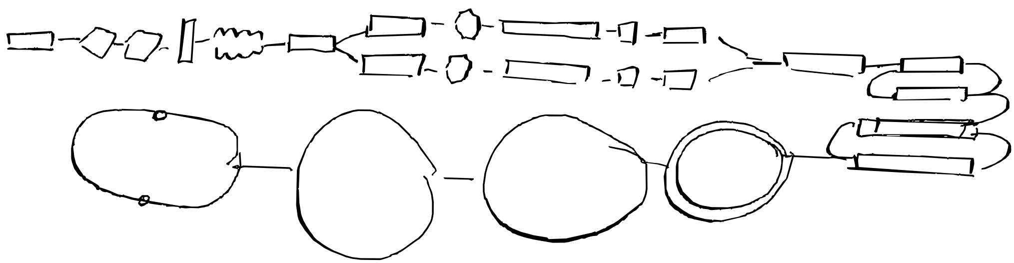
Section 1: Introduction to Muon Acceleration

1) Initial Beam From Pre-Accelerator
2) Greenfield energies of the whole system, step by step



Final Cooling ~5MeV final momentum
Long beam size ~10m

Preaccelerator: 5MeV -250MeV for relativistic beam
Very low frequencies ~5MHz

| What is frequency & gradient |

Preface: Handling beam prior to Synchrotrons

Section 2: Low Level Accelerator

1) Superconducting LINAC parameters
2) RLA design, arcs, dogbone or racetrack

LINAC: Superconducting RF, 325MHz to 1.25GeV
Via two Cryomodules 20MV/m

Q1) Start at 250MeV, end at 1250MeV for 20MV/m. What is the length?

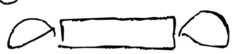
Then we want to get to 62.5GeV = 62500MeV.

Q2) Why would we want to get to 62.5 GeV.

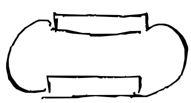
Q3) How long would this LINAC be? What's another thing we can do instead?

$62500 - 1250 = \frac{61250}{20}$
$= 3062.5m$

Recirculating Linear Accelerator (Like ERL)
Either dogbone or racetrack



Beamloading from following bunches
Symmetric optics
Long arcs



Shorter arcs
Same beam loading
More passes, lower efficiency

RLA1: 1.25GeV → 5GeV
RLA2: 5GeV → 63GeV

Gain per Pass: 0.85GeV

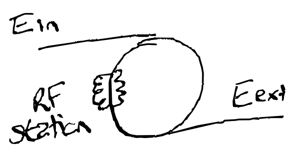
Q4) How many passes?

Need seperate arcs for each energy
~300m

Frequencies of 352 & 1056 MHz

Section 3: High Energy Acceleration

1) Introduction to synchrotrons
2) Global parameter optimisation
3) Decays and gradient
4) B-Ramps
5) NC and Hybrid RCS
6) FODO cells, orbit excursions
7) Magnet design and magnet ramp rate and power converters



$$\text{Energy Swing} = \frac{E_{ext}}{E_{in}}$$

Q5) Why is muon acceleration challenging? Decays!

First we will quantify what acceleration energy swing & average gradient is required to give us reasonable survival rate.
Then we will calculate magnetic ramp rate for a given circumference & what designs give these rates.

How do we quantify decays? Need $\frac{dN}{dE}$ rate.

$$\frac{dN}{dt} = -\lambda N = \frac{-1}{\gamma \tau_\mu} N = \frac{-m_\mu c^2}{E \tau_\mu} N$$

$$E = \gamma m c^2 \qquad \gamma = \frac{E}{mc^2}$$

$$\frac{dE}{dt} = e\bar{V}c$$ → MV/m mean field

$$\frac{dN}{dt} = \frac{dN}{dE}\frac{dE}{dt}$$

$$-\frac{m_\mu c^2}{E \tau_\mu} N = \frac{dN}{dE} \times e\bar{V}c$$

$$\frac{dN}{dE} = \frac{-m_\mu c}{e\bar{V}\tau_\mu}\frac{N}{E} = -\frac{N}{E}\frac{1}{\delta_\tau} \quad \text{where } \delta_\tau = \frac{e\bar{V}\tau_\mu}{m_\mu c}$$

After Integrating $y = \frac{N_1}{N_0} = \left(\frac{E_1}{E_0}\right)^{-\frac{1}{\delta_\tau}} = \left(\frac{E_1}{E_0}\right)^{-\frac{m_\mu c}{e\bar{V}\tau_\mu}}$

Q6) Ask for $\bar{V}$ & E_1 to aim for ($\bar{V} = 2.4\,MV/m$ throughout ring, $E_1 = 315$)

Calculate with

$$= \left(\frac{E_1}{62.5}\right)^{\left\{\frac{-105\,MeV/c^2}{\bar{V}_{MeV} \times 2.2\times10^{-6} \times 3\times10^{8}}\right\}}$$

$$\left(\frac{E_1}{62.5}\right)^{-\frac{1}{\bar{V}\times 6}} \qquad \frac{-100}{\bar{V}\times 600} = \frac{-1}{\bar{V}\times 6}$$

→ Get transmission e.g. 0.89

Next we calculate Circumference & number of turns. Both can inform magnetic ramp rate.

Assumes synchrotron full of RF

$$\text{Len RF} = \frac{E_1 - E_0}{\bar{V}}$$ 105000km

Q8) Ask for Circumference (6000km)

$$N_{turns} = \frac{\text{Tot len}}{C} \quad \text{e.g. } 17 \qquad \rho = \frac{C}{2\pi}$$

Q9) B-field of the magnets at lowest & highest energy

$$F_{Lorentz} = qE + qv \wedge B$$

$$F_{Centr} = \frac{mv^2}{\rho} \text{ (radius)}$$

$$qvB = \frac{mv^2}{\rho}$$

$$B\rho = \frac{mv}{q} = \frac{p}{q}$$

$$B\rho = 0.33\frac{p}{q}$$ (beam rigidity)

[Tm] ~ 0.3 [GeV/c]

$$B\rho = 0.3 \times 62.5$$
$$B\rho = 18.75\,Tm$$
$$B_i = \frac{0.33 p\, 2\pi}{C \times frac}$$
$$B_{inj} = 0.35\,T$$
$$B_{ext} = 1.8\,T$$

Q10) Ramp rates of these fields

Assume linear

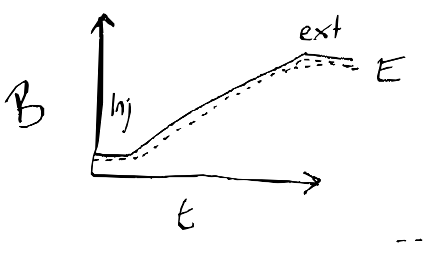
Assume rel

$$N_{turns} \qquad t_{AV} = \frac{C \times N_{turns}}{c} = 340\mu s$$

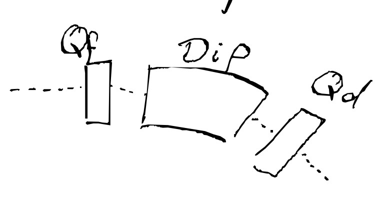
$$\frac{B_{ext} - B_{inj}}{t_{av}} = \frac{dB}{dt} = \frac{1.8 - 0.35}{340\mu s} = 4264\,T/s$$

Comment on power converters
Also for polarity

What would this look like in reality?



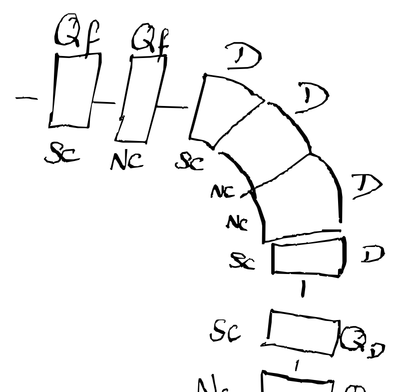
$B \propto E$ synchrotron for on-orbit beam



FODO cell Structure

What if $B_{ext} > 1.8T$. SC at ~kT/s impossible

Introduce HYBRID

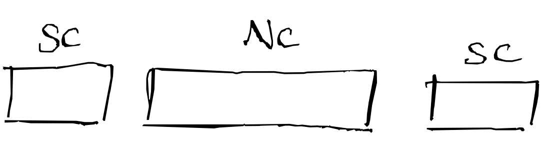


NC ramp from $-1.8T \rightarrow 1.8T$ in kT/s

SC static at 10T or 16T

Gives average fields of ~4T

Q11)



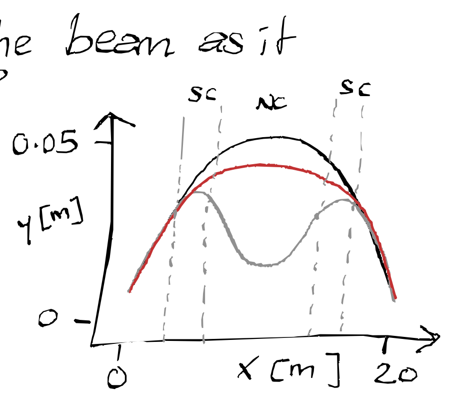
What will happen to the beam as it changes energy?

Orbit excursions!

= beam size range

= Larger magnets needed

~30 × 100 sized apertures



– Break for questions –

Section 4: RF

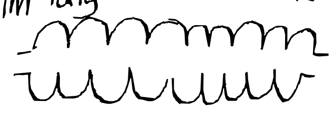
1) Number of RF stations for decay
2) Phase advance for high synchrotron tune and emittance increase
3) TESLA 1.3 GHz Cavities

Mentioned dipole packing fraction
RF also needs to be distributed around the ring

Q12) Voltage per turn if one Cavity

$$\frac{E_1 - E_0}{N_{turns}} = \frac{(314 - 62.5)}{17} = 14\ GV$$:')

Using TESLA Cavities
1.3GHz frequency
1m long



Made for ILC
Used in FELs

~20 - 46 MV/m (max theoretical)

We will use 30MV/m

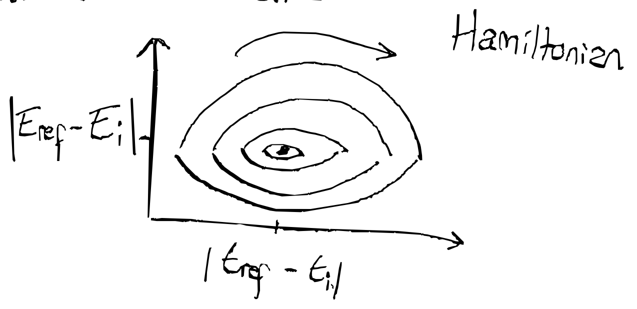
Q13) How many TESLA Cavities used?

$$\frac{E_1 - E_0}{30_{MV/m} \times N_{turns}}$$ e.g. 466

Must position in straight sections throughout accelerator

Consequences?

How we accelerate?

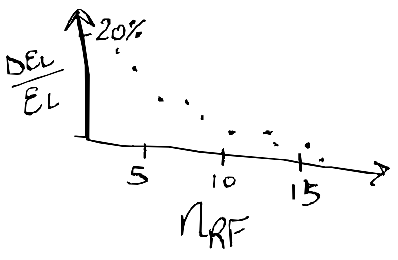


Synchrotron frequency is 1/time for one revolution.

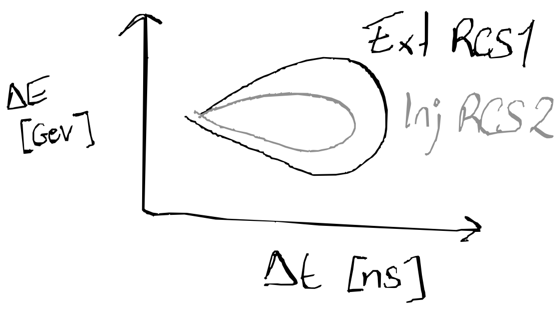
Particles need smooth motion going round phase-space

If Synchrotron tune is fast discontinuous trajectories & high A particles lost → Increases ε_L

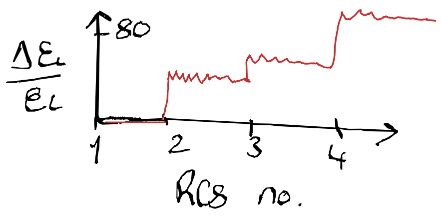
More RF stations provides smoother acceleration & therefore preserves ε_L



Matching between Sections



Q14) What happens to our beam?



Adjust phase to encompass injected beam.

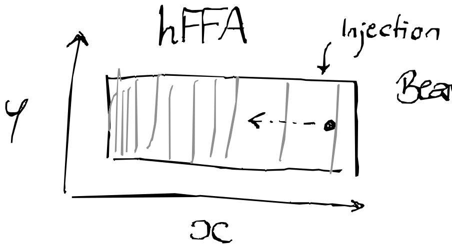
Same for All Systems/subsystems match longitudinally / transverse

HOMs

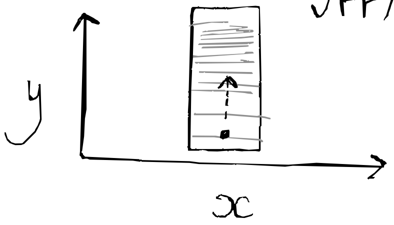
TM modes
Higher order Peaks
TESLA to Suppress
Effect on beam dynamics

Section 5: Fixed Field Accelerators

B_z Field changes in space but not time



Beam rigidity: Higher energy beam increases in radius, sees higher field



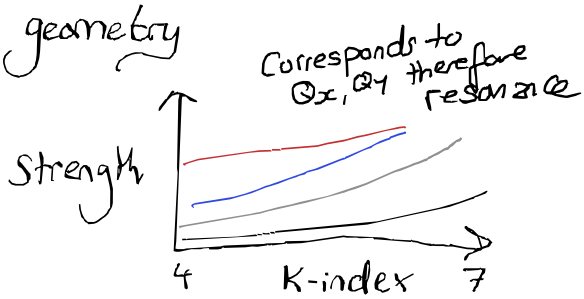
VFFA field increases vertically

SCALING FFA: Tune constant with energy i.e. Chromaticity is zero

$$B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k F(\theta) \rightarrow \text{(spiral sector)}$$

θ & k defined from magnet geometry

$$B(r,\theta) = B_0 F(\theta)\left[1 + kS + \frac{k(k-1)}{2!}S^2 + \frac{k(k-1)(k-2)}{3!}S^3 + \dots\right]$$



NON-SCALING FFA: Tune crossings occur rapidly to avoid resonances

EMMA:

electron machine for ~~muon acceleration~~ many applications

STFC Daresbury

Non-scaling offset quads

Section 6: Specific Designs

(from 2024 parameter report)

Greenfield

	C	E_{in}	E_{out}	N_{RF}	N_{turns}
RCS1	5990	62.5	314	683	17
2	5990	314	750	366	55
3	10700	750	1500	524	66
4	35000	1500	5000	2933	55

CERN

RCS1	6912	62.5	350	686	19
RCS2	26659	350	1600	1958	29
RCS3	26659	1600	3800	2017	50

Fermilab, see Kyle's talk