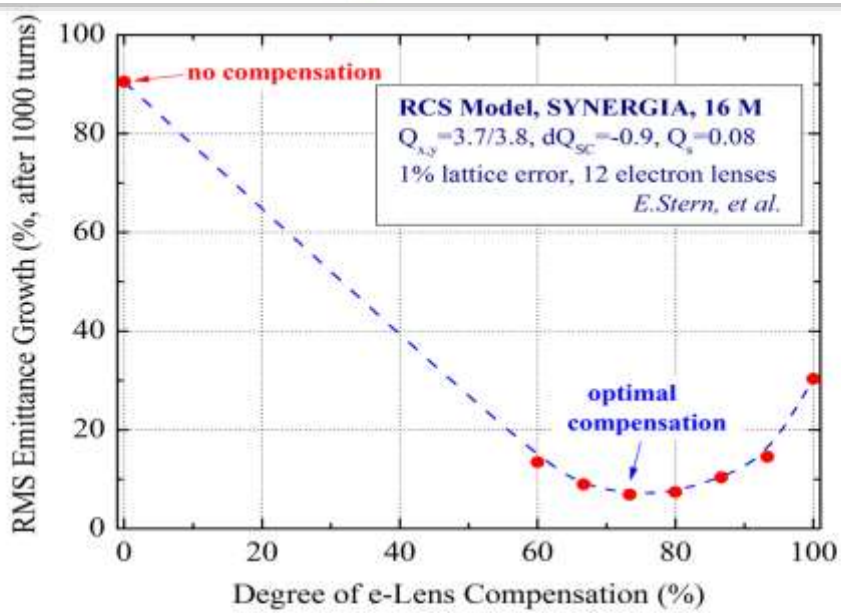
Ways to Increase "Protons Per Pulse"

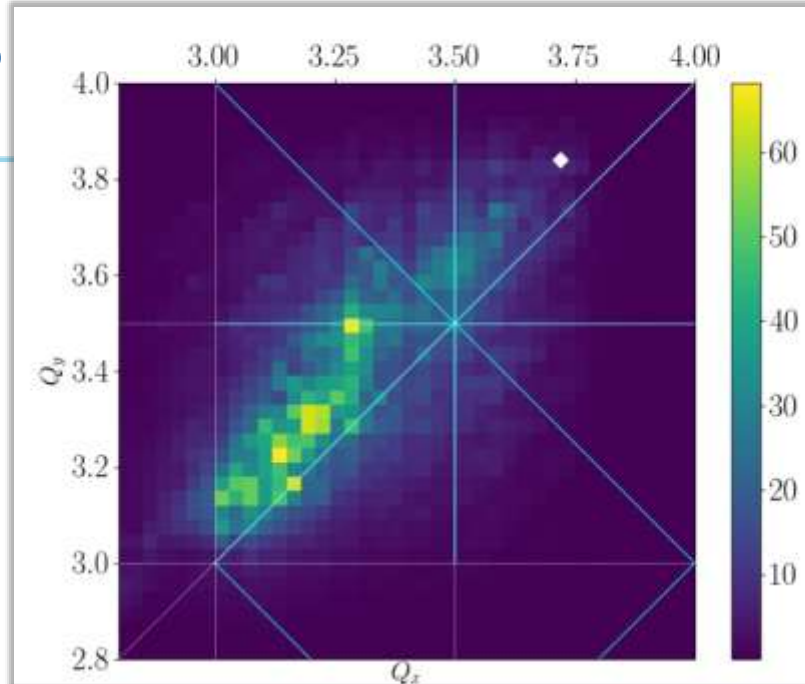
- **Increase the injection energy**:
  - Gain about $N_p \sim \beta\gamma^2$, need (often - costly) linac
- **Flatten the beams** (using 2nd harm, RF) :
  - Makes SC force uniform, $N_p \sim$ x2
- **"Painting" beams** at injection:
  - To linearize SC force across beams $N_p \sim$x1.5
- **Better collimation system beams:**
  - From $\eta \sim 80\%$ to ~95% $N_p \sim$x1.5
- **Make focusing lattice perfectly periodic**:
  - Eg P=24 in Fermilab Booster, P=3 in JPARC MR → $N_p \sim$ x1.5
- **(to be tested) Introduce *Non-linear Integrable Optics*** :
  - May reduce the losses and allow $N_p \sim$ x 1.5-2
- **(tbt) Space-Charge Compensation** by electron lenses :
  - Electrons to focus protons, may allow $N_p \sim$x1.5 - 2

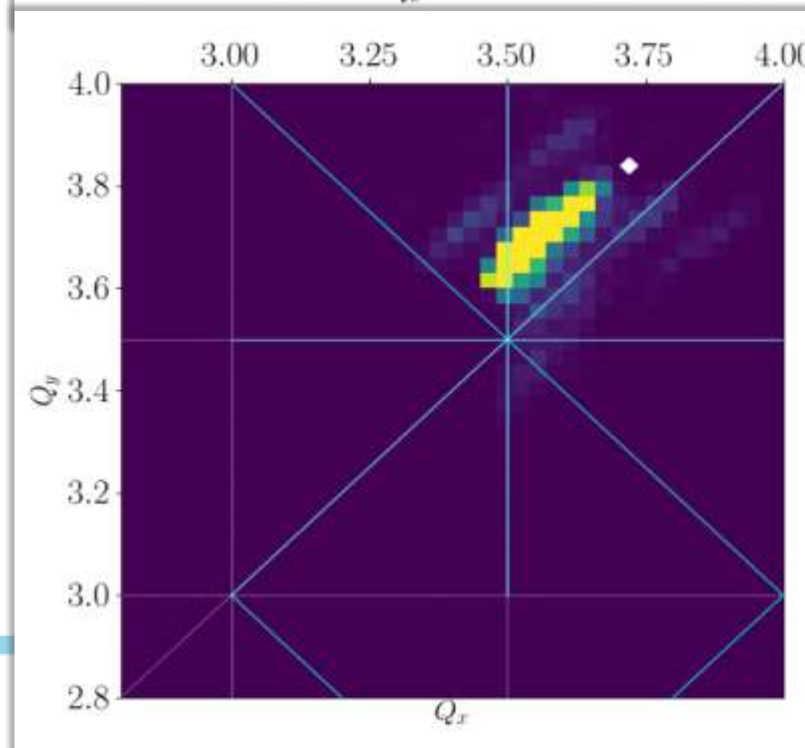


Space-Charge Compensation R&D









IOTA: *Integrable Optics Test Accelerator @ FNAL*

(COHERENT BEAM) INSTABILITIES

Instabilities

- Beam instabilities are driven by the **electromagnetic interaction with the accelerator environment** (-> **wakefields/impedances**) and by **electron clouds**.
- Above a certain **intensity threshold** the beam's oscillation amplitude increases exponentially and the beam is either **lost at the wall** (transverse instabilities) or from the RF bucket (longitudinal) and/or the **emittance increases**.
- Presently, heat loads and instabilities are one of the **main beam quality and intensity limitation** in particle accelerators for high intensity and brightness !
- Finding "**cures" for instabilities** is one of the major challenges in beam physics and accelerator technology for **future machines**.
- **High energy beams:** Beam instabilities are a 'current effect'. However, synchrotron radiation, photoelectrons or other high energy effects affect instability thresholds.

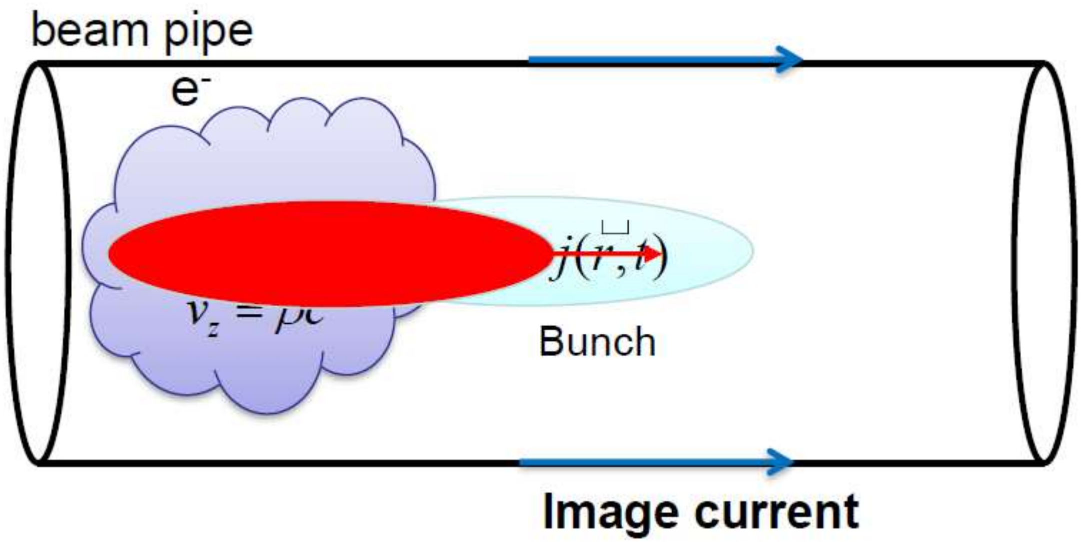
Maxwell's equations and Lorentz Force

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$



+ boundary conditions at the walls

Impulse approximation

$$c\Delta \boldsymbol{p} = q\int_{-\infty}^{\infty} (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\, ds$$

The EM force continuously acting on a test charge is lumped in a single kick after the passage through the structure.

Rigid bunch approximation

$$j = \beta_0 c \rho e_z$$

The beam traverses the structure rigidly.

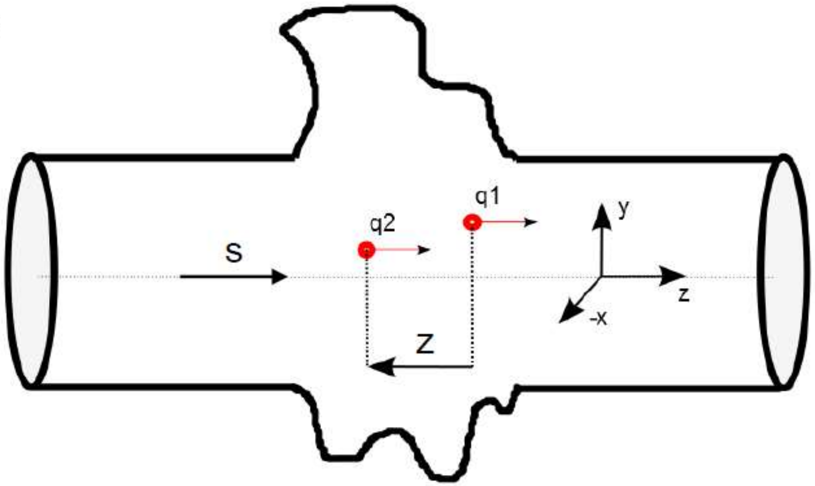
EM forces due to : a) wake fields and impedances, b) electron cloud, c) beam-beam, d) etc

See more in USPAS lectures

Wake-fields

$$W(\mathrm{r}_2, \mathrm{r}_1, z) = -\frac{1}{q_1}\int_{-\infty}^{\infty} [\mathrm{E} + \mathrm{v} \times \mathrm{B}]\left(\mathrm{r}_2, z, t = \frac{z+s}{c}\right)ds$$

T. Weiland and R. Wanzenberg, “Wake Fields and Impedances,” in CERN Accelerator School (CAS), 1993. 20, 28, 99
L. Palumbo, V. G. Vaccaro, and M. Zobov, “Wake Fields and Impedance,” in Cern Accelerator School, 1994. 20



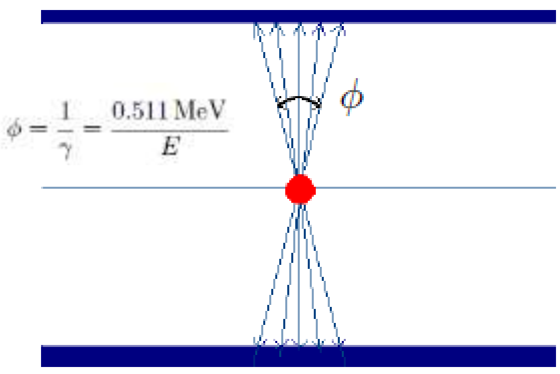
Longitudinal:

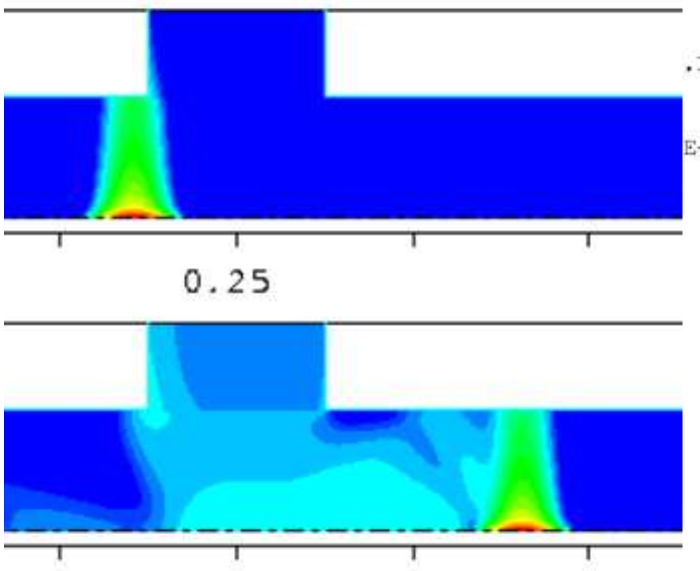
$$W_{\parallel}(z) = -\frac{1}{q_1}\int_{-\infty}^{\infty} E_z\left(\mathrm{r}_2 = 0, z, t = \frac{z+s}{c}\right)ds$$

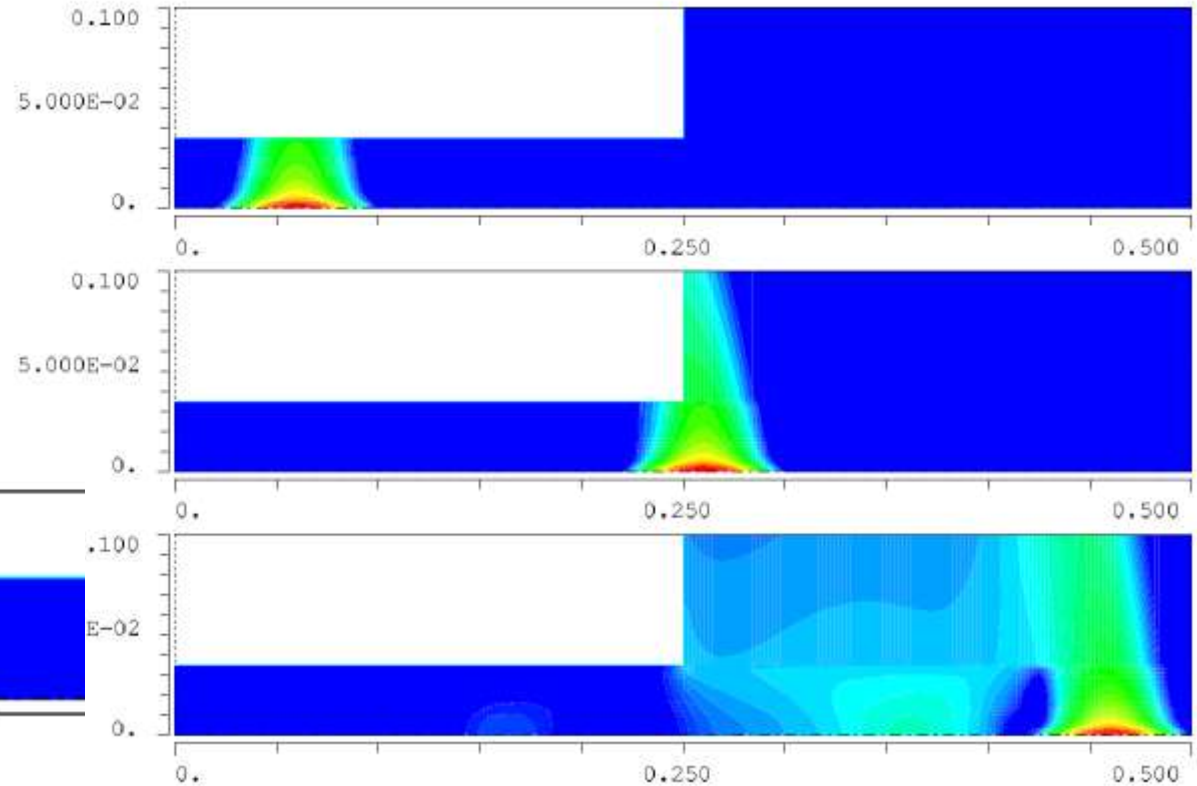
Transverse:

$$W_{\perp}(z) = -\frac{1}{q_1 d_1}\int_{-\infty}^{\infty} [\mathrm{E} + \mathrm{v} \times \mathrm{B}]_{\perp}\left(\mathrm{r}_2 = 0, z, t = \frac{z+s}{c}\right)ds$$

Wake-fields - Examples







Wake fields behind a bunch generated at a step-out transition from a small to a larger beam pipe

Wake fields in a cavity

What if we have many particles

- Wake-functions

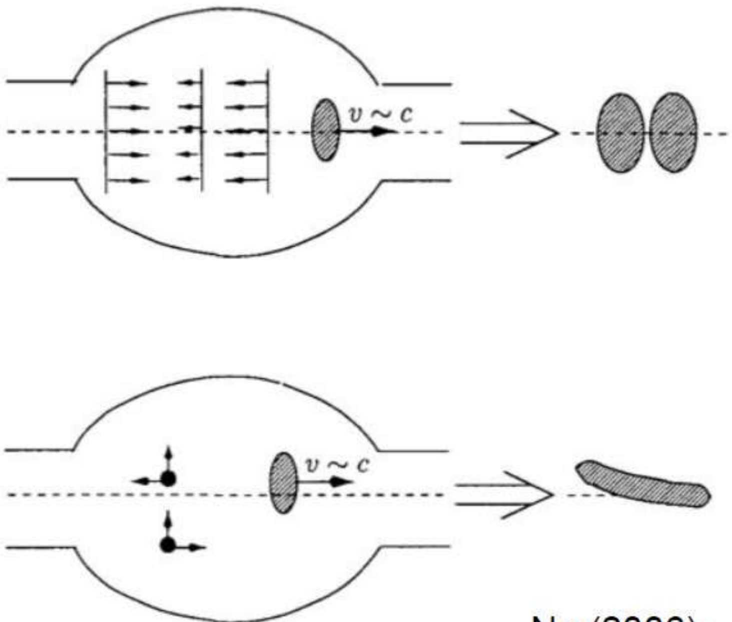
Longitudinal: $\int_0^L F_z ds = -q^2 W_\parallel(z)$

For a test particle in a bunch:

$$\int_0^L F_z ds = qV \qquad V = -q\int W_\parallel(z-u)\lambda(u)\,du$$

(Voltage kick or Wake potential)

Line density: $\lambda(z) = \dfrac{dN}{dz}$



Ng (2006)

Transverse: $\int_0^L F_\perp ds = -q^2 r W_\perp(z)$

For a test particle in a bunch:

$$\frac{1}{\gamma_0 mc^2}\int_0^L F_x ds = \Delta x' \qquad \Delta x' = -\frac{q^2}{\gamma_0 mc^2}\int W_\perp(z-u)\lambda(u)\bar{x}(u)\,du$$

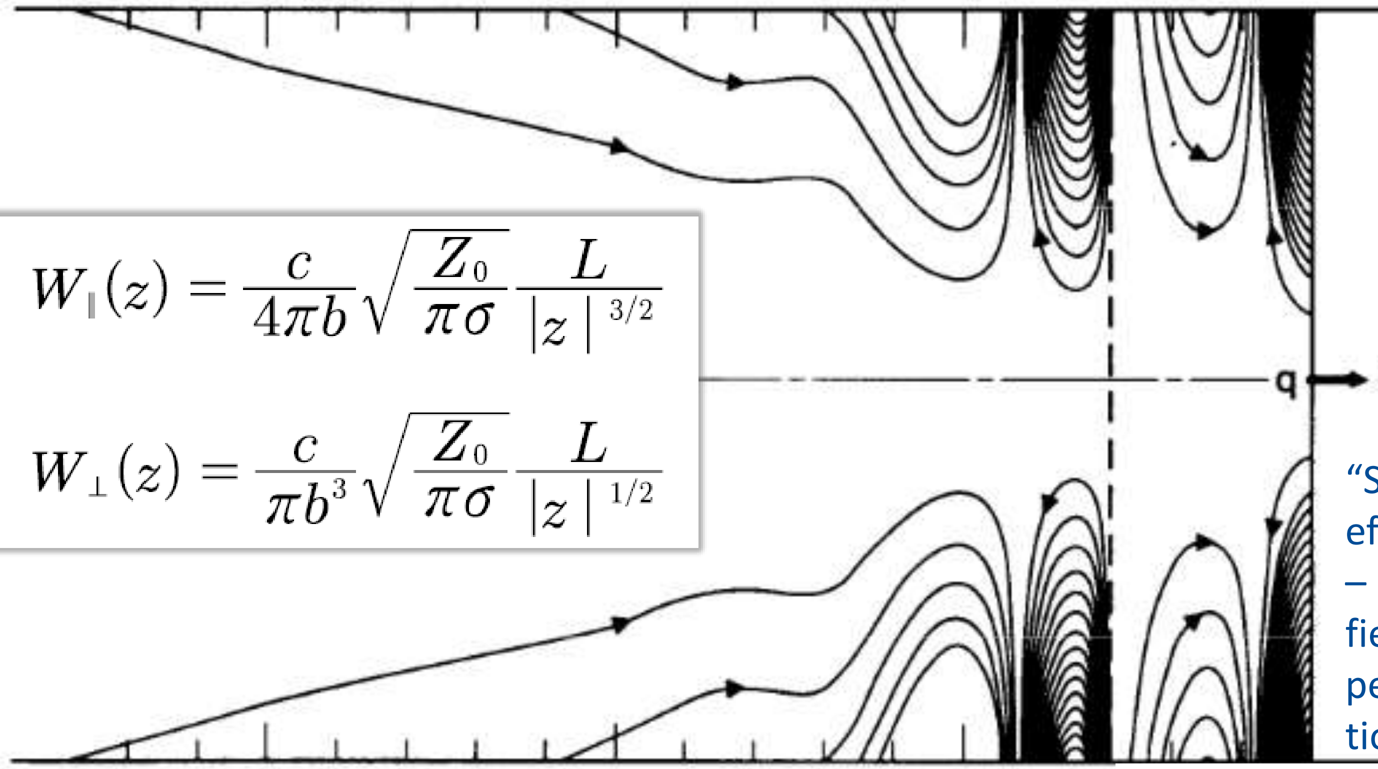
(horizontal kick)

$\bar{x} = \langle x \rangle$: local bunch offset

Even "simple" resistive wall leaves wakes

Longitudinal: $$W_{\parallel}(z) = \frac{c}{4\pi b}\sqrt{\frac{Z_0}{\pi\sigma}}\frac{L}{|z|^{3/2}}$$

Transverse: $$W_{\perp}(z) = \frac{c}{\pi b^3}\sqrt{\frac{Z_0}{\pi\sigma}}\frac{L}{|z|^{1/2}}$$



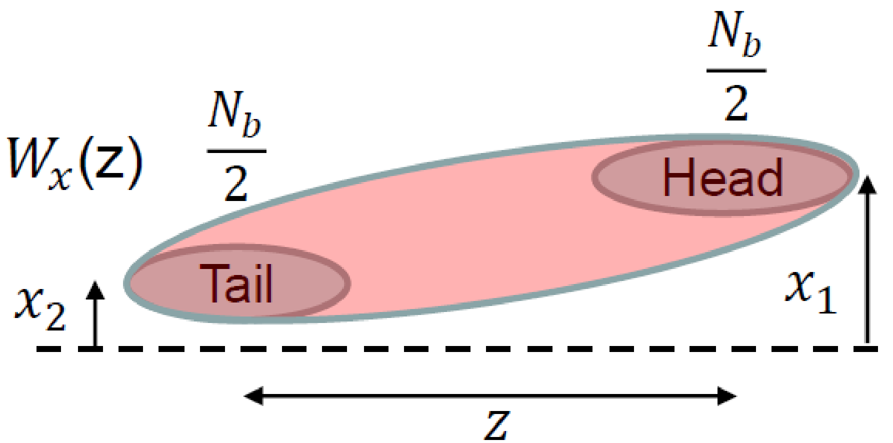
"Skin-effect" – EM field penetration depth

$$\delta r = \sqrt{\frac{c|z|}{\pi\sigma}}$$

Figure 2.3. Wake electric field lines in a resistive wall pipe generated by a point charge q. The field pattern shows oscillatory behavior in the region $|z| \lesssim 5(2\chi)^{1/3}b$ (or $|z| \lesssim 0.35$ mm for an aluminum pipe with $b = 5$ cm). The field line density to the left of the dashed line has been magnified by a factor of 40. (Courtesy Karl Bane, 1991.)

Key points: a) longitudinal wakefield leads to particle energy loss and pipe heating; b) transverse wake is defocusing for vacuum beam pipe (focusing in case of electron cloud)

Consequences: two-particle model



Two-particle coupled betatron oscillations:

$$x_1'' + \varkappa x_1 = 0$$

$$x_2'' + \varkappa x_2 = \frac{q^2 N_b W_x(z)}{2LE_0} x_1 \qquad \varkappa = \frac{Q_x^2}{R^2}$$

New coordinates:

$$\tilde{x}_l = x_l + i\frac{x'_l}{\varkappa} \qquad l = 1,2$$

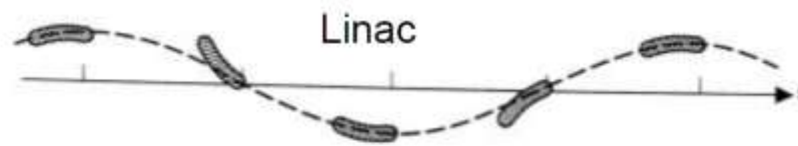
Solution:

Linear growth !

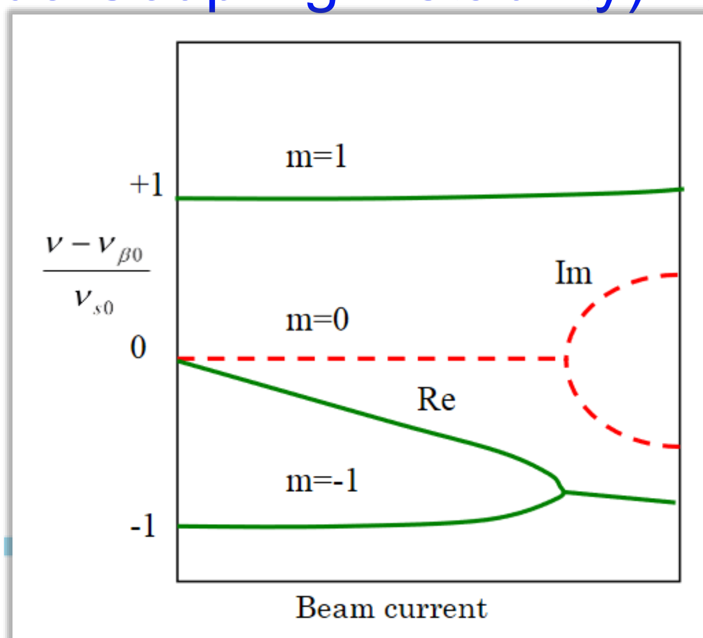
$$\tilde{x}_1(s) = \tilde{x}_1(0)\, e^{-i\varkappa s}$$

$$\tilde{x}_2(s) = \tilde{x}_2(0)\, e^{-i\varkappa s} - \boxed{i\frac{q^2 N_b W_x(z)}{4E_0 L\varkappa}\tilde{x}_1(0)\, s e^{-i\varkappa s}}$$

In linacs: Beam-break up (BBU) instability



In rings: Head-tail instability (aka TMCI = Transverse Mode Coupling Instability)



Intensity Limits and Cures

Beampipe heating is important for cryo – may limit on $N_b\ I_b$

Instabilities severely limit either single bunch current I_b or total beam current $N_b\ I_b$

Cures employed so far:

1) Reduce wakes/impedances – no discontinuities in beam pipe, better conducting materials, etc
2) In linacs – *BNS damping*= introduce energy difference btw head and tail of the bunch (RF phase choice) leading to slight difference in the betatron oscillation frequencies
3) In rings
    1) Feedback dampers (might not work for single bunch instabilities)
    2) introduce betatron frequency spread via chromaticity $dQ=Q'(dP/P)$ (does not always work) or octupoles $dQ{\sim}Oct{*}\sigma^2$ (mostly worked so far) or electron beams for Landau damping (next gen colliders)

Intensity Limits and Cures

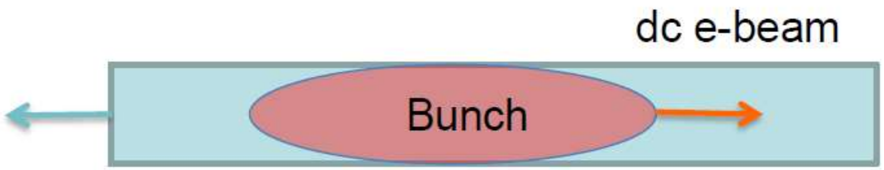
168 LHC octupoles for Landau Damping

Tune shifts (integrated):

$$\Delta Q_x = a_x J_x - b_{xy} J_y$$
$$\Delta Q_y = a_y J_y - b_{xy} J_x$$

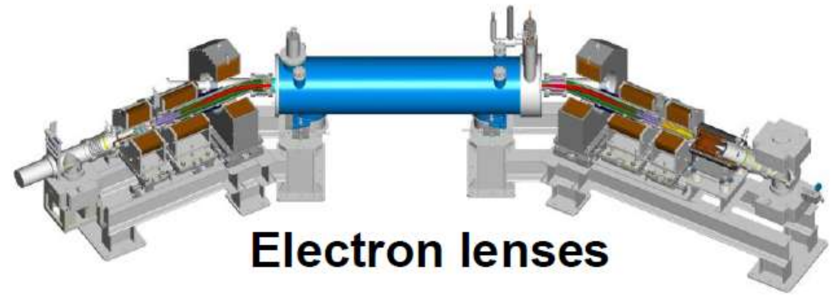
Concern is that these octupolse are so nonlinear that they reduce *Dynamic Aperture* of the collider
→ affect lifetime

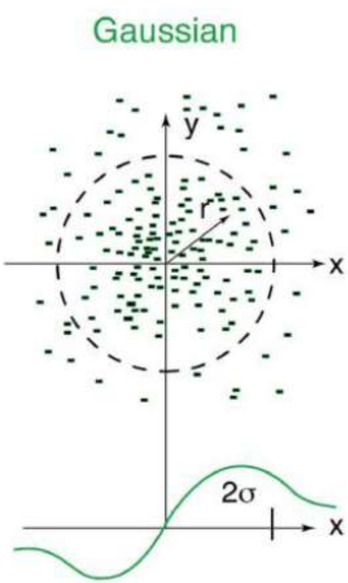
Landay Damping by Electron Lenses



Matched transverse beam radii.

Similar to the beam-beam force !





Gaussian electron beam provides a nonlinear tune shift.

Tune shift induced by a counter-propagating electron beam:

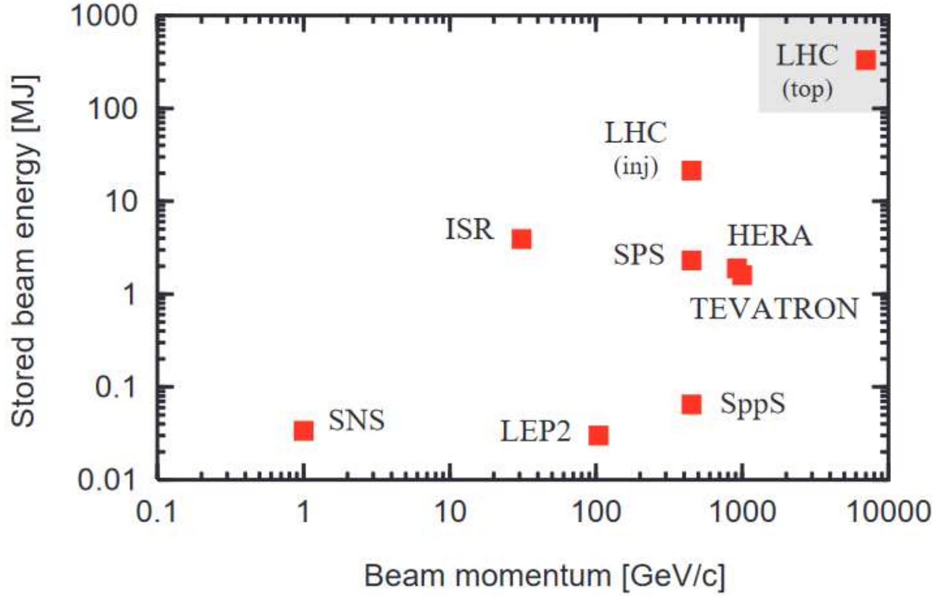
$$\Delta Q_x^e = \frac{1+\beta_e}{\beta_e}\frac{I_e l r_p}{2\pi e c \varepsilon_x}$$

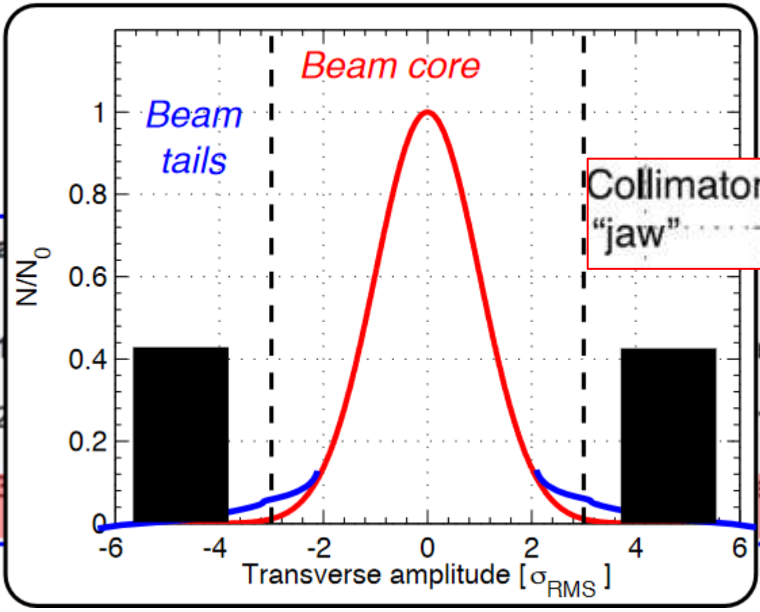
V. Shiltsev et al., PRL (2017)

Example: One e-lens (l=2 m, I_e=1 A) in LHC would provide a tune spread similar to the 168 octupoles.

Collimation

- To protect from enormous beam power (and power density) of high energy accelerators and colliders – events and processes:
  - Injection errors
  - Instabilities
  - Losses due to beam-beam, beam-gas, intrabeam scattering, etc
  - Synchrotron radiation photons
- Protect magnets, RF and detectors !

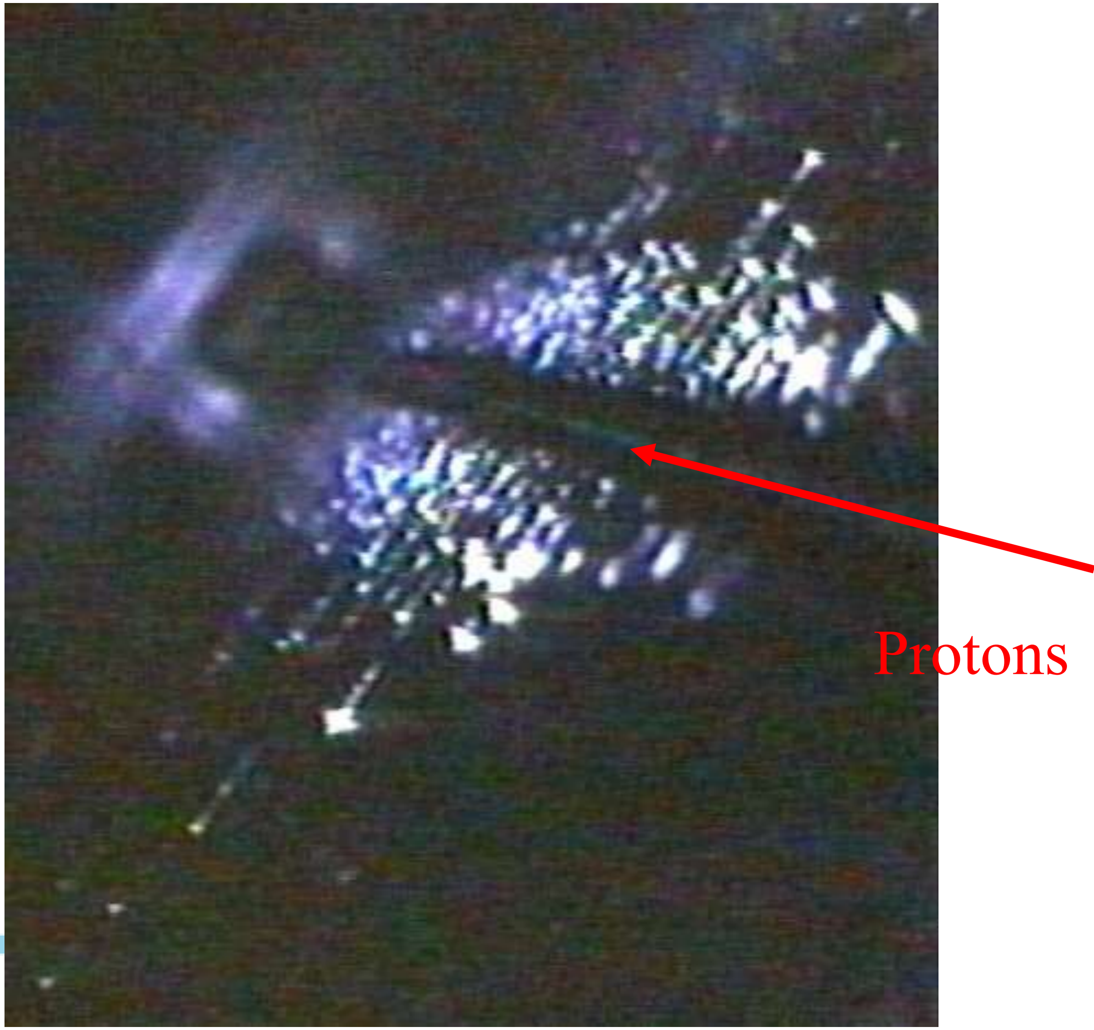




Collimators

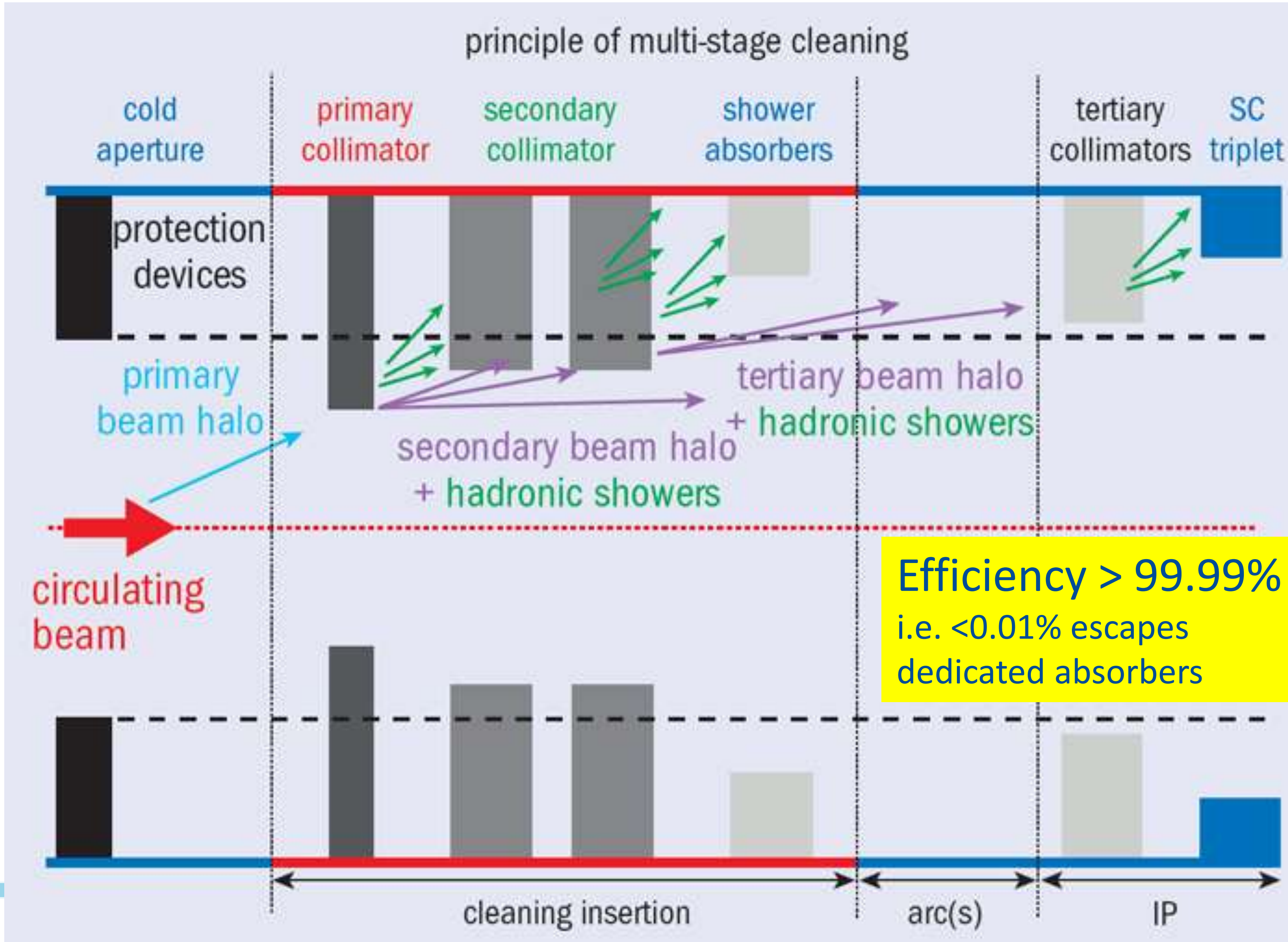
- Tevatron 12 collimators:
  - Hor and Vert
  - Proton and antiproton
  - 4 primaries
    - 5 mm W
  - 8 secondaries
    - 1.5 m stainless steel
    - Flat to <25 micron
    - As close as few mm to the
- Efficiency 95-99%
  - reduction of background in CDF and D0 detectors x20-100

Damage to E03 1.5m Collimator



 see lectures NM1-4

(Most Sophisticated) LHC Collimation

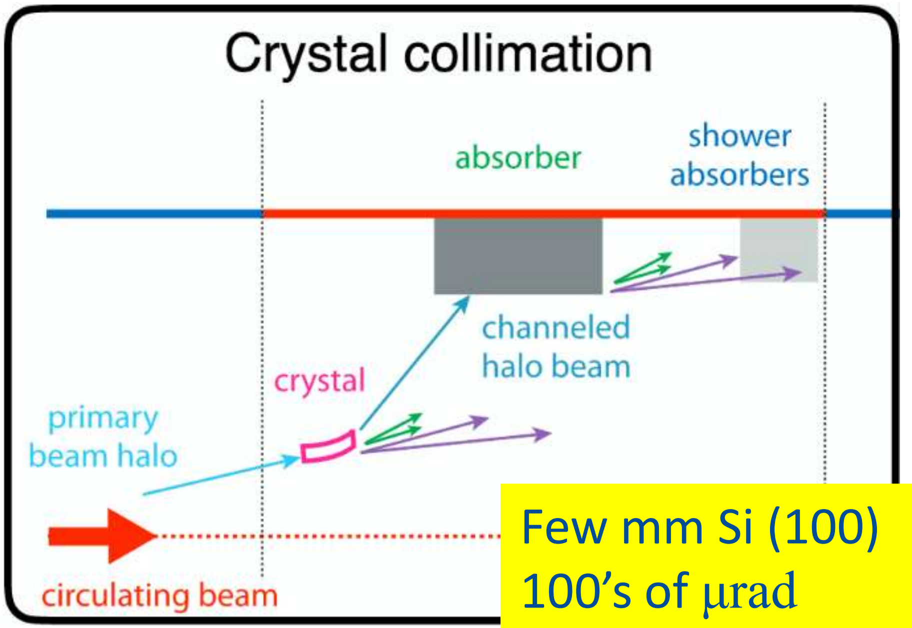


Efficiency > 99.99%
i.e. <0.01% escapes dedicated absorbers

Collimation Challenges and Cures

- Too many, too close to beams → large wakefields/impedance
- Can be damaged/destroyed …. NEW METHODS

Bent crystal collimation

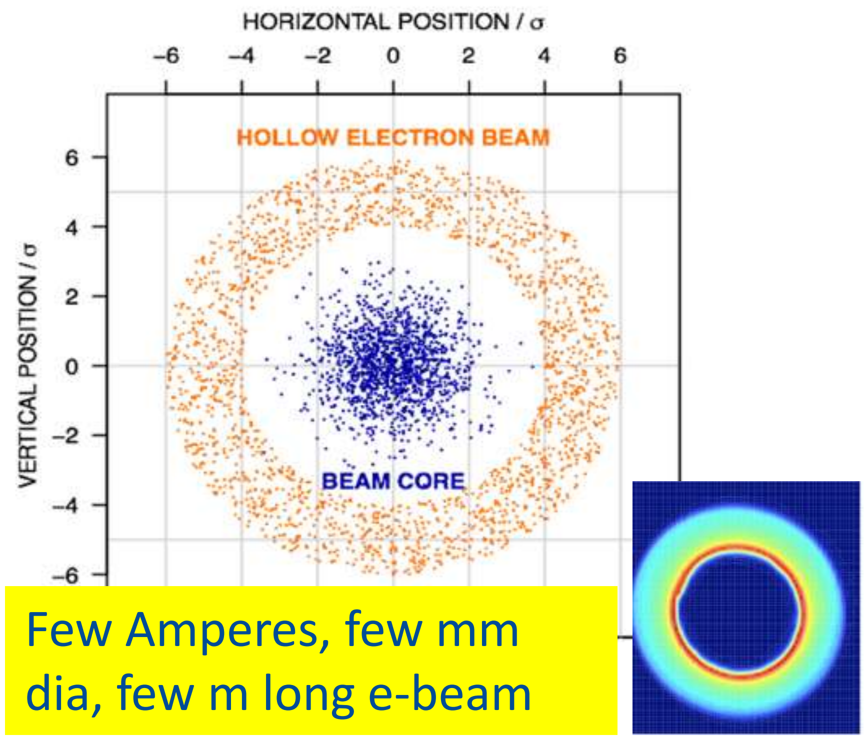


Few mm Si (100)
100's of μrad

Makes bigger deflection → better interception of scattered particles

Tested at the Tevatron and LHC

Hollow e-beam collimation

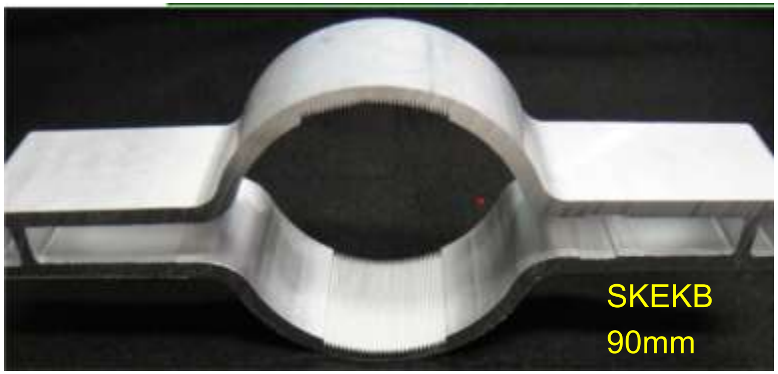


Few Amperes, few mm dia, few m long e-beam

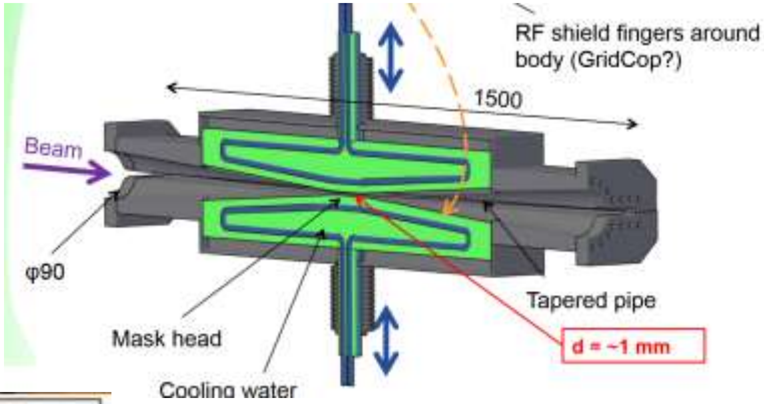
Soft "penetrable" & fast diffusor → undamageable. Tested at the Tevatron and being built for LHC

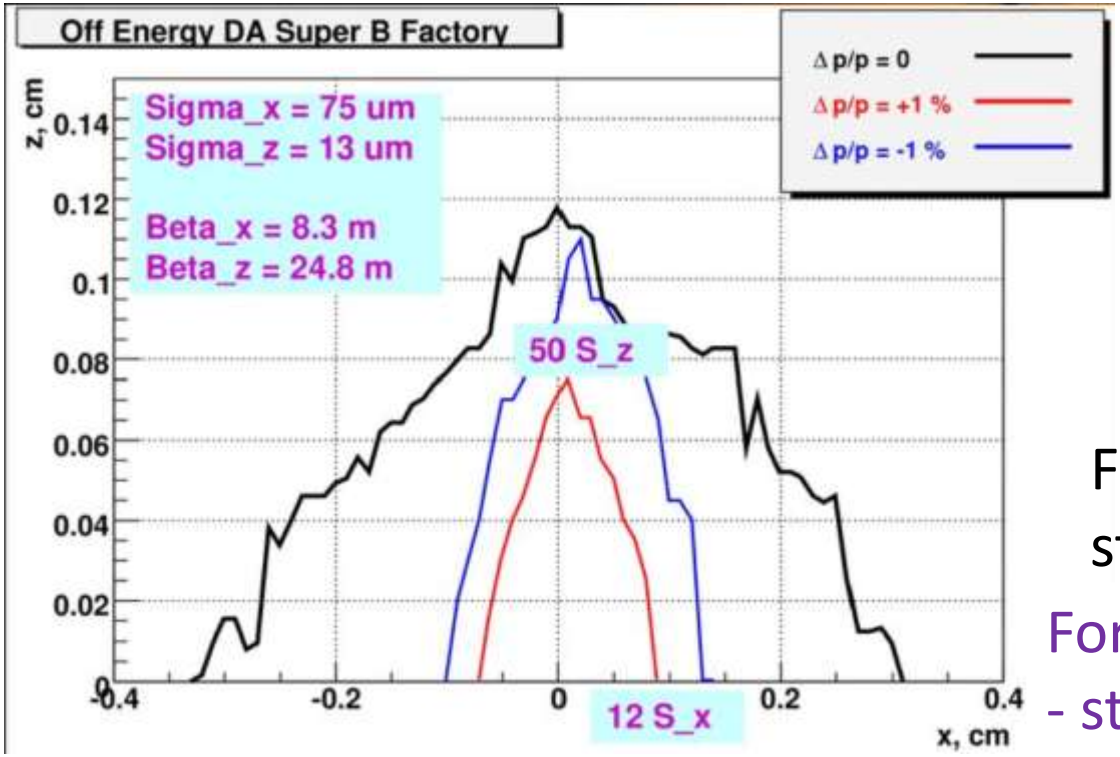
Aperture and Dynamic Aperture

Physical beam pipe ~60-100 mm …10’s-100’s σ
Often coated (eg TiN) and/or grooved (ecloud)



Collimators - losest to beam – 5-10’s of σ
Often coated (eg TiN) and/or grooved (ecloud)





The **dynamic aperture** is the stability region of phase space in an accelerator – dependent on nonlinearities and chromatic effects

For proton machines - stability over O(1e9) turns

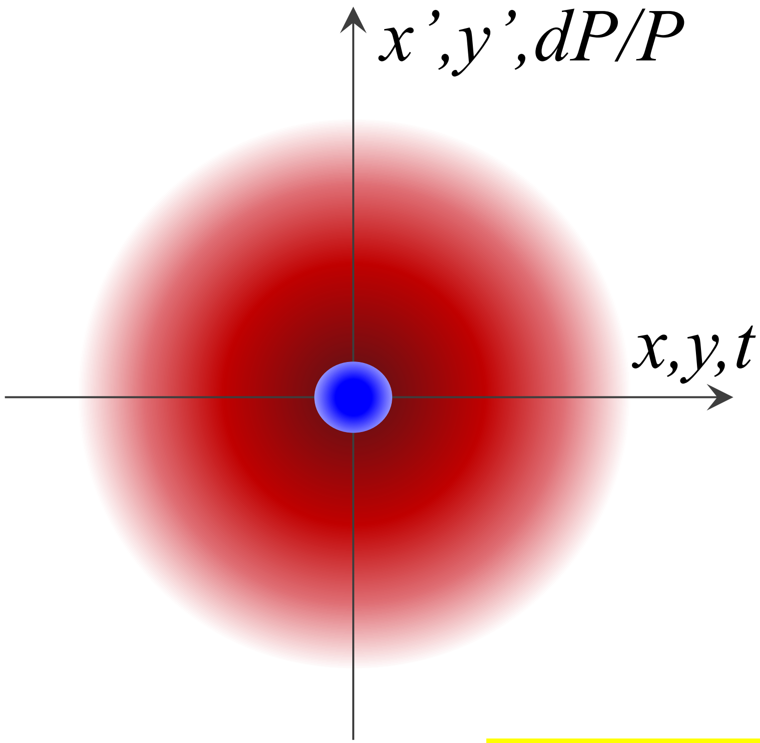
For electron/muon machines - stability over O(1e3) turns

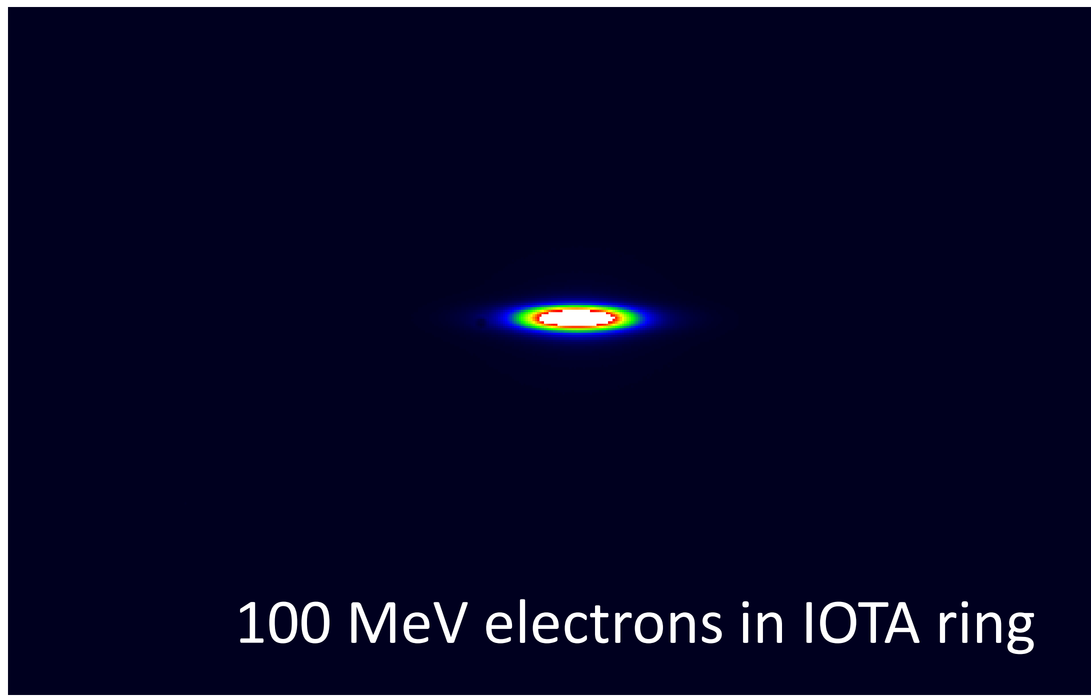
Beam Cooling

$$\mathcal{L} = f_{\text{coll}} \frac{N_1 N_2}{4\pi\sigma_x^* \sigma_y^*}$$

Beam Phase Space Density Increase

- As needed for a collider
- Forbidden by the *Liouville theorem* in non-dissipative systems





Ideally - "6D-Cooling"

Diffusion and Cooling (1)

Diffusion equation for beam distribution function *f(J,t), J*- action variable

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial t}\left(D(J)\frac{\partial f}{\partial J}\right)$$

In the presence of cooling:

$$\frac{d\varepsilon_n}{dt} = \beta\gamma\frac{dD(J)}{dJ} - \frac{\varepsilon_n}{\tau_{\text{cool}}}$$

where for example:

Dipole noise For a single dipole steering error randomly fluctuating each revolution of the accelerator with rms value θ_{rms}, the emittance growth rate is

$$\frac{d\epsilon_N}{dt} = \frac{1}{2} f_0 (\gamma v/c)\beta_0 \theta^2_{\text{rms}} \qquad (10)$$

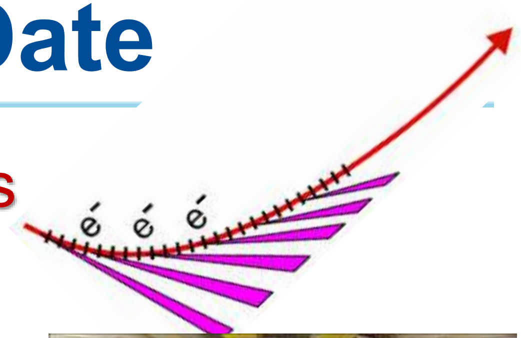
where β_0 is the β-function at the location of the error, and f_0 is the revolution frequency.

Coulomb scattering If the scattering is due to small angle Coulomb interactions between the beam particles and other material in the beam chamber, then

$$\frac{d\epsilon_N}{dt} = \frac{1}{2} f_0 \langle\beta\rangle \left(\frac{13.6\,\text{MeV}}{mc^2}\right)^2 \frac{z}{\gamma(v/c)^3}\frac{\ell}{X_0} \qquad (12)$$

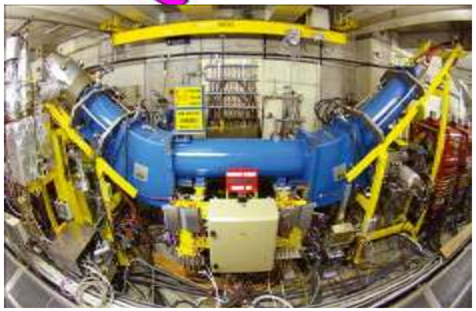
where mc^2 is the rest energy of a beam particle, z its charge, and X_0 is the radiation length of the

Beam Cooling Methods to Date



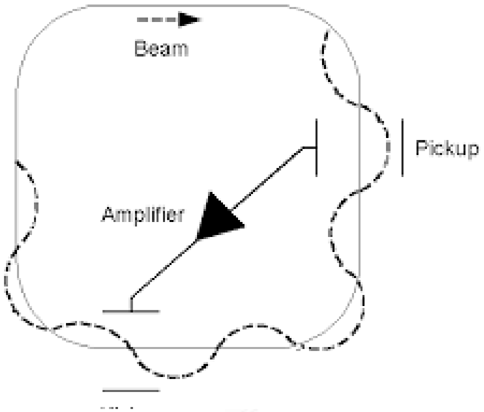
Synchrotron Radiation Damping – since 1960's

- common in all e+/e- rings



Electron Cooling – since 1970's

- Widely used to cool ions and antiprotons
- 0.1 - 8 GeV/n (50 keV – 4 MeV electrons DC)



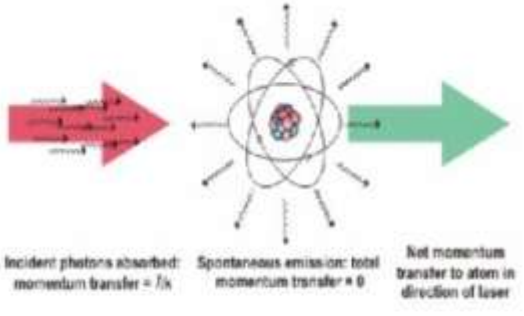
Stochastic Cooling – since 1970's

- Widely used to cool ions and antiprotons
- 0.1-100 GeV/n (up to 10 GHz feedback BW)

Laser Cooling – since 1990's

$$\Omega = \gamma\omega_{21}(1 - \beta\cos\theta)$$

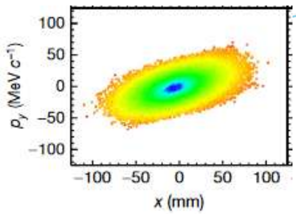
- Works for some highly charged ions
- 0.1-0.5 GeV/n, deep cooling, spectroscopy



Lectures VL13-14

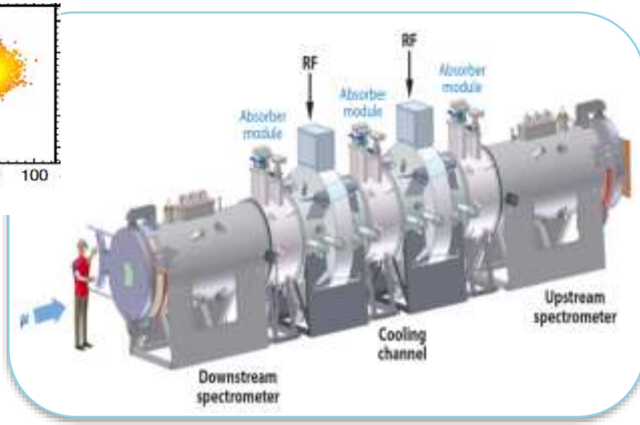
Recent Beam Cooling Breakthroughs

2019 - Ionization cooling of muons (140 MeV/*c*, RAL, UK)

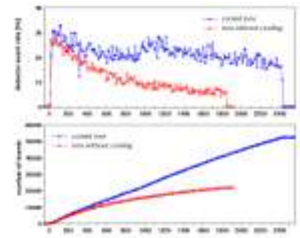


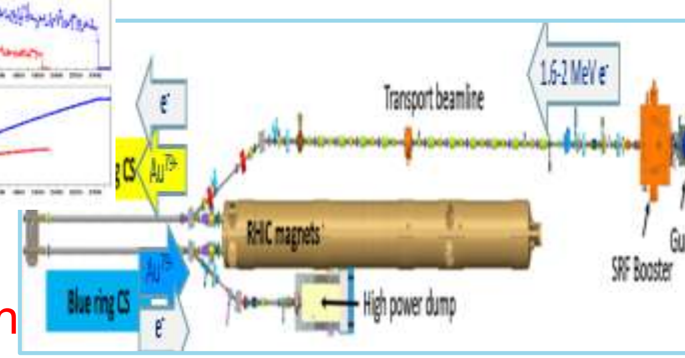
MICE

~10% in one pass



2020 – "Bunched" electron cooling of ions (γ~5, BNL)

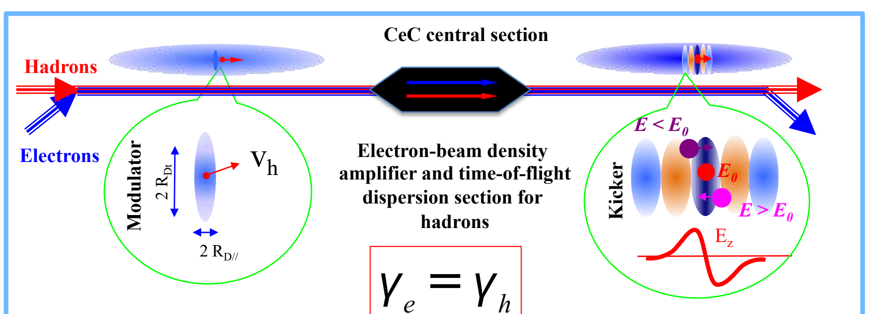




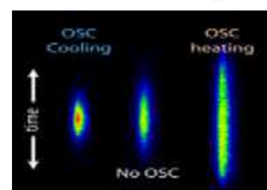
RHIC

RF e-gun

2025 – Coherent Electron cooling of ions (26.5 GeV/*n*, RHIC) – ongoing PoP exp't at BNL



2021 – Optical Stochastic cooling e- (100 MeV, FNAL)



IOTA

THz bandwidth

Thanks for Your Attention !

Questions !?

Literature

- W.Herr, CAS school

https://cds.cern.ch/record/941319/files/p379.pdf

- V.Lebedev, V.Shiltsev, Tevatron Book Ch.8

https://indico.cern.ch/event/774280/attachments/1758668/2915590/2014_Book_AcceleratorPhysicsAtTheTevatro.pdf

- Proc. 2013 ICFA mini-workshop on "Beam-Beam Effects in Hadron Colliders"

https://indico.cern.ch/event/189544/

- Past schools :
  - A. Chao, The beam-beam instability, SLAC-PUB-3179 (1983).
  - L. Evans, The beam-beam interaction, CAS Course on proton-antiproton
  - colliders, in CERN 84-15 (1984).
  - L. Evans and J. Gareyte, Beam-beam effects, CERN Accelerator School, Oxford
  - 1985, in: CERN 87-03 (1987).
  - A. Zholents, Beam-beam effects in electron-positron storage rings, Joint
  - US-CERN School on Particle Accelerators, in Springer, Lecture Notes in
  - Physics, 400 (1992).

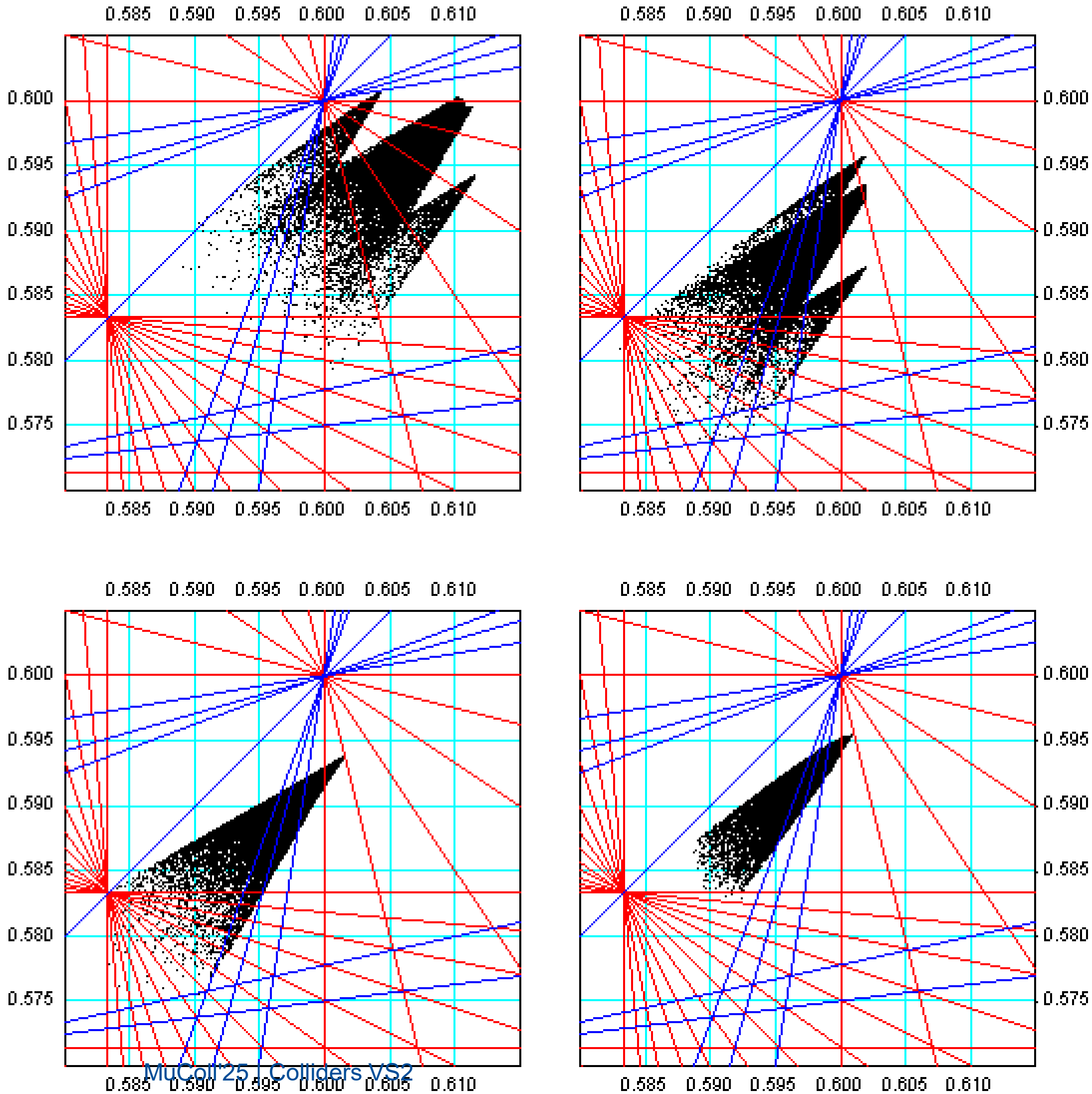
Comprehensive *JUAS-book* (2371 pages – all topics!)
https://doi.org/10.23730/CYRSP-2024-003.

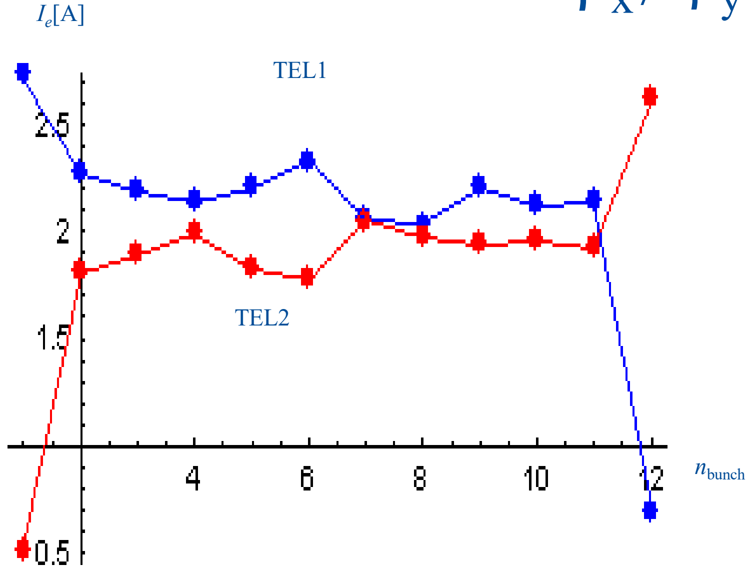
Instabilities:
A.Chao, *Physics of collective beam instabilities in high energy accelerators* (1993)
https://www.slac.stanford.edu/~achao/wileybook.html

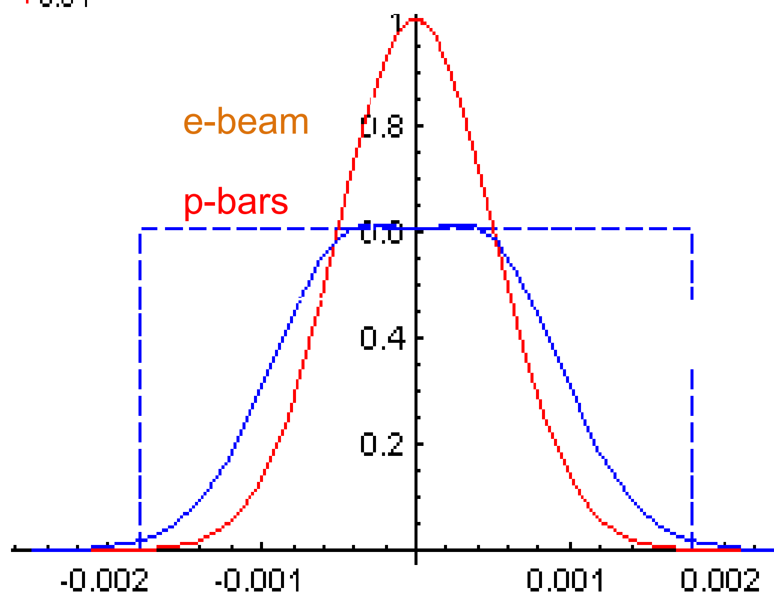
Many useful articles:
S.Myers, H.Schopper *Accelerators and Colliders* (2013, open access)
https://link.springer.com/book/10.1007/978-3-030-34245-6

Can also be used for Long-Range Beam-Beam Compensation

vary the currents bunch-by-bunch in two e-lenses installed at $\beta_x \neq \beta_y$







Beam-Beam Effects

- Remember:

$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f B}{4\pi\sigma_x\sigma_y} \quad = \quad \frac{N_1 N_2 f B}{4\pi \cdot \ \sigma_x\sigma_y}$$

- Overview: which effects are important for
- present and future machines (LEP, PEP,
- Tevatron, RHIC, LHC, ...)
- Qualitative and physical picture of the effects

Fields and Forces (1)

- Start with a point charge q and integrate over the particle distribution.
- In rest frame only electrostatic field: $E \neq 0$ while B=0
- Transform into moving frame and calculate
- Lorentz force

$$E_{||} = E'_{||}, \quad E_{\perp} = \gamma \cdot E'_{\perp} \quad \text{with :} \quad \vec{B} \;=\; \vec{\beta} \times \vec{E}/c$$

$$\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$$

- Note that *F≈0* if velocities are collinear

Fields and Forces (2)

- Derive potential *U(x, y, z)* from Poisson equation:

$$\Delta U(x,y,z) = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

- The fields become:

$$\vec{E} \;=\; -\nabla U(x,y,z)$$

- Example Gaussian distribution:

$$\rho(x,y,z) = \frac{Ne}{\sigma_x\sigma_y\sigma_z\sqrt{2\pi}^3}\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}-\frac{z^2}{2\sigma_z^2}\right)$$

A Common Example: Gaussian

- For 2D case the potential becomes:

$$U(x, y, \sigma_x, \sigma_y) = \frac{ne}{4\pi\epsilon_0} \int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2 + q)(2\sigma_y^2 + q)}} dq$$

- Can derive *E* and *B* fields and therefore forces
- Also easy for uniform distribution: *E* and *B* scale linear with *r* for *r<a*, and *1/r* for *r>a…* easy for simple easily integrable axisymmetric distributions
- For arbitrary distribution (non-Gaussian):
  - difficult (or impossible, numerical solution required)

Further Simplification: Round Gaussian

- Round beams: $\sigma_x = \sigma_y = \sigma$
- Only components Er and B are non-zero
- Force has only radial component, i.e. depends only on distance r from bunch center, i.e. $r^2 = x^2 + y^2$.

$$F_r(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

Beam-Beam Kick

- Kick $\Delta r'$ - angle by which the particle is deflected during the passage
- Derived from force by integration over the collision assume: $\mathbf{m_1 = m_2}$ **and** $\beta_1 = \beta_2$

$$\Delta r'$$

$$F_r(r,s,t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0 r\sigma_s}\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]\cdot\left[\exp(-\frac{(s+vt)^2}{2\sigma_s^2})\right]$$

→ Newton's law $$\Delta r' = \frac{1}{mc\beta\gamma}\int_{\infty}^{\infty} F_r(r,s,t)dt$$

Beam-Beam Kick

- Using the classical particle radius: $r_0 = e^2/4\pi\epsilon_0 mc^2$

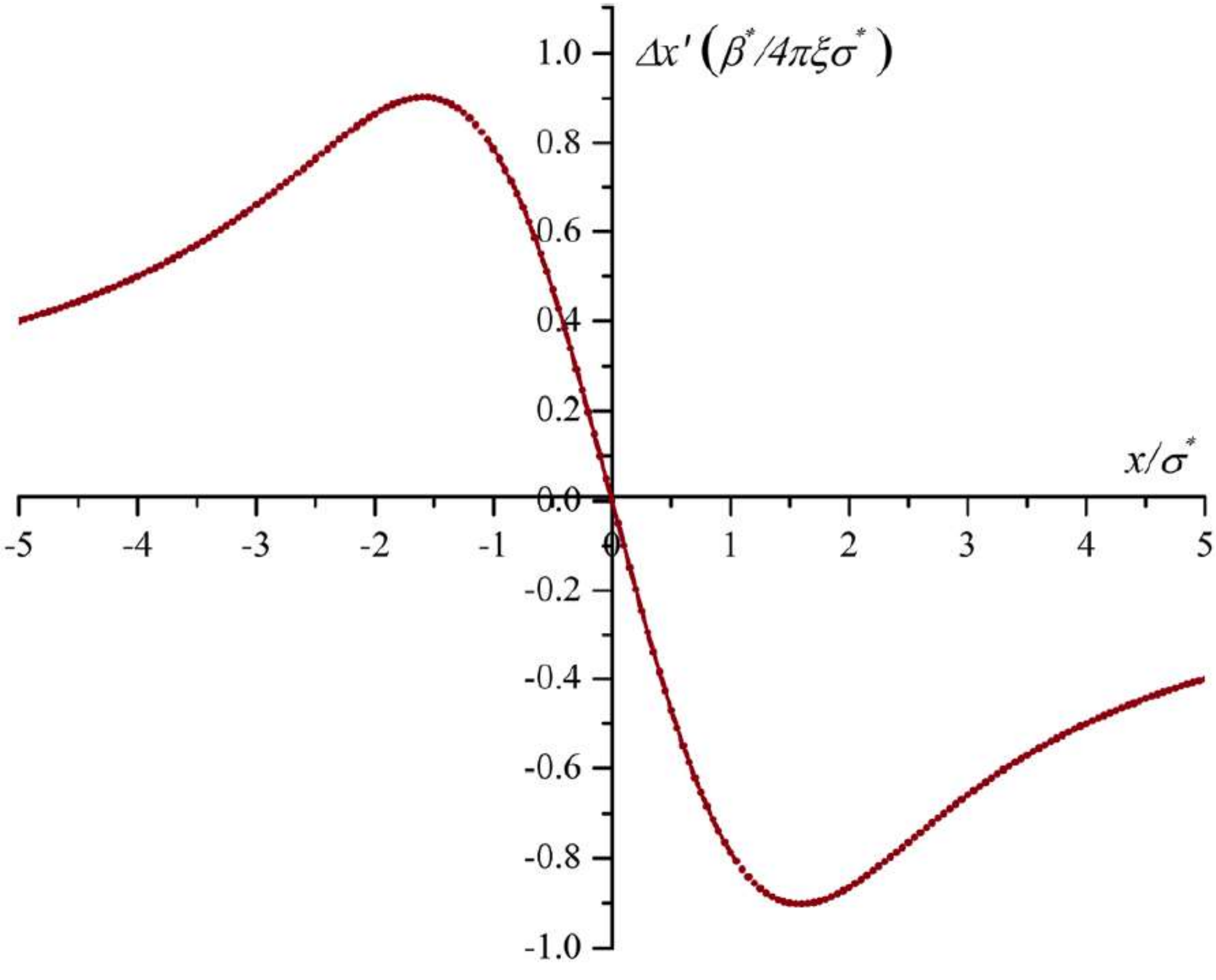
- we get radial kick and in Cartesian coordinates:

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta x' = -\frac{2Nr_0}{\gamma} \cdot \frac{x}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta y' = -\frac{2Nr_0}{\gamma} \cdot \frac{y}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Beam-Beam Kick



Kick(force) varies strongly with amplitude:

- linear inside → like quadrupole → tune shift amplitude independent at << sigma
- 1/r outside the beam core → amplitude dependent tune shift

Highly nonlinear btw 1 and 3 sigma:

- contains many high order multipoles

Beam-beam strength parameter → tuneshift

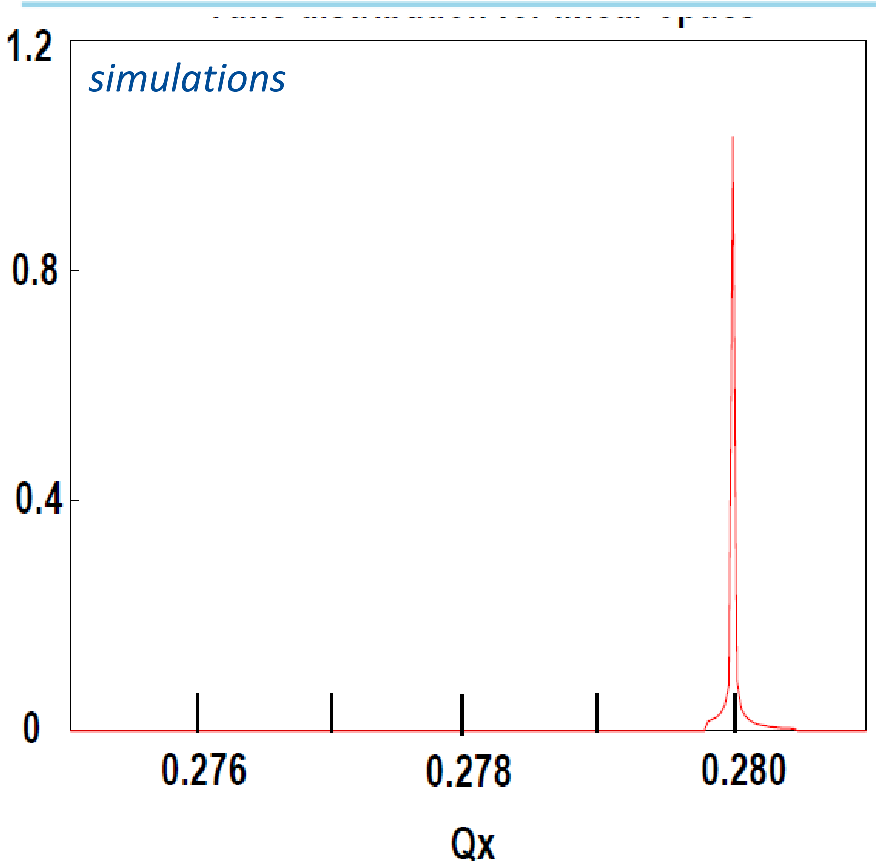
- Slope of force at zero amplitude → proportional to (linear) tune shift $\Delta\,\mathbf{Q}_{bb}$ from beam-beam interaction
- This defines: *beam-beam parameter* ξ
- For head-on interactions we get:

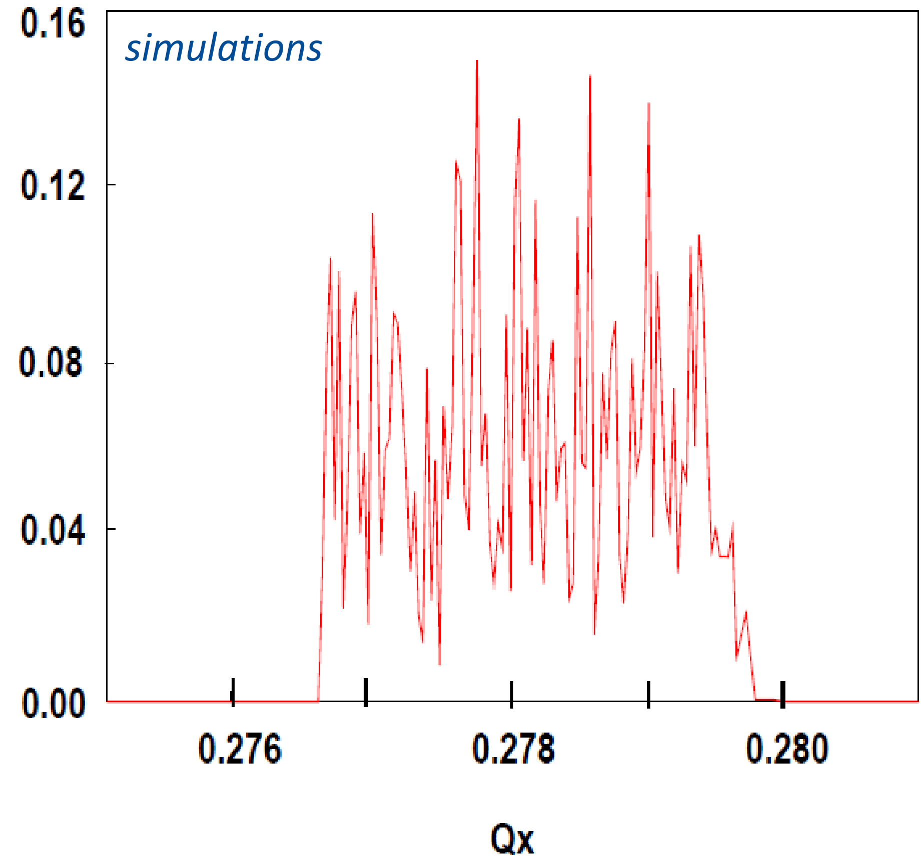
$$\xi_{x,y} = \frac{N \cdot r_0 \cdot \beta_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

- so far: only an additional "quasi-quadrupole" BUT non-linear part of beam-beam force scales with ξ

Note that for flat beams $\sigma_x >> \sigma_y$ $\xi_y >> \xi_x$

Tune Spectra: with/w.o. Beam-Beam





Linear force →
all particles have same tune
→ one line in the spectrum of transverse oscillations

Non−linear force →
particles with different amplitudes have different frequencies (tunes)
We get frequency (tune) spectra
Width of the spectra: ~ξ

In Reality – Even More Complex

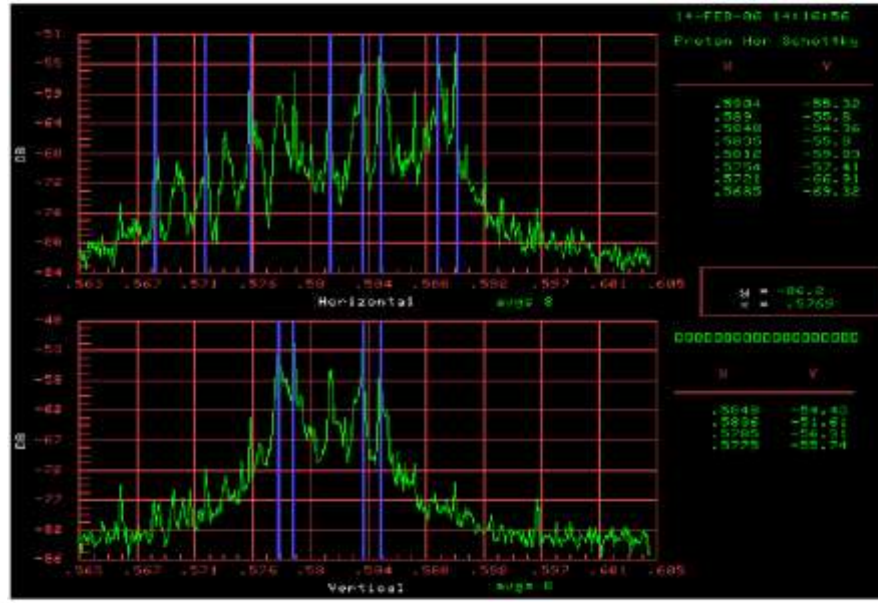
Tevatron 980 GeV p and

980 GeV antiprotons (pbars)

Colliding with ***ξ~0.028***

Force is focusing → tuneshift is positive

Measured with 21MHz Schottky monitors

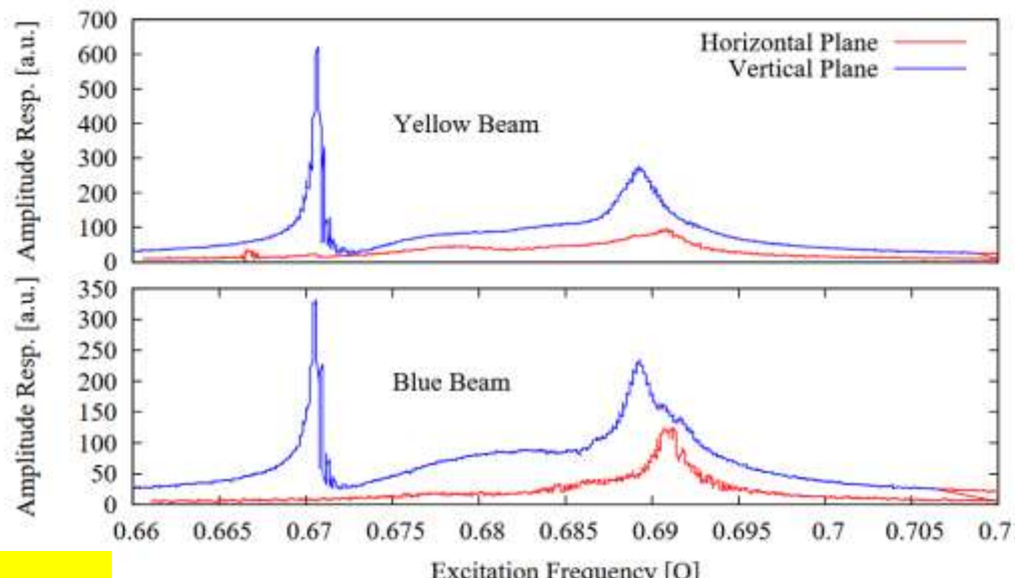


RHIC 100 GeV p + 100 GeV p

Colliding with ***ξ~0.020***

Force is de-focusing → tuneshift is negative

Measured with BTF (beam transfer function) monitor



…more on that later

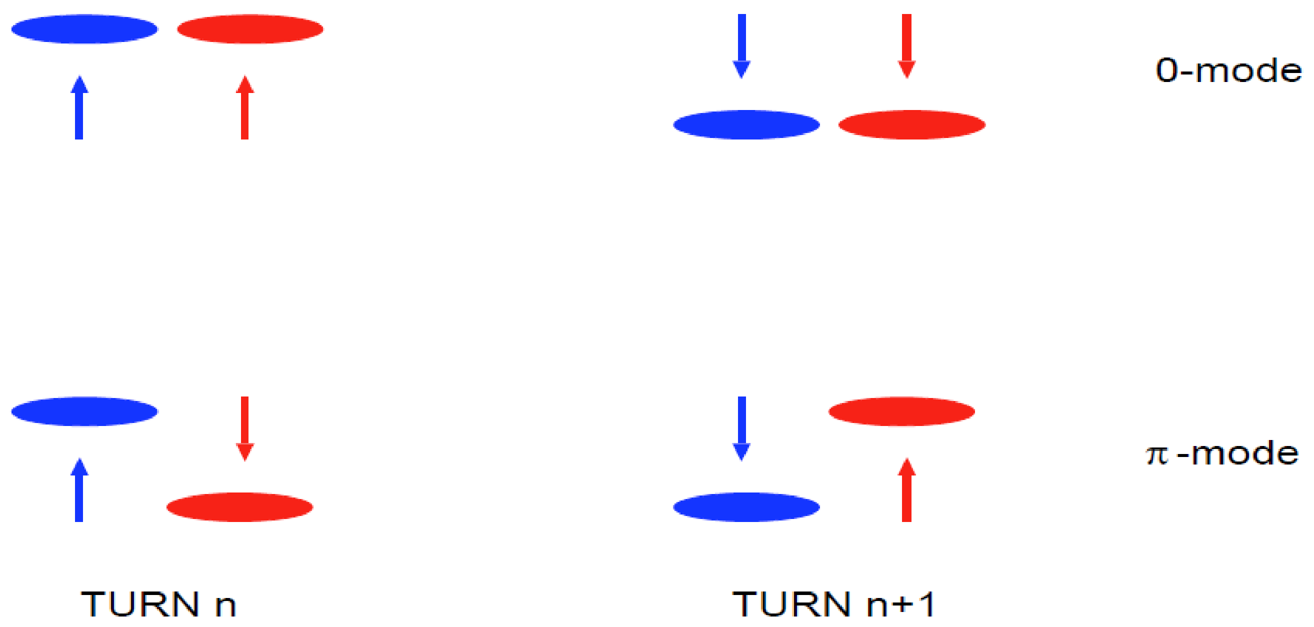
Complications : Strong-Strong vs Weak-Strong

- Both beams are very strong (strong-strong):
  - Both beam are affected and change due to beam-beam interaction
  - Examples: LHC, LEP, RHIC, ...
- One beam much stronger (weak-strong):
  - Only the weak beam is affected and changed due to beam-beam interaction
  - Examples: SPS collider, Tevatron (early in Run II) , ...

Incoherent vs Coherent Beam-Beam Effects

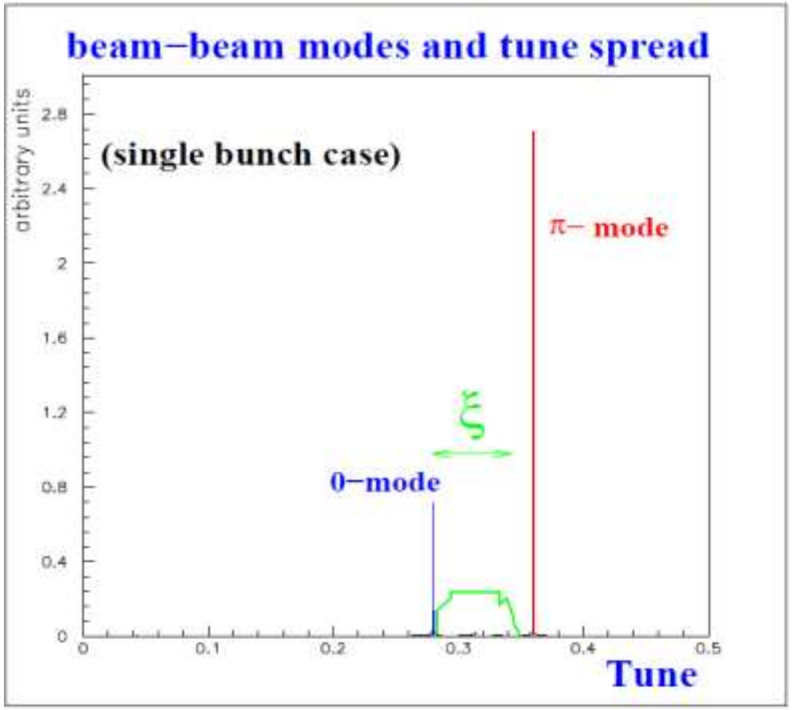
- Incoherent (single particle effects):
  - Single particle dynamics - treat as a particle passing through a static electromagnetic lens
  - Basically, non-linear dynamics effects:
    - unstable and/or irregular motion ("chaos")
    - beam size blow up or bad lifetime
    - Very bad: unequal beam sizes (studied at SPS, HERA, Tevatron)
- Coherent (bunches affected as a whole):
  - Collective modes
  - Bunch-by-bunch differences in:
    - Orbits
    - Tunes
    - Chromaticities

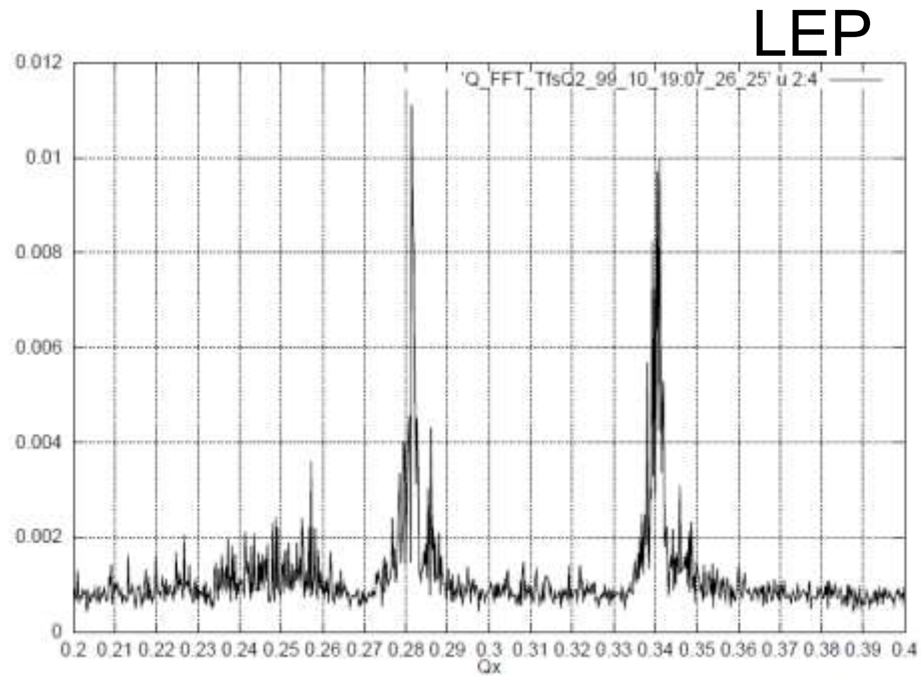
Coherent Beam-Beam: Modes



- Coherent mode: two bunches are "locked" in a coherent oscillations
  - 0-mode is stable (Mode with NO tune shift)
  - π-mode can become unstable (Mode with LARGEST tune shift)

Coherent Beam-Beam: Modes





- 0−mode is at unperturbed tune
- π−mode is shifted by 1.1 − 1.3 · ξ
- Incoherent spread between [0.0,1.0] · ξ

Two modes clearly visible
Can be distinguished by phase relation, i.e.
sum and dierence signals

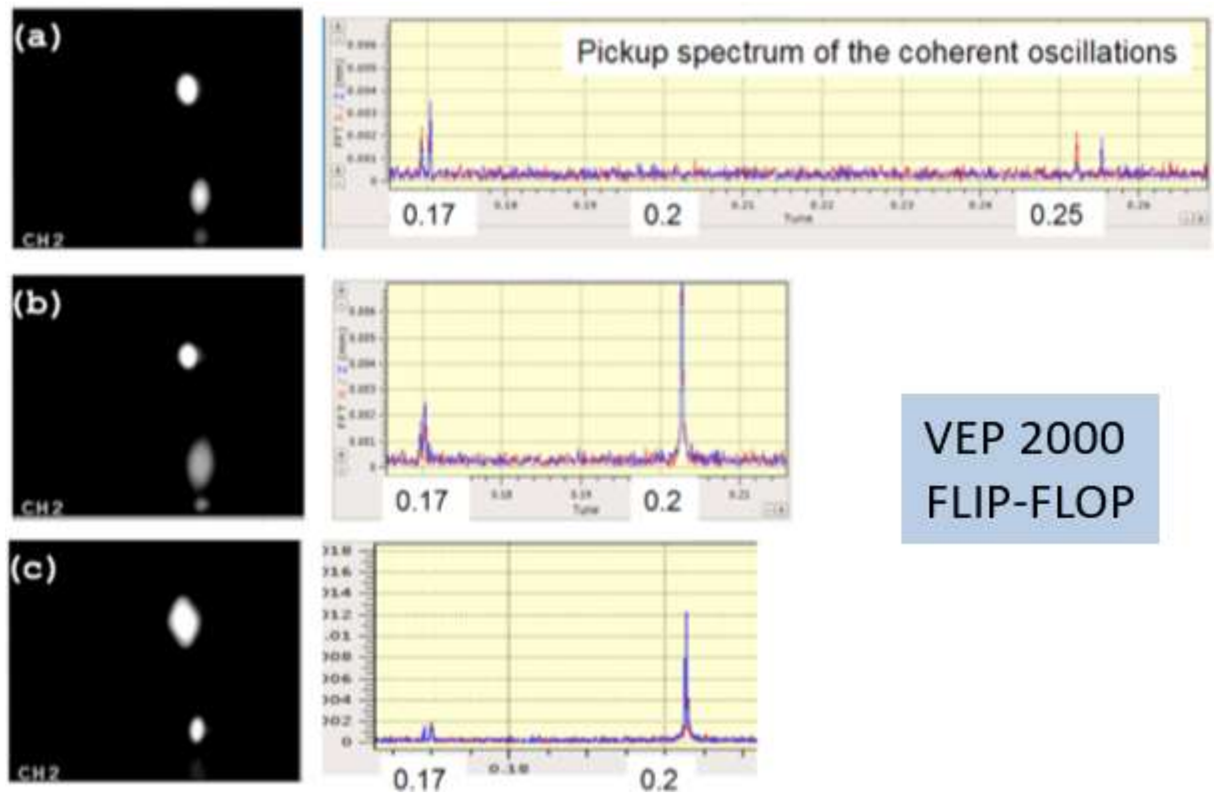
Coherent Beam-Beam: Flip-Flop

Bunch sizes get bigger or smaller out of phase

(PEP-II, VEPP-2000, etc)

The intensity threshold for the flip-flop depends on:

- asymmetry in beam intensities
- x-y coupling



3D Flip-Flop effects triggered by non-linearities of lattice. π-mode on 1/5 resonance. The effect have shown a strong sensitivity to X-Y coupling, beta unbalance and bunch length → main limitation in VEPP 2000.

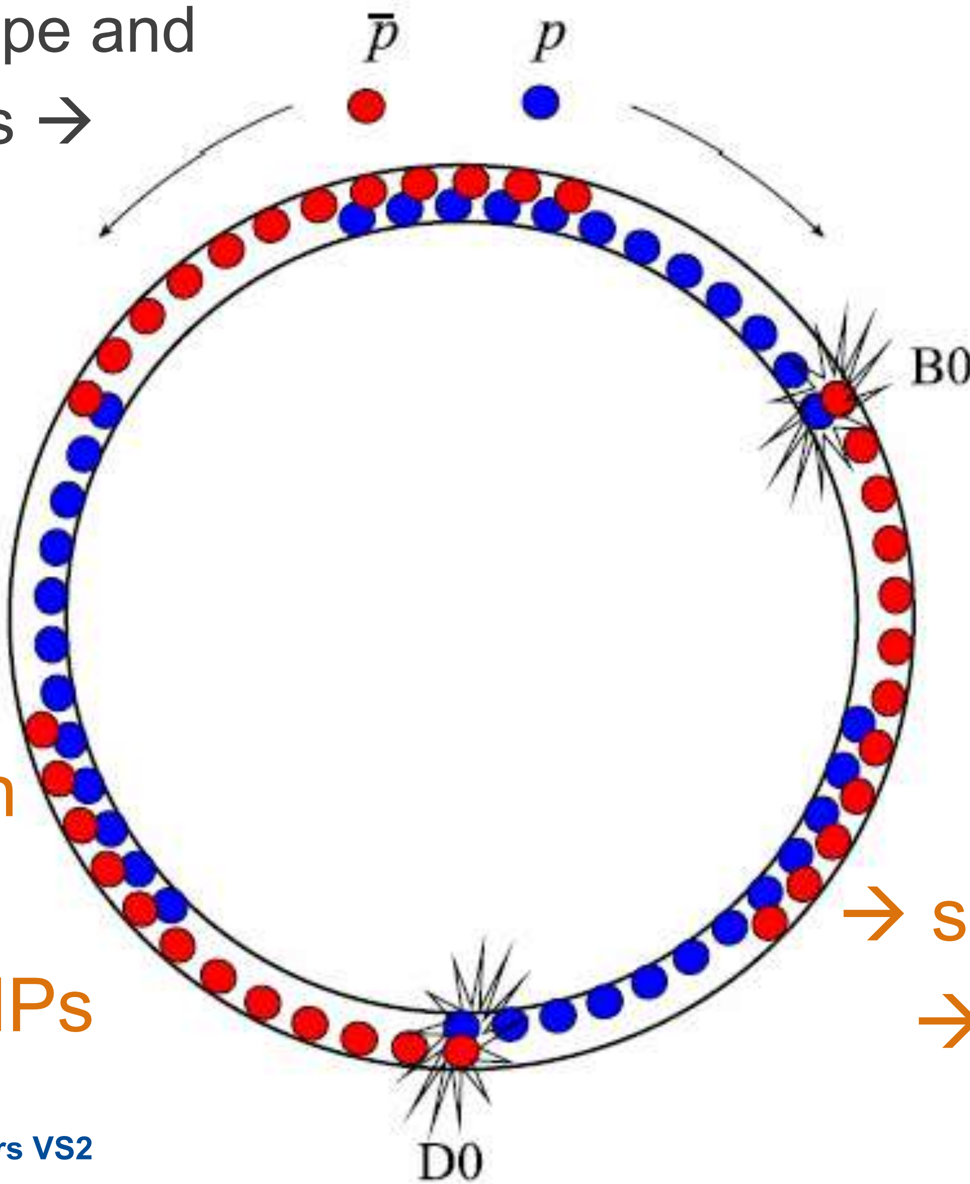
Multi-Bunch Operation: Need and Issues

$$\mathcal{L} = \frac{N_1 N_2 f \cdot B}{4\pi\sigma_x\sigma_y}$$

- How to collide many bunches (for high *L*) ??
- Must avoid unwanted collisions !! Otherwise $\xi \rightarrow 2B\xi$
- Separation of the beams:
  - Pretzel/helix scheme (SPS,LEP,Tevatron)
  - Bunch trains (LEP, PEP)
  - Crossing angle (LHC)

Tevatron: 36 proton x 36 antiproton

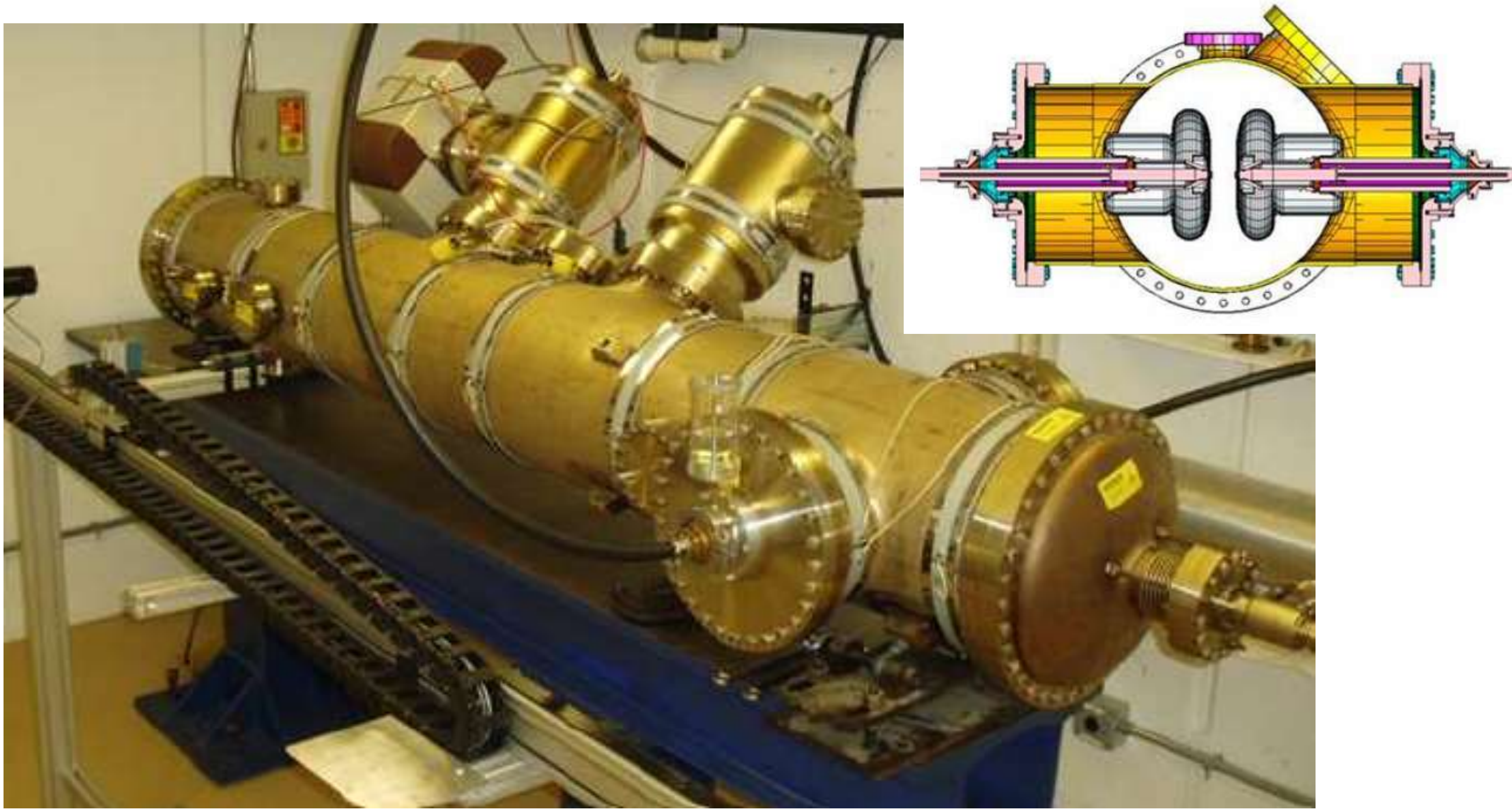
Same beam pipe and magnetic fields → same orbits → 72 IPs



396 ns bunch separation → 59 m btw IPs

Need only 2 → separate at 70 → Electric field

Tevatron High Voltage Electrostatic Separators



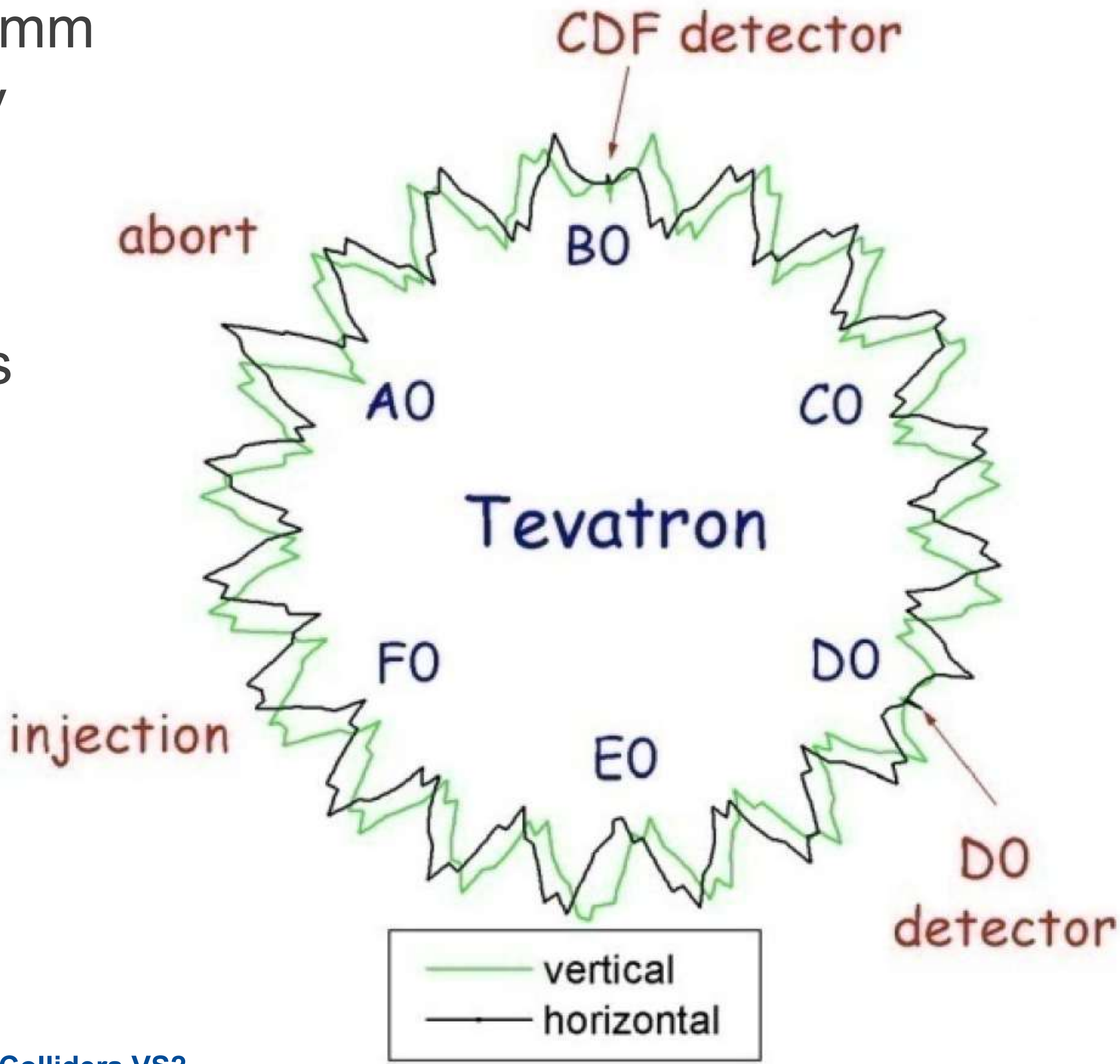
300 kV over 50 mm gap; 3 m ; 24 of them (H/V)

Tevatron Helix

24 electrostatic separators are used

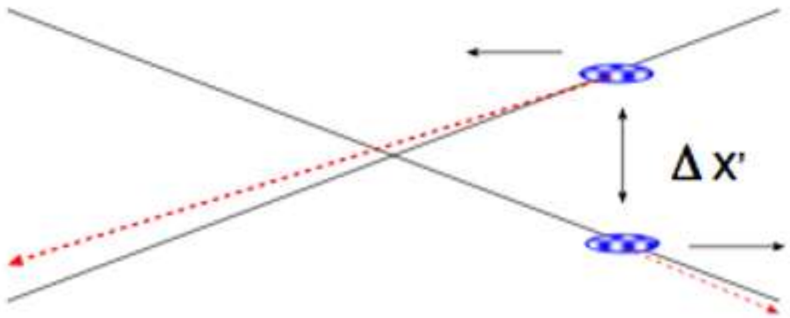
size 12-15 mm
at 150 GeV

6-8 mm
at collisions



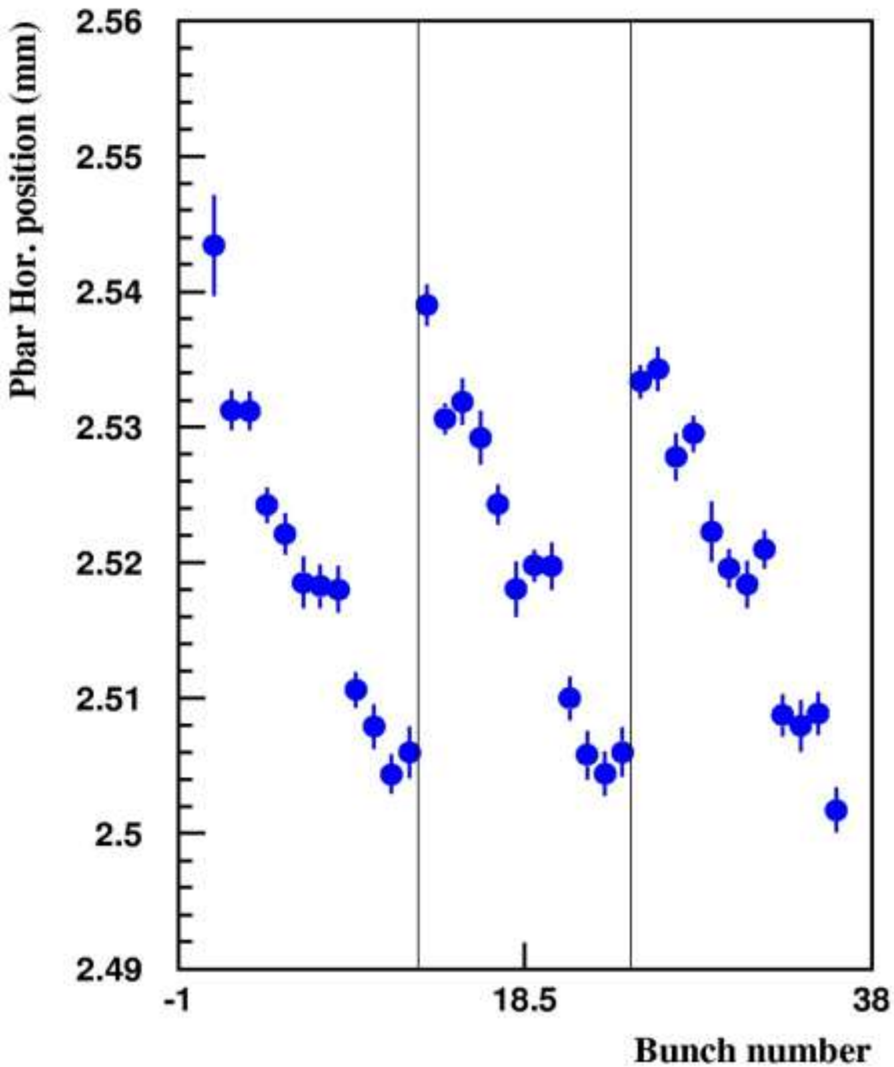
All beam indicators become bunch dependent due to long-range beam-beam effects

$$\Delta x' = \frac{const}{d}[1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \ldots]$$



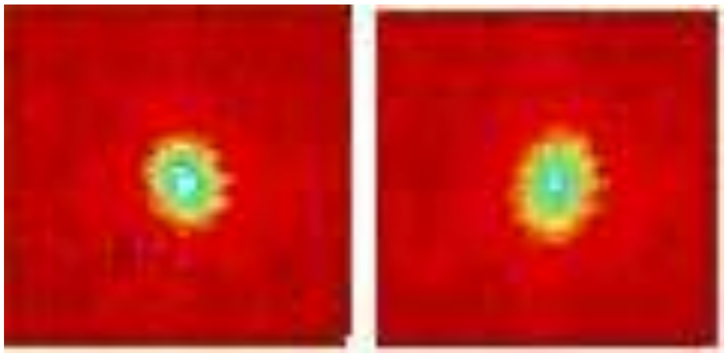
- Orbits
- Tunes, couplings
- Chromaticities
- In both – protons and pbars
- Have 3-fold symmetry (trains of 12)

Long-range B-B Seen at Low-Beta (980 GeV)

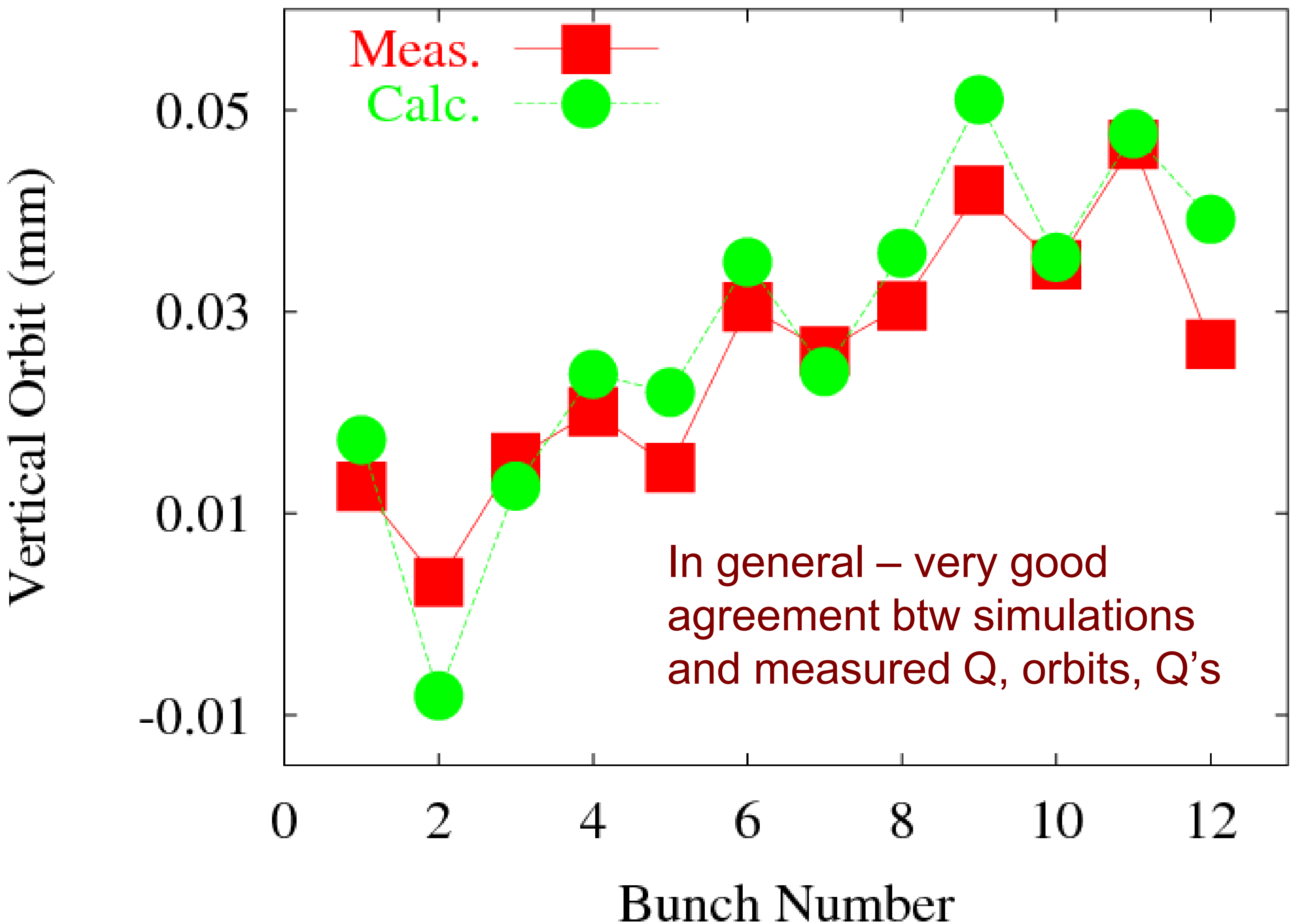


- Synchrotron light monitors show 40 micron b-by-bunch hor pbar orbit variation along the bunch train with 3-train symmetry (4 microns for protons)
- Also indicate coupling differences →

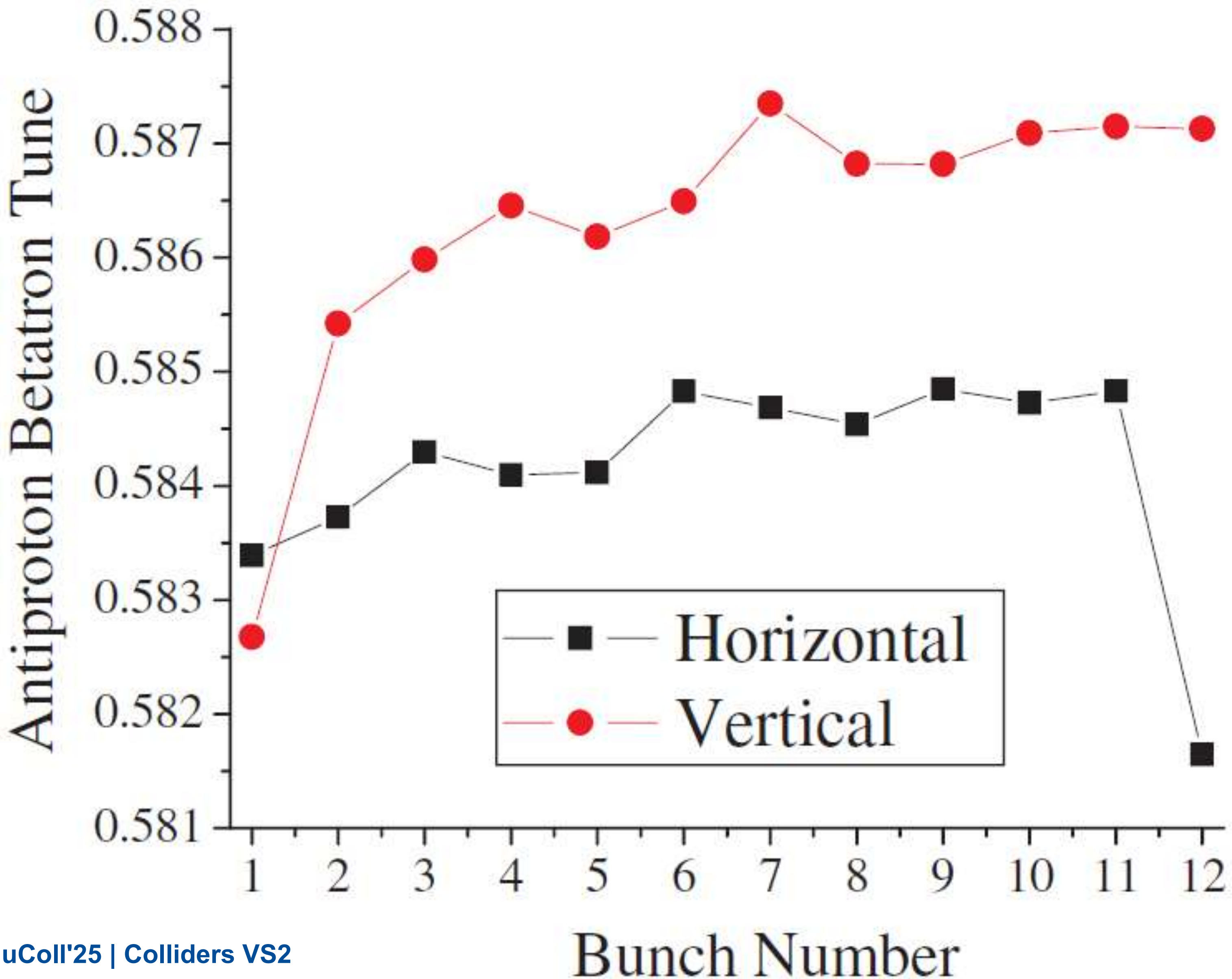
Bunch #1 Bunch #8



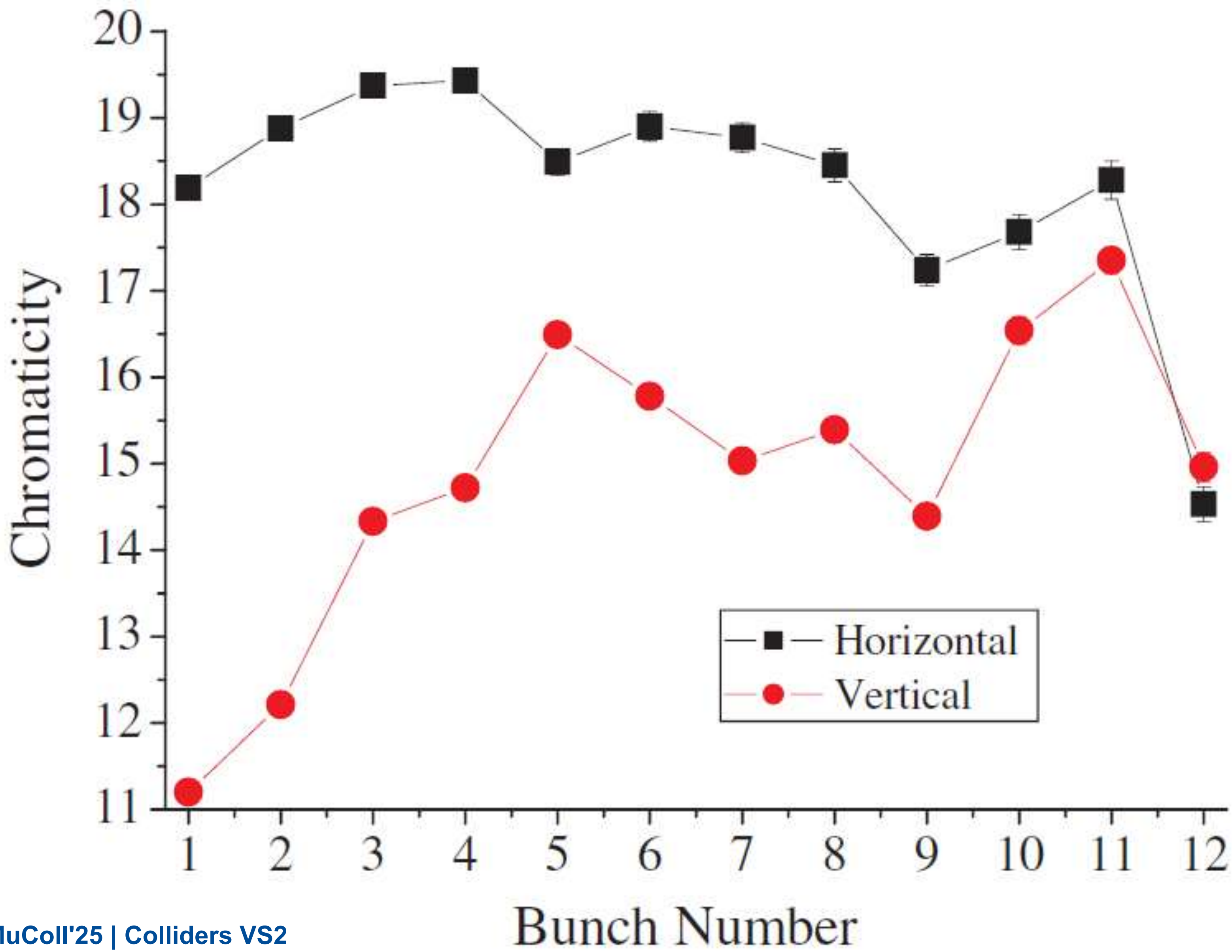
Antiproton Vertical Orbit



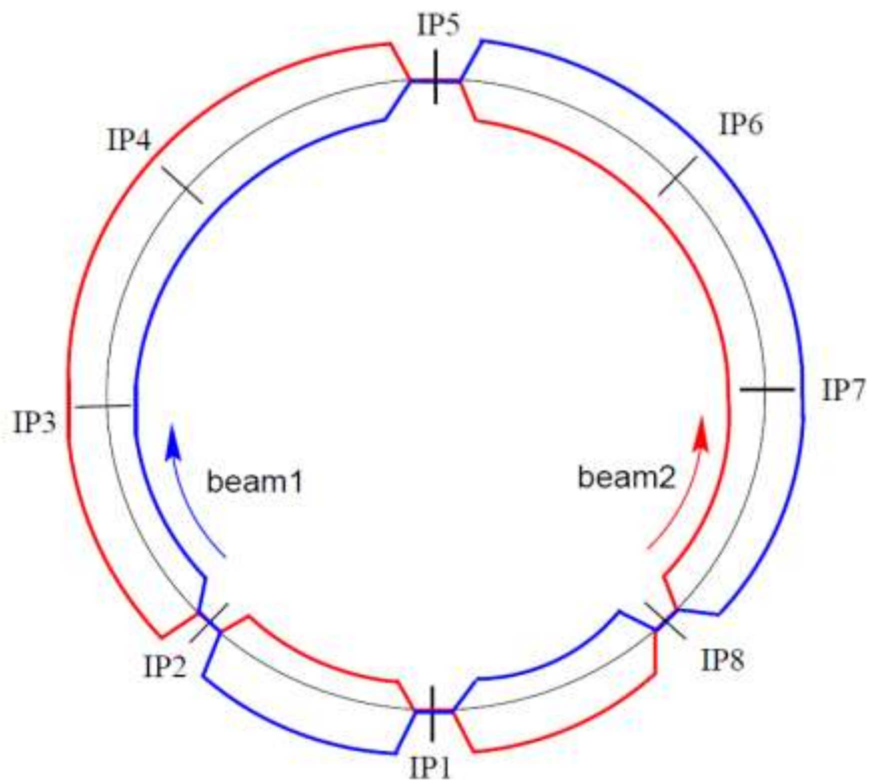
Pbar Bunch Tunes in Collisions

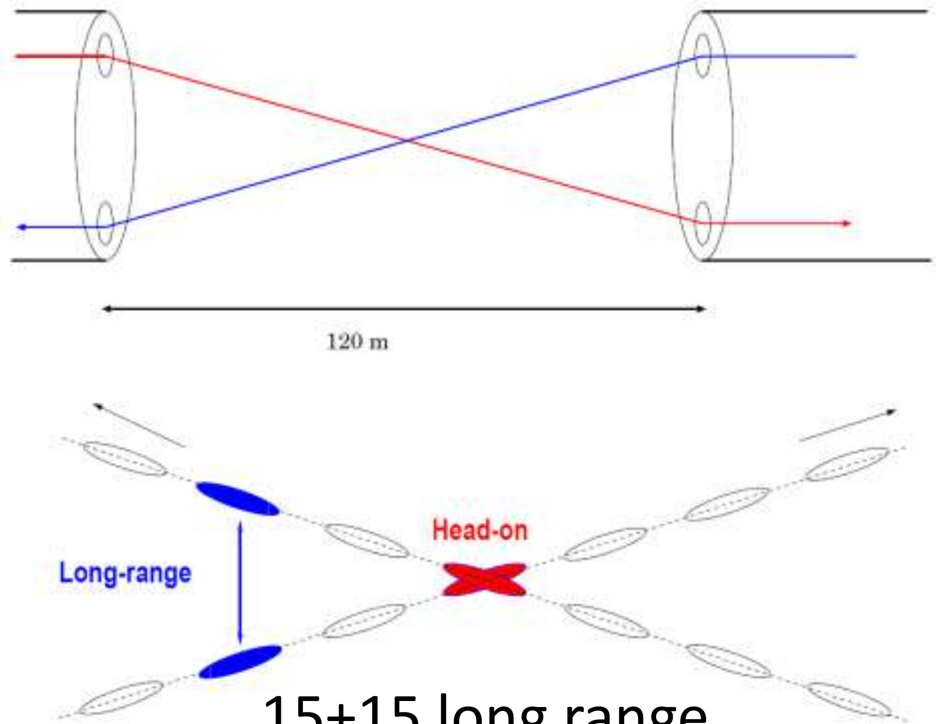


Pbar Bunch Chromaticity in Collisions



In the LHC

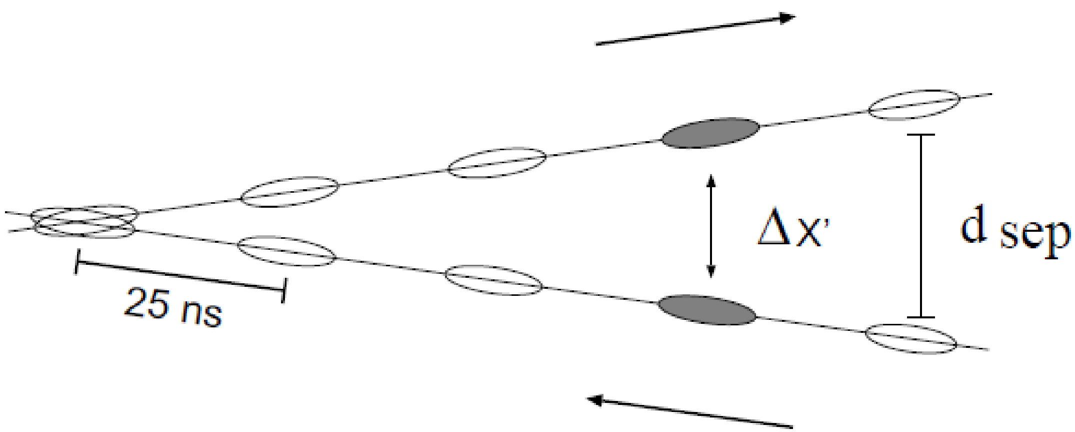




15+15 long range interactions (6-12 σ)

- 2808 p bunches in each beam, every 25 ns
- Two beams in separate beam pipes except in common chamber around 4 experiments
- Local separation via two horizontal and two vertical crossing angles

Parasitic Beam-beam Kicks



For horizontal separation d:

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2} \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$(with: \quad r^2 = (x+d)^2 + y^2)$$

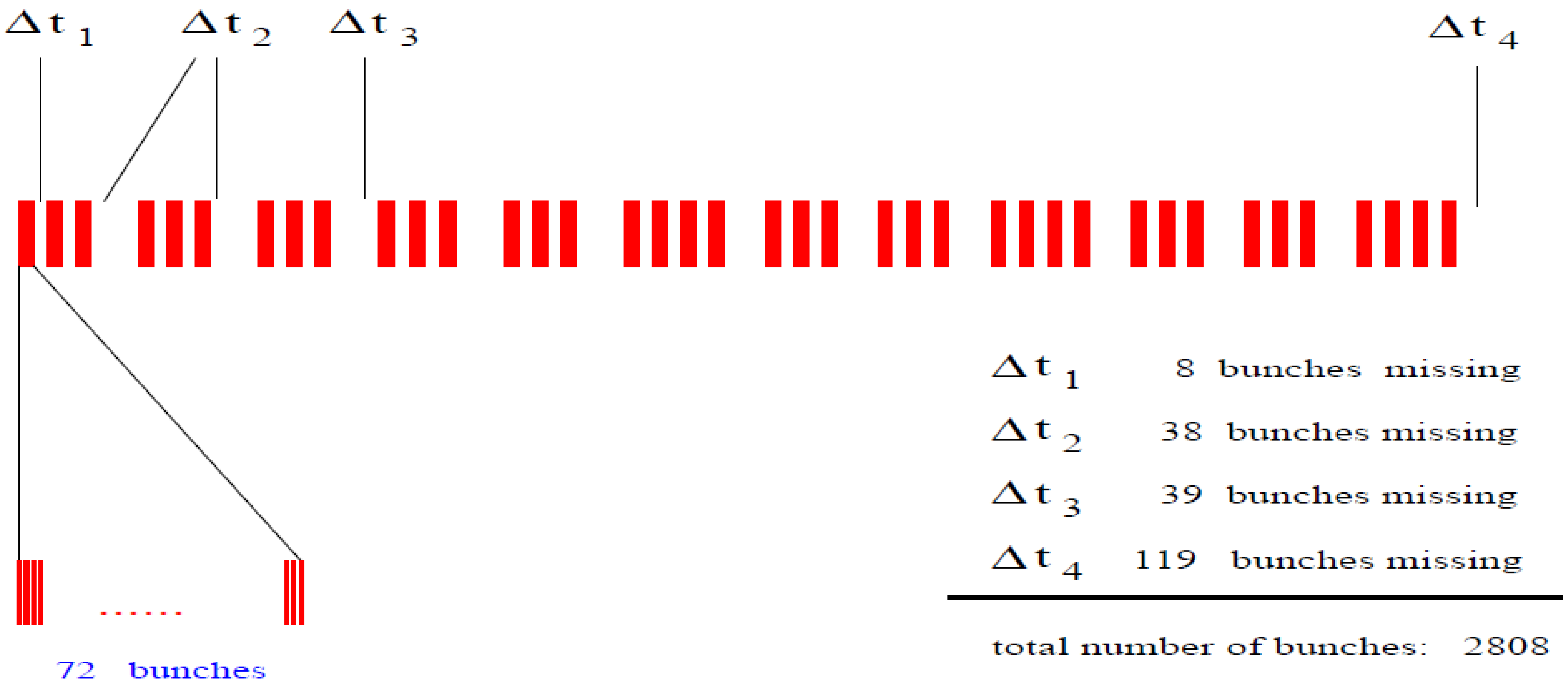
In LHC 15 collisions on each side, 120 in total!

Effects depend on separation, eg tuneshift

$$\Delta Q \propto -\frac{N}{d^2}$$

PAMCMAN bunches due to gaps

- Average orbit and tune variations can be corrected, but:



LHC bunch filling not continuous: holes for injection, extraction, dump ..

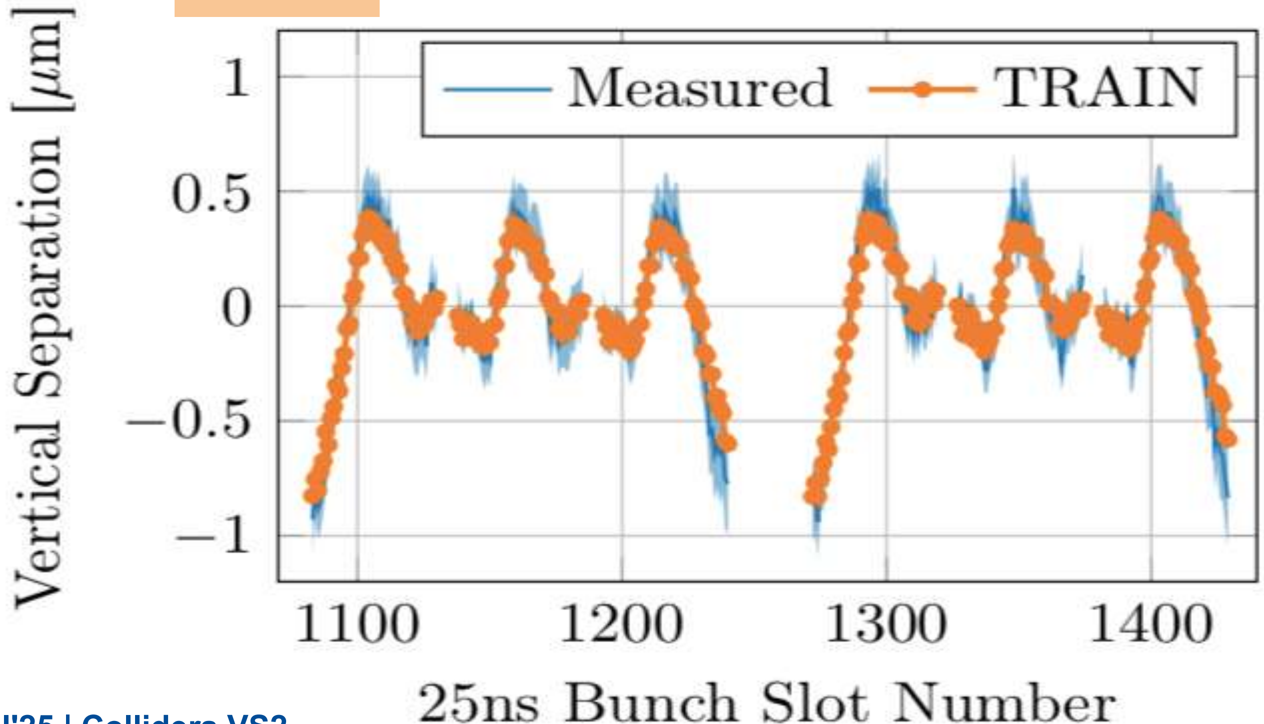
"Only" 2808 of 3564 possible bunches circulate ! 1756 "holes"

"Holes" meet "holes" at the interaction point - But not always ...

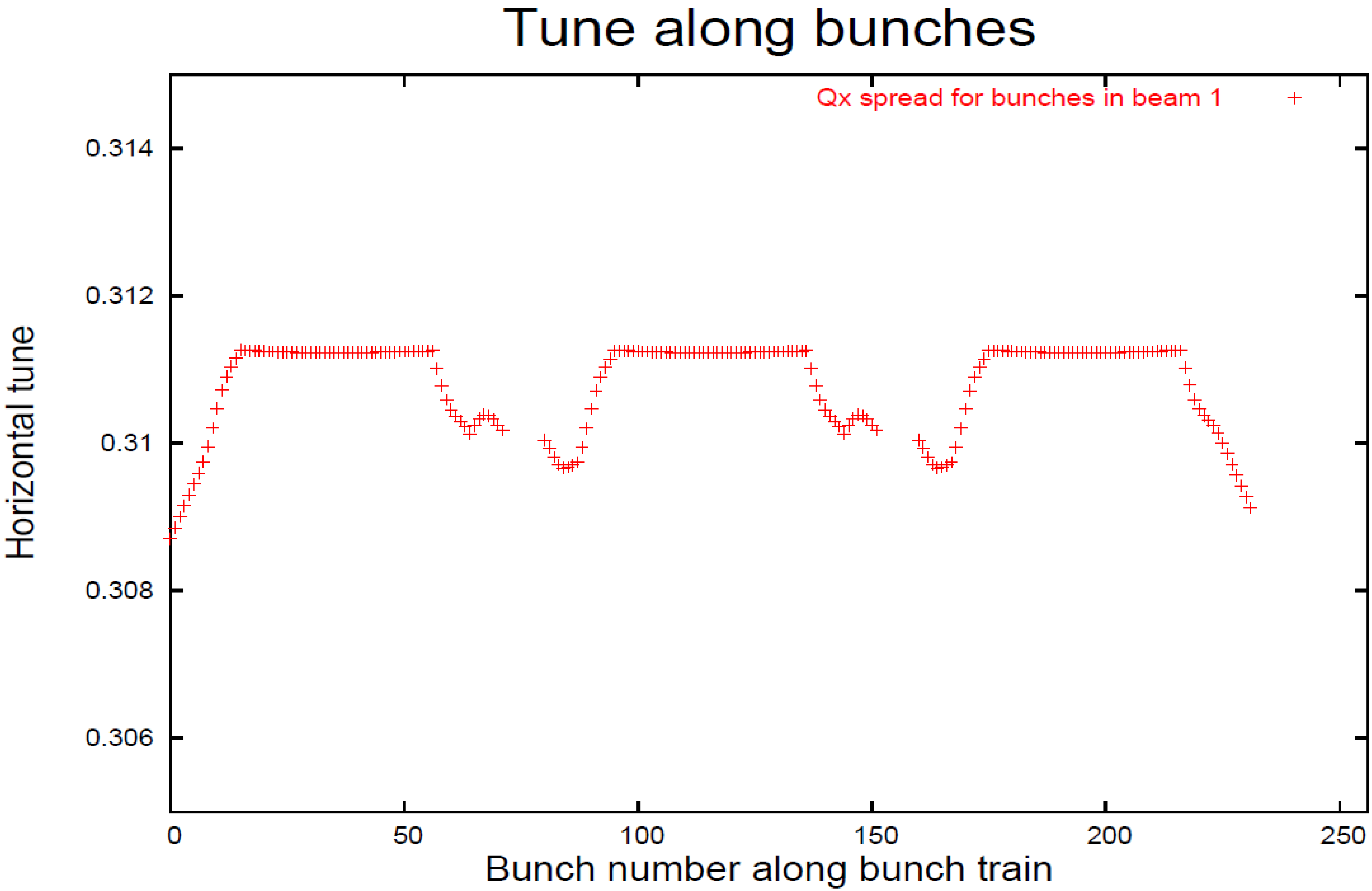
Effect of PACMAN bunches (end of train)

- Some bunches can meet a hole/holes (at beginning and end of bunch train) →
- They see fewer unwanted interactions in total: between 120 (max) and 40 (min) long range collisions → Different integrated beam-beam effect for different bunches

LHC

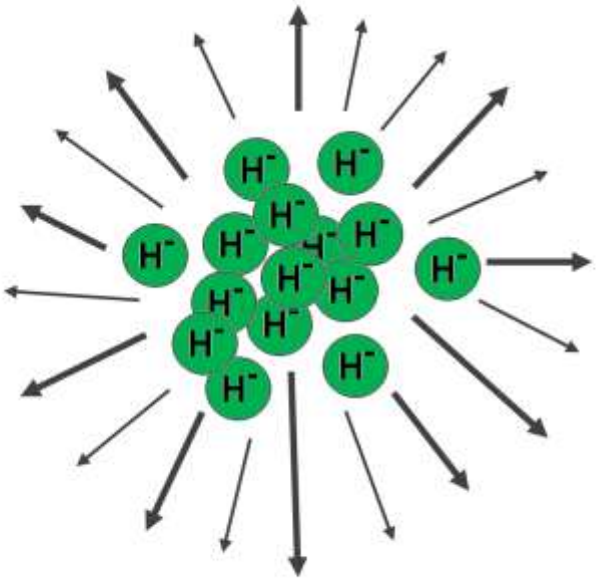


Tune Spread - too large for safe operation

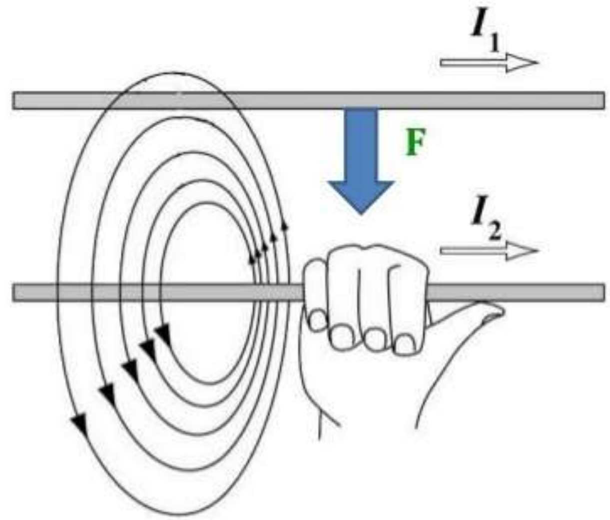


Transverse Interaction of Co-Moving Charges

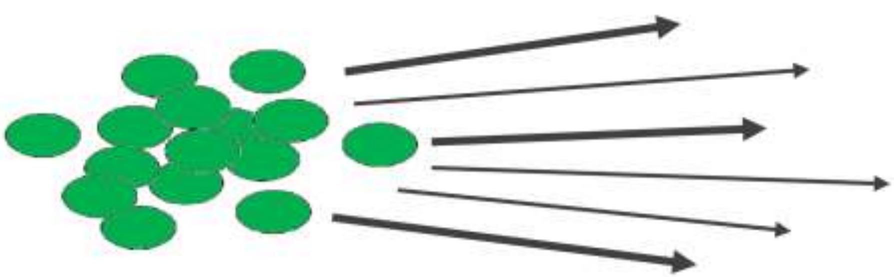
Electric Repulsion:



Magnetic Attraction:



Weakened Repulsion with Acceleration:



$$\vec{E}_\perp = \gamma E'_\perp$$

$$\vec{B}_\perp c = \beta(\hat{z} \times \vec{E}_\perp)$$

$$\vec{F}_\perp = q(E + v \times B)_\perp$$

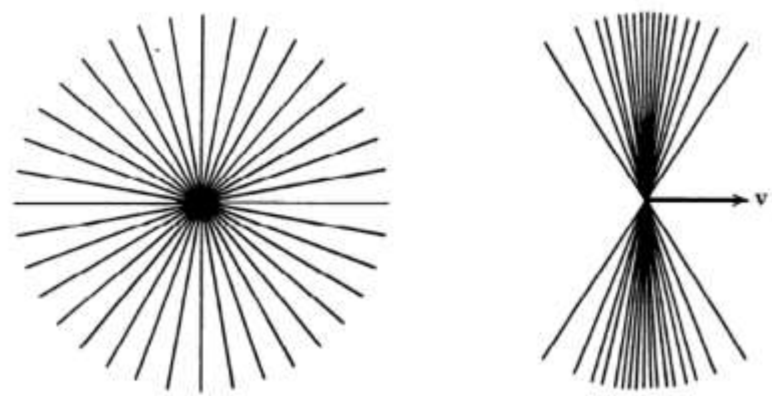
$$\vec{F}_\perp = q(1 - \beta^2)E_\perp = \frac{q}{\gamma^2} E_\perp$$

Space-charge Core vs Tail

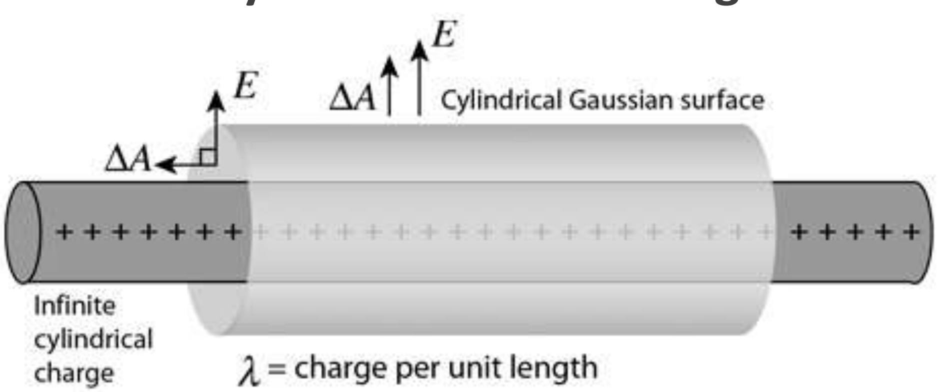
Transverse space-charge forces much stronger than longitudinal space-charge.

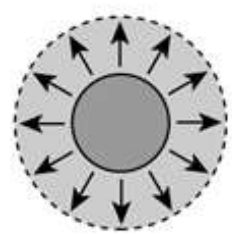
Transverse and Longitudinal charge distribution can be written as separable functions: $$\rho(x,y,z) = \lambda(z)\rho_\perp(x,y)$$

Relativistic Distortion of EM Fields "Pancake-ification"



Gaussian cylinder for a line-charge:





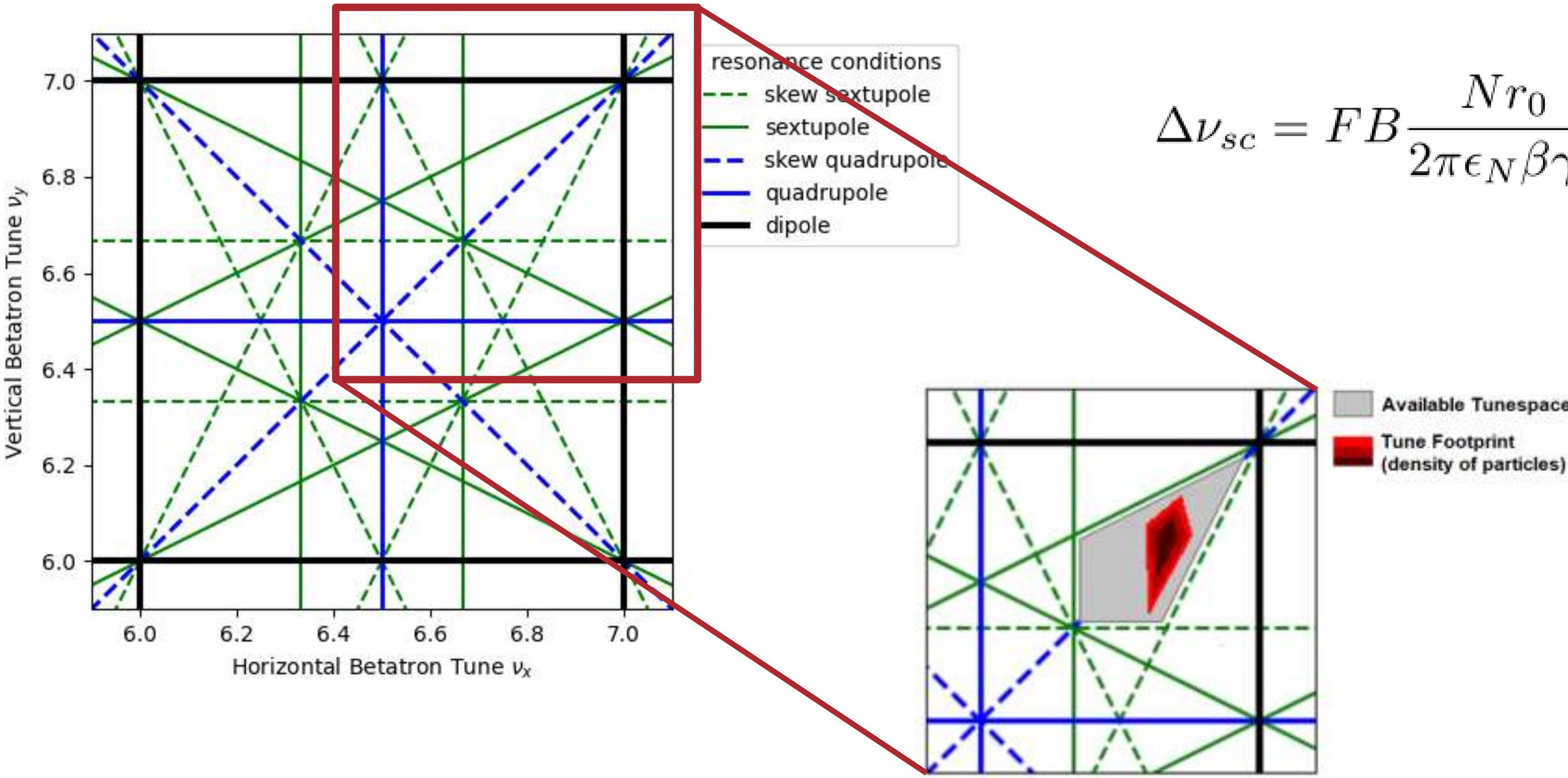
For $r < R$

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

For $r \geq R$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Tune Diagrams



$$\Delta\nu_{sc} = FB\frac{Nr_0}{2\pi\epsilon_N\beta\gamma^2}$$

Summary

The linear transverse dynamics of a particle accelerator are governed by Hill's Equation, which is a time-varying harmonic oscillator.

We calculate the trajectory of individual particles through the many individual magnets of a particle accelerator using transfer matrices.

Transfer matrices are also used for the beam size and oscillation phase, which are represented by Courant-Snyder parameters.

There are chromatic effects, resonances, and space-charge effects that complicate the process of designing and operating a particle accelerator.

Some backup slides on longitudinal dynamics.