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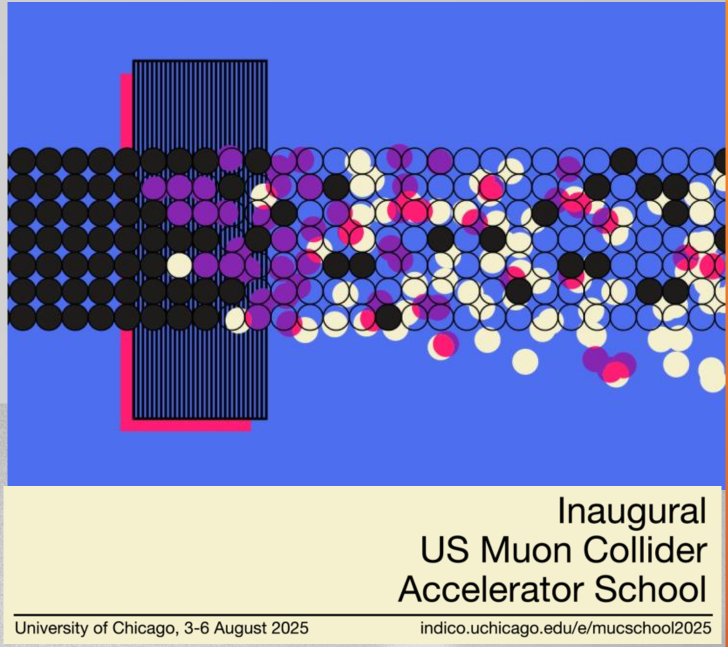
August 6, 2025



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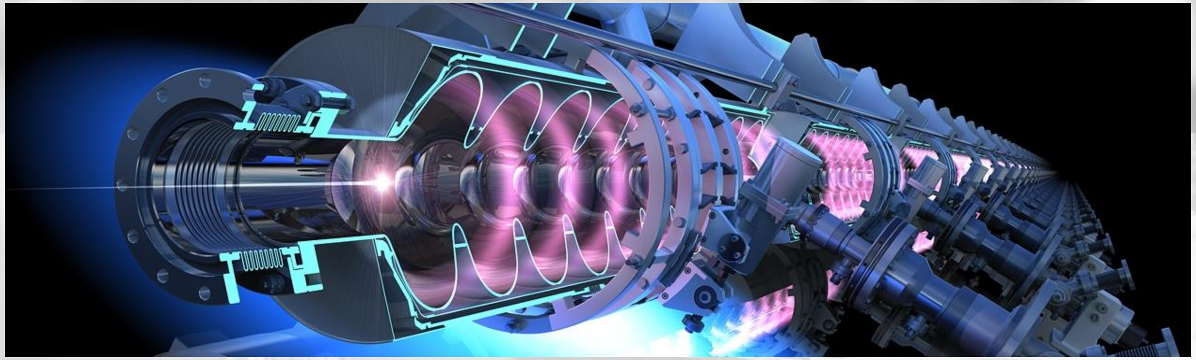
RF Technology



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U.S. DEPARTMENT of ENERGY

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Lecture outline

1. RF acceleration basics

2. RF cavities

3. Figures of merit

4. Enter superconductivity

5. RF for Muon Collider

6. Further reading

1

RF acceleration basics

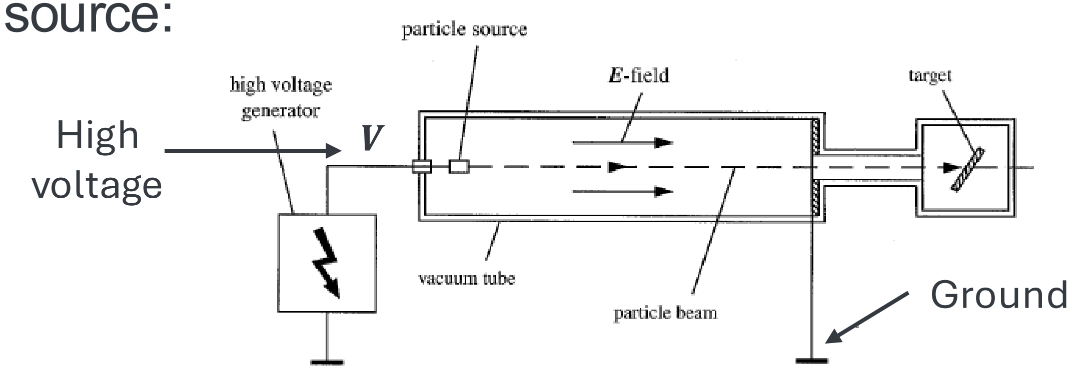
Lorentz force, DC field acceleration

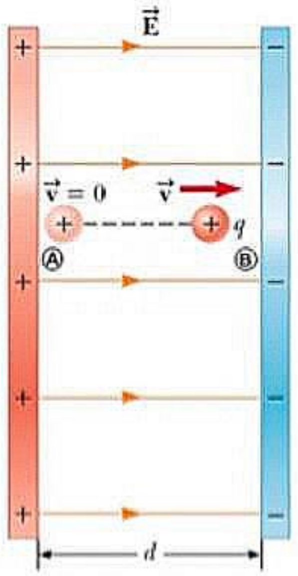
Dynamics of a charged particle (with a charge q) accelerated by electromagnetic field is described by the Lorentz force equation

$$\vec{F} = \frac{d\vec{p}}{dt} = q\left[\vec{E}(\vec{r},t) + \vec{v} \times \vec{B}(\vec{r},t)\right]$$

The electric field accelerates the particle while the magnetic field bends its trajectory.

Consider a constant electric (electrostatic) field between two electrodes with one electrode containing a particle source:



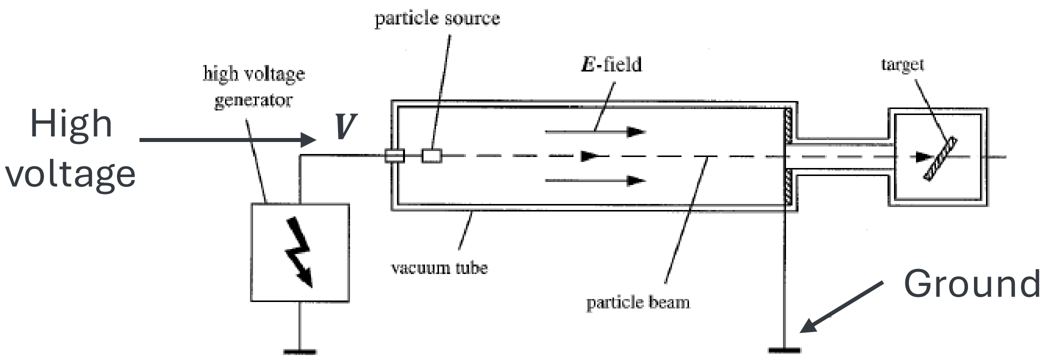


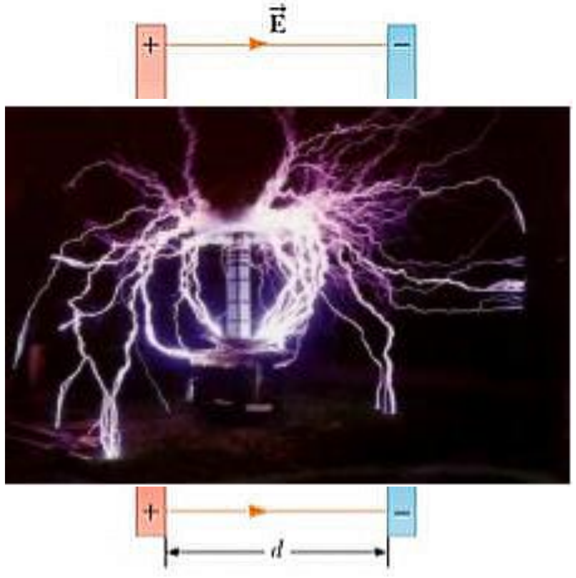
The energy gain is

$$\Delta\mathcal{E} = q\int_0^d \vec{F}dz = q \cdot Ed = q \cdot \Delta V = q \cdot V$$

What limits the particle energy in an electrostatic accelerator?

DC acceleration limitation

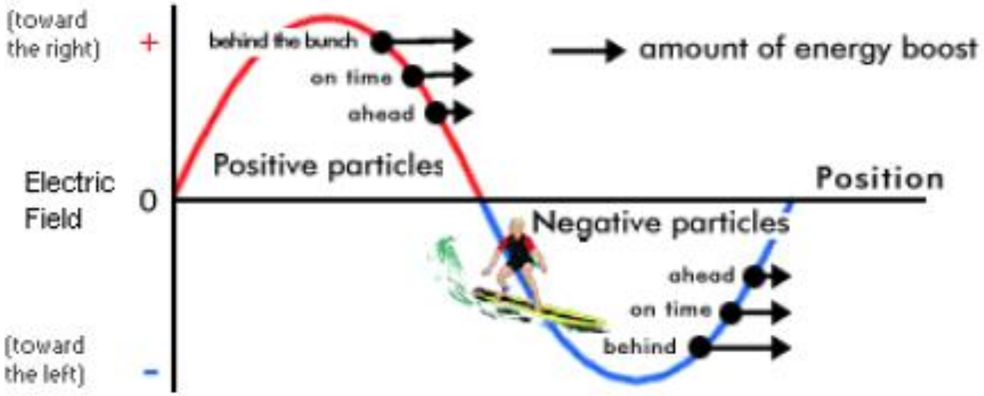




- The energy limit is given by the maximum possible voltage. At this voltage, electrons and ions are accelerated to such high energies that they produce new electrons and ions when hit the surface. An avalanche of charge carriers (corona formation) causes a large current and breakdown of the voltage.
- Maximum DC voltages of several MV are technically possible

RF acceleration

- Key idea: use of rapidly changing high frequency voltages instead of electrostatic voltages avoids corona discharge and breakdown → much higher accelerating voltages possible.
- We can EITHER add voltages from several accelerating gaps connected into a linear chain → linear accelerator, or linac, OR re-use voltage from an accelerating gap (or several gaps) by turning the particle around → circular accelerator.
- Particles must have correct phase relation with the accelerating voltage.
- Need structure with low losses for efficient acceleration (remember from the first lecture on colliders that $P_{RF} = P_{beam} + P_{loss}$) and high-power RF sources to drive the structures.



RF accelerator types

Linear accelerators (linacs) for science

Typically, medium- to large-size systems

Examples: SLAC 2-mile linac, SNS, European XFEL, LCLS-II

Recirculating linacs (including ERLs)

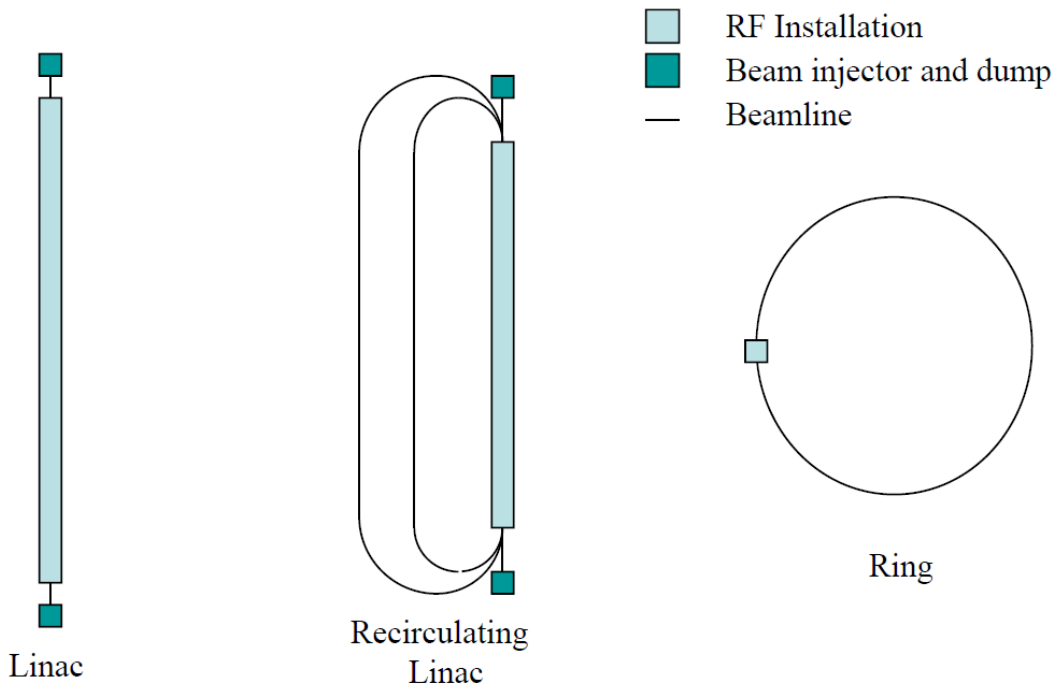
Typically, small- to medium-size systems, CW mode of operation, SRF

Example: CEBAF

Circular accelerators

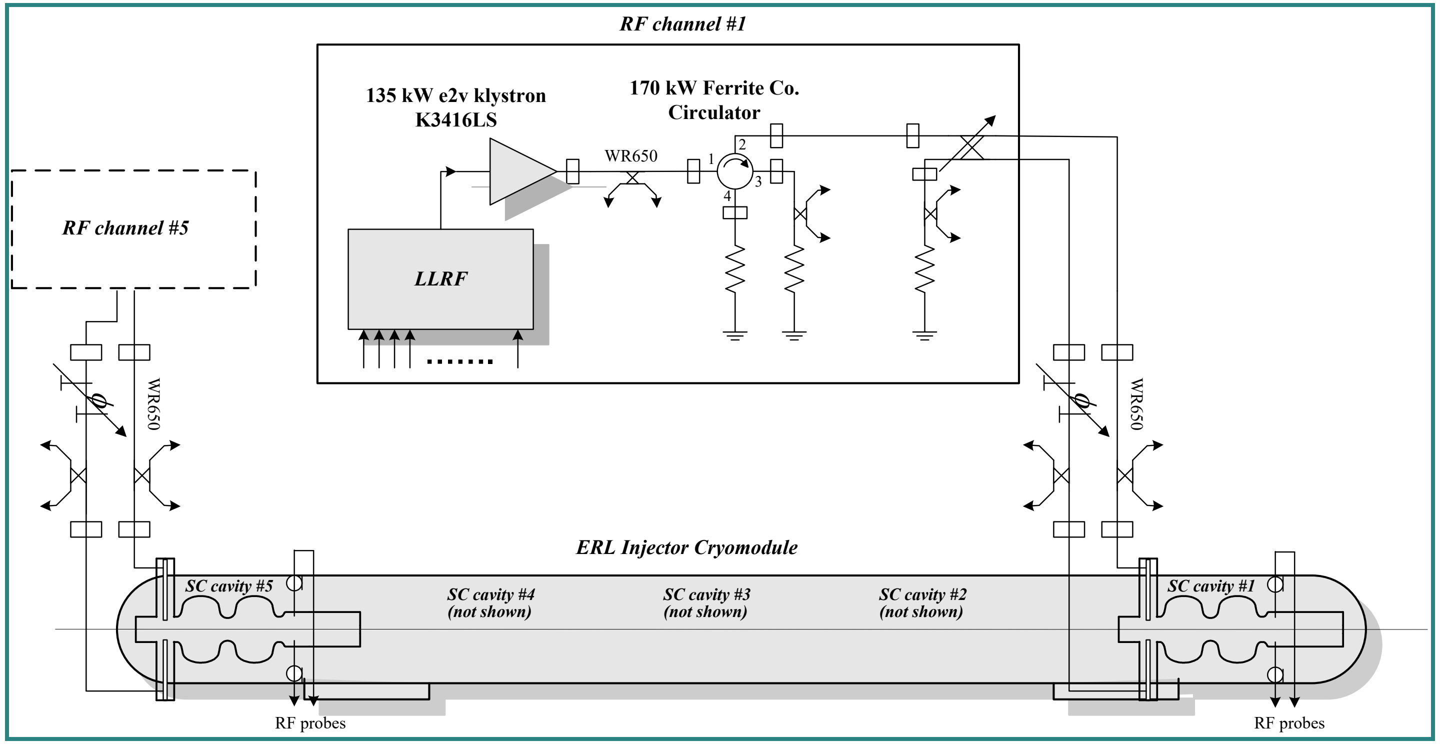
Typically, small-size system (except for high energy lepton colliders)

Examples: MI (Fermilab), SPS (CERN), LHC, RHIC, EIC, SuperKEKB, Light sources, FCC



Typical RF system architecture

Accelerator RF systems consist of several subsystems employing different technologies



- Accelerating cavities (normal conducting or superconducting)
- RF power generation (e.g., klystrons, solid state amplifiers)
- Transmission lines to connect RF amplifiers to cavities
- Fundamental power couplers (an interface between the transmission line and the cavity)
- Low level RF (LLRF) to maintain stable amplitude and phase of the cavity voltage and synchronize RF systems
- Cavity frequency tuners to keep the cavity on resonance
- Other subsystems (vacuum, instrumentation, cryogenics, …)

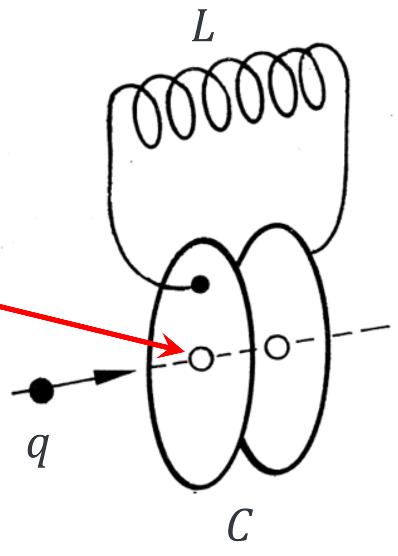
In this lecture we will concentrate on accelerating RF cavities

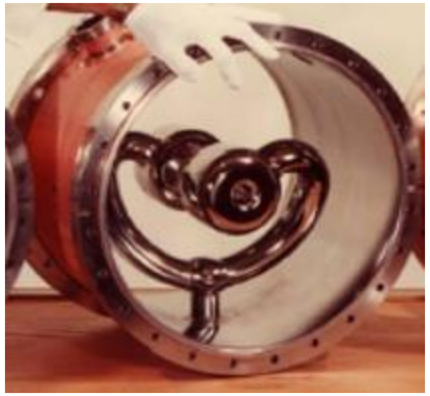
2

RF cavities

RF resonator

- We need to create an oscillating electric field along the particle trajectory.
- The simplest form of an RF resonator is $(R)LC$ circuit.
- To use such a circuit for particle acceleration, it must have opening for beam passage where the electric field is maximum (through the capacitor).
- As particles are accelerated in vacuum, the structure must provide vacuum space:
  - A ceramic vacuum break (a ceramic tube between the two electrodes of the capacitor) can be used to separate the beam line vacuum from the rest of the resonator;
  - Or the resonant structure can be enclosed in a vacuum vessel.
- However, such simple structures are not very efficient and not suitable for high frequencies.



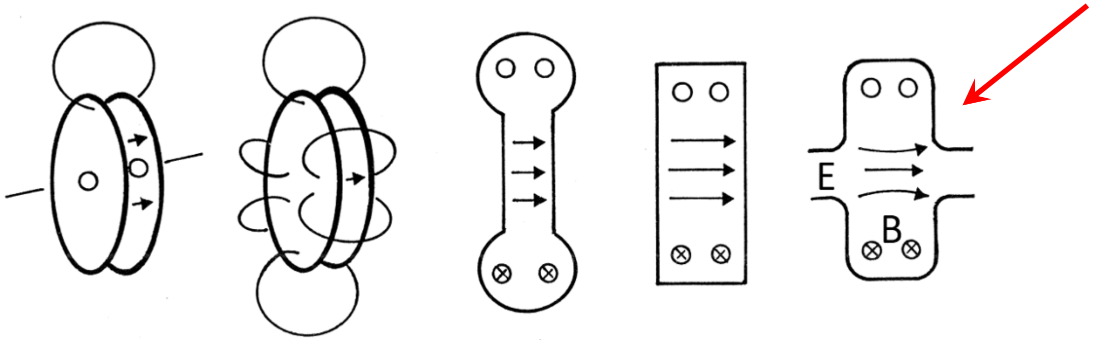


Cavity resonators

Alternatively, we can use *cavity resonators*.

Metamorphosis of the LC circuit into an accelerating cavity

1. Proceed to enclose the volume, reducing L and increasing ω: $\omega = 1/\sqrt{LC}$
2. Increase the distance between the electrodes, lowering C, hence further increasing ω. (Arriving at cylindrical, or "pillbox" cavity geometry, which can be solved analytically.)
3. Add beam tubes to let particle pass through, thus completing an accelerating cavity.



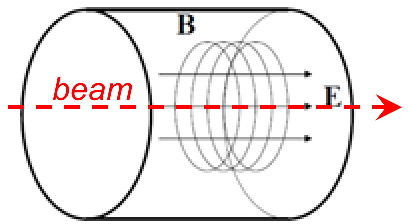
The LC circuit and a resonant cavity share common aspects:

- Energy is stored in the electric and magnetic fields.
- Energy is periodically exchanged between electric and magnetic field.
- Without any external input, the stored power will eventually all turn into heat in the cavity walls (RF losses) or radiated out of the cavity.

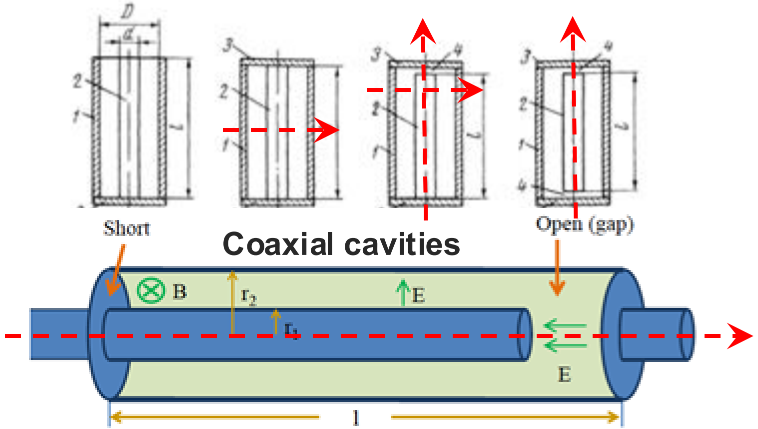
Creating a cavity resonator

A cavity resonator is a closed metal structure that confines EM fields in the RF or microwave region of the spectrum.

- Such cavities act as resonant circuits with extremely low losses. The quality (Q) factor for cavities made of copper is typically of the order of ten thousands compared to a few hundreds for resonant circuits made with inductors and capacitors at the same frequency.
- Resonant cavities can be made from closed (or short-circuited) sections of a waveguide or coaxial line.

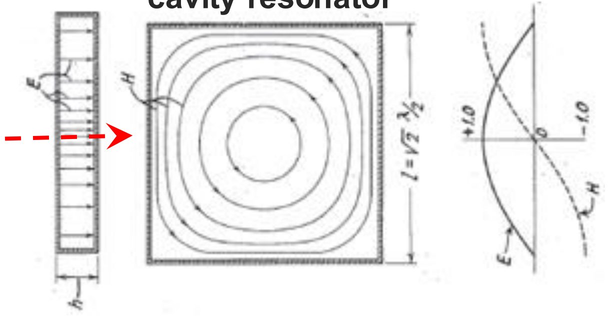


Cylindrical (pillbox) cavity operating in TM010 mode



Coaxial cavities

Rectangular cavity resonator

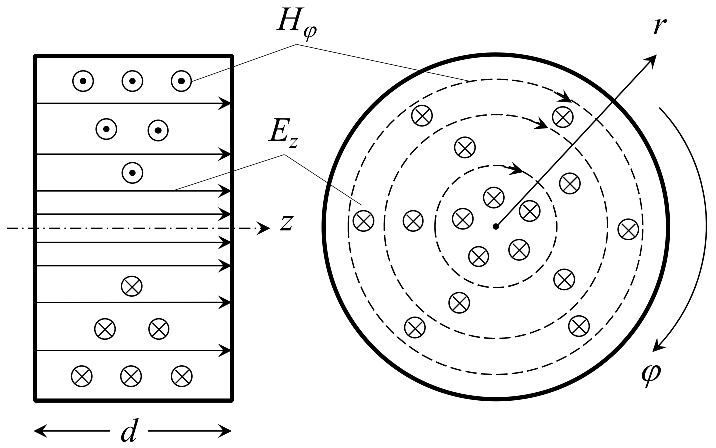


- Ferrite-loaded cavities are used at low frequencies to make cavities compact and allow very wide frequency tuning range.
- EM energy is stored in the cavity; the only losses are due to finite conductivity of cavity walls and dielectric/ferromagnetic losses of material filling the cavity.
- The cavity wall structure can be made stiff to allow its evacuation.

Pillbox cavity

Fields in the cavity are solutions of the electromagnetic wave equations:

$$\left(\nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$



subject to the boundary conditions

$$\hat{n} \times \vec{E} = 0$$

$$\hat{n} \cdot \vec{H} = 0$$

(perfectly conducting walls.)

An infinite number of solutions (eigenmodes) belong to two families of modes with different field distribution and eigenfrequencies:

- TE modes have only transverse electric fields ($\boldsymbol{E_z = 0}$);
- TM modes have only transverse magnetic fields ($\boldsymbol{H_z = 0}$).

The modes are classified as TM_{mnp} (TE_{mnp}), where integer indices m, n, and p correspond to the number of sign variations that E_z (H_z) has along φ, r, and z directions, respectively.

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TM modes

For acceleration, the particles need longitudinal electric field along their path (z axis), thus only TM modes can be used.

The TM_{010} mode of a pillbox cavity (usually the lowest frequency mode) is called the fundamental mode. This mode is used for acceleration.

All other modes are "parasitic" as they may cause unwanted effects (beam instabilities, emittance dilution, additional power losses, etc.) These modes are called Higher-Order Modes (HOMs).

The complete set of eigenmodes is described by

$$E_z = E_0 \cos\left(\frac{p\pi z}{d}\right) J_m\left(\frac{\nu_{mn} r}{R}\right) \cos(m\varphi)$$

$$E_r = -E_0 \frac{p\pi R}{\nu_{mn} d} \sin\left(\frac{p\pi z}{d}\right) J'_m\left(\frac{\nu_{mn} r}{R}\right) \cos(m\varphi)$$

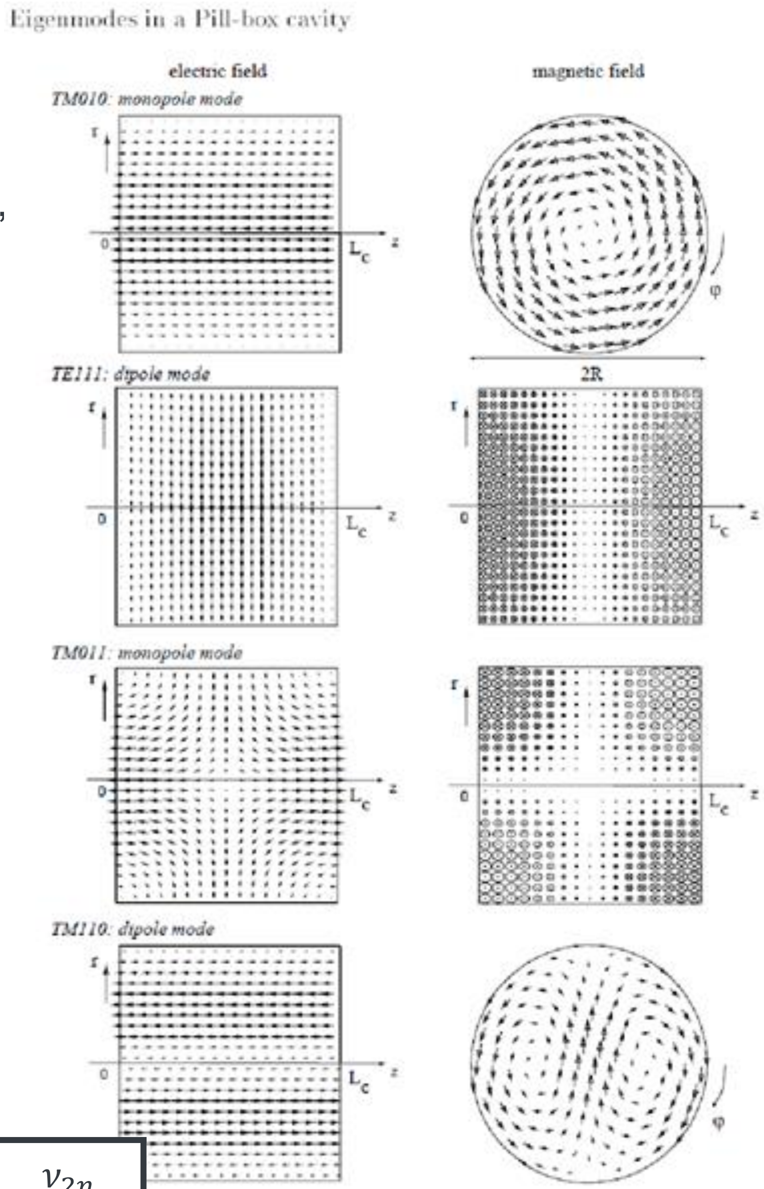
$$E_\varphi = E_0 \frac{mp\pi R^2}{r\nu_{mn}^2 d} \sin\left(\frac{p\pi z}{d}\right) J_m\left(\frac{\nu_{mn} r}{R}\right) \sin(m\varphi)$$

$$H_r = iE_0 \frac{m\omega_{mnp} R^2}{Z_0 c r \nu_{mn}^2} \cos\left(\frac{p\pi z}{d}\right) J_m\left(\frac{\nu_{mn} r}{R}\right) \sin(m\varphi)$$

$$H_\varphi = iE_0 \frac{\omega_{mnp} R}{Z_0 c \nu_{mn}} \cos\left(\frac{p\pi z}{d}\right) J'_m\left(\frac{\nu_{mn} r}{R}\right) \cos(m\varphi)$$

$$\lambda_{mnp} = \frac{2\pi c}{\omega_{mnp}} = \frac{1}{\sqrt{\left(\frac{\nu_{mn}}{2\pi R}\right)^2 + \frac{p^2}{4d^2}}}$$

ν_{mn} is the n-th root of the Bessel function $J_m(x)$



n	ν_{0n}	ν_{1n}	ν_{2n}
1	2.405	5.520	8.654
2	3.832	7.016	10.174
3	5.135	8.417	11.620

Fundamental mode

For the fundamental mode we have

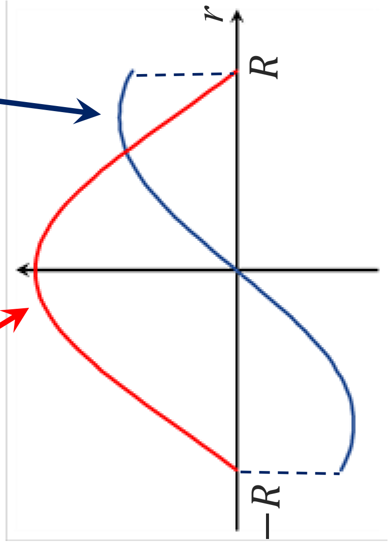
$$E_z = E_0 J_0\left(\frac{2.405\,r}{R}\right)\,e^{i\omega t},$$

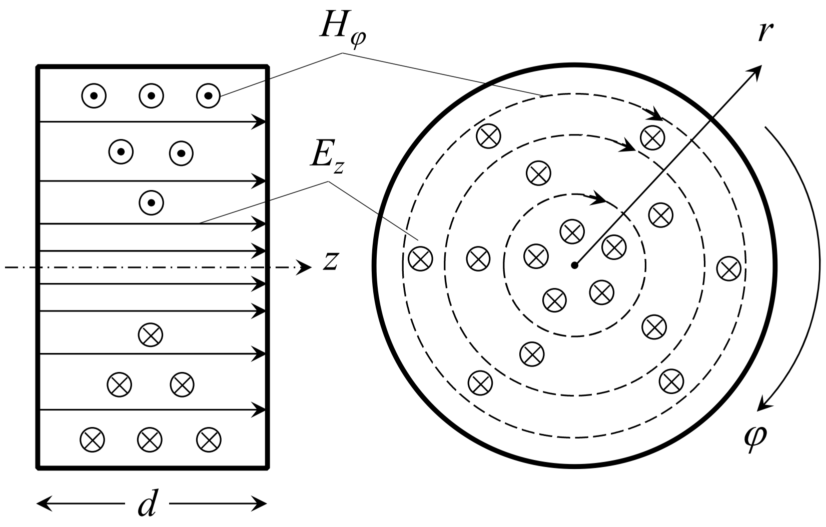
$$H_\varphi = -i\frac{E_0}{\sqrt{\mu/\varepsilon}}J_1\left(\frac{2.405\,r}{R}\right)\,e^{i\omega t},$$

$$\lambda_{010} = 2.61R,\ \omega_{010} = \frac{2.405c}{R}$$

The magnetic field is concentrated near the cylindrical wall and contributes to RF losses.

The electric field, concentrated near the axis, is used for acceleration.





3

Figures of merit

Accelerating voltage, gradient, transit time

Assuming that a charged particle is moving along the z axis, we can calculate the maximum energy gain in the pillbox cavity as

$$\Delta\mathcal{E}_{max}(r=0,\varphi) = qV_c = q\left|\int_{-\infty}^{\infty} E_z\, e^{i\omega t} dz\Big|_{t=\frac{z}{v}}\right| = qE_0\left|\int_0^d e^{i\omega z/\beta c} dz\right| = qE_0 d\,\frac{\sin\left(\frac{\omega d}{2\beta c}\right)}{\frac{\omega d}{2\beta c}} = qE_0 d\cdot T,$$

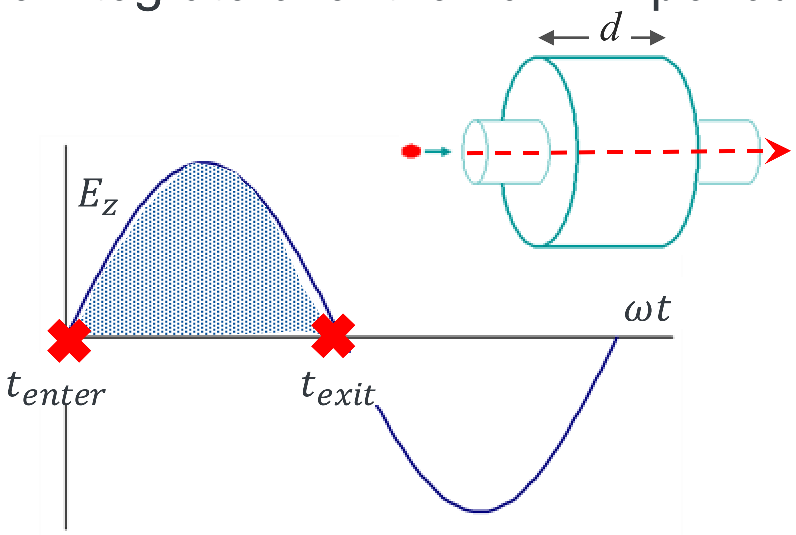
where we defined the accelerating voltage V_c and transit time factor T. The transit time factor depends on the cavity length d and the particle velocity βc.

T reaches maximum $T_{max} = 2/\pi$ at $d = \beta\lambda/2$, when we integrate over the half RF period:

$$t_{exit} - t_{enter} = \frac{d}{\beta c} = \frac{T_{RF}}{2} \quad\Rightarrow\quad V_c = E_0 d\,\frac{2}{\pi}$$

The accelerating gradient is defined as

$$E_{acc} = \frac{V_c}{d} = \frac{V_c}{\beta\lambda/2} = \frac{2}{\pi}E_0$$



The cavity length is not as easy to specify for shapes other than pillbox, so usually it is assumed to be $d = \beta\lambda/2$ per accelerating gap.

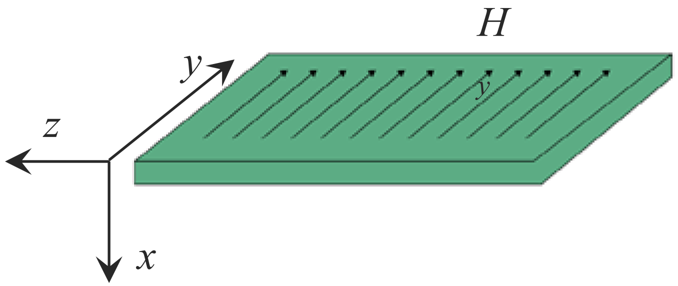
Skin depth in normal conductors

Real cavities has wall made of materials with finite conductivity (typically copper), so that

$\vec{J} = \sigma_c \vec{E}$ (Ohm's law).

In this case the tangential component of the electric field is not zero on the cavity surface and the EM field penetrates inside the metal (exponentially decaying).

Considering the cavity wall as a locally planar surface yz,



We can start from the Maxwell's equations

$$\vec{\nabla} \times \vec{E} = -i\mu_0\omega\,\vec{H}, \quad \vec{\nabla} \times \vec{H} = (\sigma_c + i\omega\varepsilon_0)\,\vec{E}$$

and neglect the displacement current ($\omega\varepsilon_0 \ll \sigma_c$), getting

$$\nabla^2\vec{H} = -i\mu_0\omega\sigma_c\vec{H} \text{ with the solution } H_y(x) = H_y(0)\,e^{-\frac{(1+i)}{\delta}x}.$$

We see that the magnetic field penetrates the conductor, and its intensity is decaying exponentially along the skin depth

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_c}} \propto \omega^{-1/2}$$

Surface resistance and power losses

The electric field $E_z(x)$ and current density $J_z(x)$ behave similarly, i.e., there is a small tangential component of the E-field decaying into the lossy conductor

The total current flowing past a unit width on the surface is found by integrating $J_z(x)$ from the surface to infinite depth:

$$I_S = \int_{-\infty}^{0} J_z(x) \cdot dx = \frac{J_z(0)\,\delta}{1+i}$$

The surface impedance for a unit length and unit width is defined as

$$Z_s \equiv \frac{E_z(0)}{H_y(0)} = \frac{1+i}{\sigma_c \delta} = R_s + iX_S$$

The real part of the RF surface impedance – responsible for losses – is called RF surface resistance. The losses per unit area are

$$P'_{loss} = \frac{1}{2} R_S H_{y0}^2,$$

and the total power dissipating in the cavity walls is

$$P_{loss} = \frac{1}{2} \oint_S R_s \left|\vec{H}_s\right|^2 dS = \{\text{if } R_s \text{ is constant over the cavity surface}\} = \frac{1}{2} R_s \oint_S \left|\vec{H}_s\right|^2 dS$$

Quality factor

The total energy stored in the electromagnetic field of a cavity is the sum of energy stored in the electric and magnetic fields. Given the sinusoidal time dependence and the 90° phase shift between the electric and magnetic fields, the energy oscillates back and forth between the electric and magnetic field. Then the stored energy in a cavity is given by

$$U = U_E + U_H = \frac{1}{2}\varepsilon \int_V \left|\vec{E}\right|^2 dv = \frac{1}{2}\mu \int_V \left|\vec{H}\right|^2 dv.$$

An important figure of merit is the (intrinsic) quality factor:

$$Q_0 = \frac{\omega_0 U}{P_{loss}} = 2\pi \frac{U}{T_0 P_{loss}} = 2\pi \frac{\text{stored energy}}{\text{energy loss per cycle}} = \omega_0 \tau_0$$

The quality factor is determined by both material property and cavity geometry.

The stored energy in the cavity dissipates the following way:

$$\frac{dU}{dt} = -\frac{\omega_0}{Q_0} U \xrightarrow{\text{yields}} U(t) = U_0 e^{-\frac{\omega_0 t}{Q_0}} = U_0 e^{-\frac{t}{\tau_0}}$$

For normal conducting (NC) cavities, $Q_0 \sim 10^4$; and for superconducting (SC) cavities at 2 K, $Q_0 \sim 10^{10}$

Geometry factor

If the surface resistance does not vary over the cavity surface, we may write the expression for quality factor as

$$Q_0 = \frac{\omega_0 U}{P_{loss}} = \omega_0 \frac{\mu \int_V \left|\vec{H}\right|^2 dv}{R_s \int_S \left|\vec{H}\right|^2 ds} = \frac{G}{R_s}$$

The ratio of the two integrals is determined only by the cavity geometry, thus we defined a new parameter G, called the geometry factor, as

$$G = \omega_0 \frac{\mu \int_V \left|\vec{H}\right|^2 dv}{\int_S \left|\vec{H}\right|^2 ds}$$

The geometry factor depends only on the cavity shape and the field configuration of an electromagnetic mode, but not on the cavity size.

It is useful for comparing the performance of different cavity shapes.

The geometry factor of pillbox cavity is G = 257 Ohm. For a copper cavity operating at 1.5 GHz, we have:

$\sigma_c = 5.8\times10^7$ S/m, $\delta = 1.7$ μm, $R_s = 10$ mOhm, and $Q_0 = G / R_s = 25{,}700$

Shunt impedance and R/Q

The shunt impedance characterizes losses in a cavity by connecting the accelerating voltage and power loss:

$$R_{sh\_acc} = \frac{V_c^2}{P_0}$$

Note that this ***accelerator definition*** is twice that of the usual *circuit theory* convention: $R_{sh\_circuit} = \frac{V_c^2}{2P_0}$.

And, to add to the confusion, a common definition *in linacs* is $r_{sh} = \frac{E_{acc}^2}{P_0{}'}$, where $P_0{}'$ is the power dissipation per unit length and hence, the linac shunt impedance is in ohms per meter

A related quantity is the geometric shunt impedance, R_{sh}/Q_0, or simply R/Q, which is independent of the surface resistance and the cavity size:

$$\frac{R}{Q} = \frac{V_c^2}{\omega_0 U}$$

The R/Q is frequently used as a figure of merit in determining the level of mode excitation by bunches of charged particles passing through the cavity.

For the pillbox cavity, $R/Q = 196$ Ohm.

Dissipated power and frequency dependences

The power dissipated in the cavity walls can be expressed in several forms:

$$P = \frac{V_c^2}{R_{sh}} = \frac{V_c^2}{Q_0 \cdot R/Q} = \frac{V_c^2}{G \cdot R/Q} \cdot R_s$$

To minimize the losses, we need to maximize the denominator. The denominator in the last expression includes only material-independent figures of merit, $G \cdot R/Q$. This new parameter can be used during cavity shape optimization.

For normal conductors, $R_s = 1/\sigma_c \delta \propto \omega^{1/2}$, hence the power per unit length and unit area scales as

$$\frac{P}{L} \propto \frac{1}{G \cdot R/Q} \cdot \frac{E_{acc}^2 R_s}{\omega} \propto \omega^{-1/2}, \qquad \frac{P}{A} \propto \omega^{1/2}.$$

One can reduce the power loss in a pulsed mode.

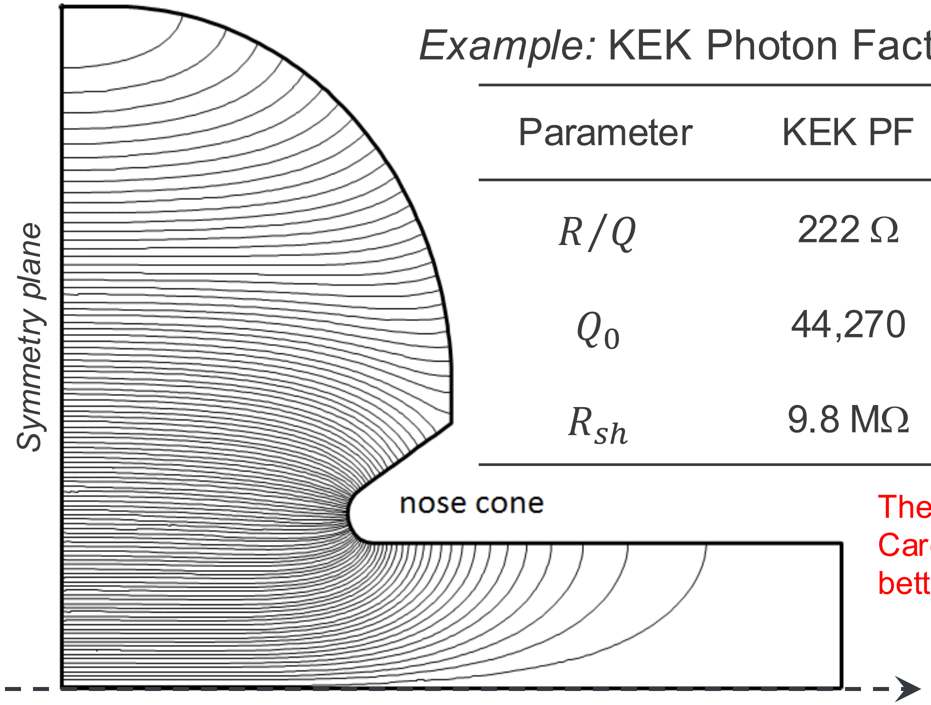
High frequency NC structures can reach gradients $\gtrsim 100$ MV/m but only at low duty factor.

In CW regime, NC cavities can operate at relatively low voltages only.

Optimal cavity shapes

Most accelerating cavities do not quite look like pillboxes — their shape is typically axisymmetric around the beam axis, and they feature transitions to the beam, but otherwise their shapes vary.

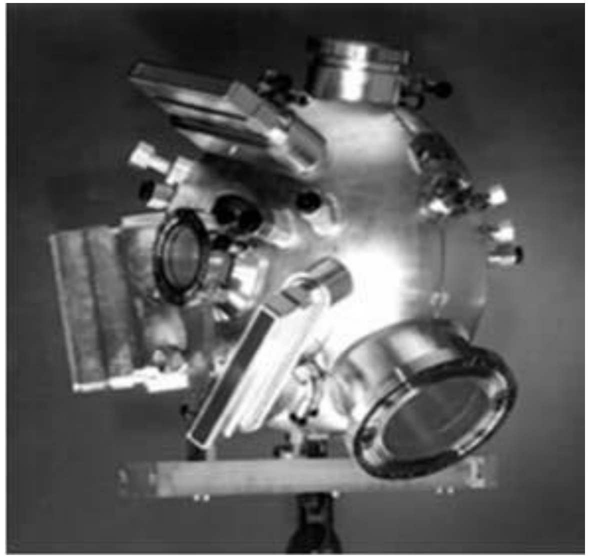
Very often single cell NC cavities feature round outer shell and a nose cones near beam pipes. Nose cones increase transit time factor, round outer shape minimizes losses.



Example: KEK Photon Factory 500 MHz cavity

Parameter	KEK PF	pillbox
R/Q	222 Ω	196 Ω
Q_0	44,270	44,054
R_{sh}	9.8 MΩ	8.6 MΩ

The simple pillbox cavity is a good reference. Careful optimization can provide slightly better parameters.

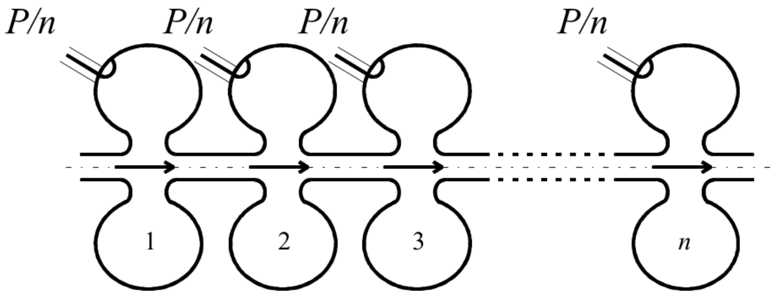


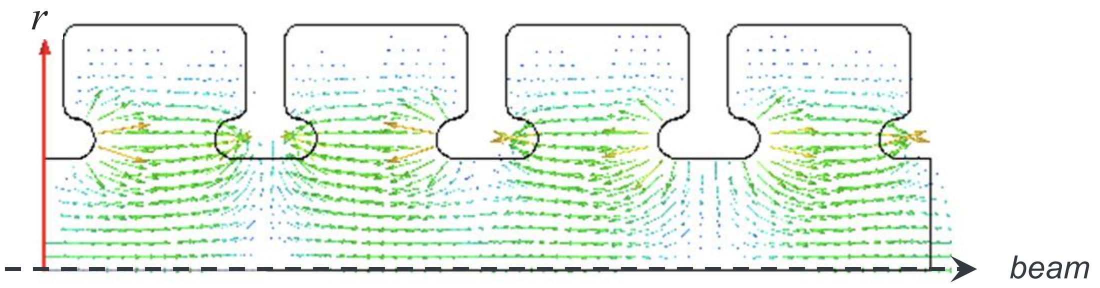
476 MHz SLAC B Factory cavity, max. accelerating voltage of ~ 0.8 MV, dissipation ~ 100 kW, strong HOM damping

Multicell structures

Using single cell cavities is not always optimal.

1. The total power is constant. Consider n single-gap cavities with the same shunt impedance. If we split the power evenly, each cavity will produce voltage $\sqrt{R_{sh}(P/n)}$. With correct phasing of the RF, the total accelerating voltage will be the sum of individual voltages: $V_{c\_total} = \sqrt{(nR_{sh})P}$. Thus, the effective shunt impedance is nR_{sh}, a significant increase.





Electric field lines of the π-mode of a periodic structure. This standing-wave cavity has a cell length of $\beta\lambda/2$ for a particle speed βc.

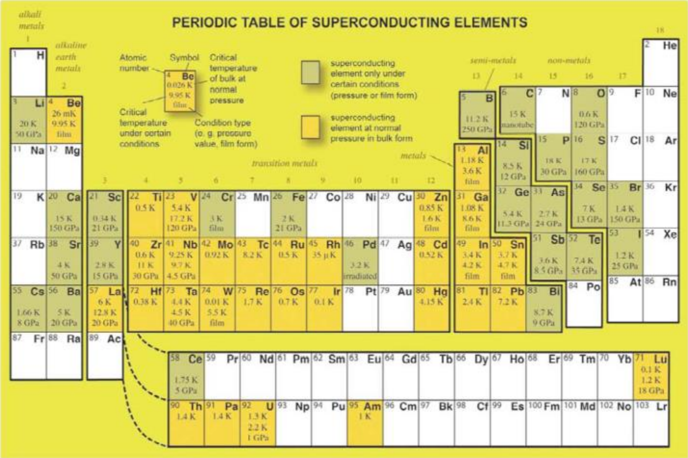
2. Combine individual gaps in one structure. → Standing-wave or traveling-wave accelerating structures. Benefits: one vacuum structure, no need for RF power splitters, one power coupler per structure, more compact accelerator. Drawbacks: more difficult to fabricate, higher parasitic HOM impedance.

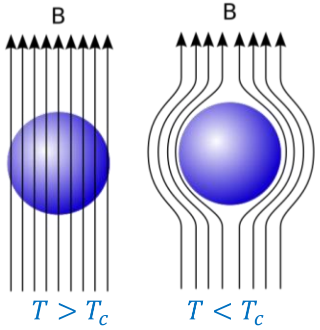
4

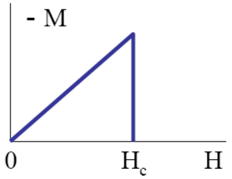
Enter superconductivity

What is superconductivity (very briefly)?

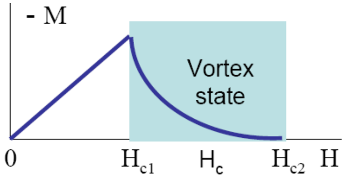
- DC resistivity of some materials drops to zero below a critical temperature T_c.
- Most of the known elements can become superconducting either at normal pressure in bulk form or under certain conditions (high pressure of film form).
- In addition to T_c, the superconducting state has critical magnetic field H_c (Type I superconductors), above which the material becomes normal conducting.
- Below H_c the superconductor is in the Messner state (ideal diamagnetic).
- Type II superconductors have two critical magnetic fields. Above H_{c1} but below H_{c2} the external magnetic field partially penetrates the material as separate vortices (Vortex state).
- The magnetic field doesn't stop abruptly but penetrates the material with exponential attenuation characterized by the London penetration depth λ_L (20-50 nm).







Complete Meissner effect in type-I superconductors



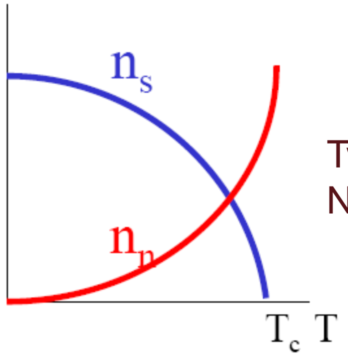
High-field partial Meissner effect in type-II superconductors

- Type-I: Meissner state B = H + M = 0 for $H < H_c$; normal state at $H > H_c$
- Type-II: Meissner state B = H + M = 0 for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

Simplified explanation of zero DC resistance

Normal conducting state

Electrons carrying electric current encounter resistance due to collisions and scattering off impurities, lattice vibrations (phonons).



Two-fluid model: coexisting SC and NC ”liquids” with densities

$$n_s(T) + n_n(T) = n$$

Superconducting state

As the material cools down, the lattice vibration slows. A moving electron attracts nearby atoms, creating a positive wake behind. A second electron is attracted by this wake, forming a weak bond with the first electron (Cooper pair). The Cooper pair is a boson; thus, all Cooper pairs can occupy the same ground energy level. They form a coherent state. Some Cooper pairs are broken at $T > 0$ due to phonon interaction. The supercurrent does not encounter resistance.

What if AC field is applied?

Two fluid model considers both superconducting and normal conducting components:

- At $0 < T < T_c$ not all electrons are bonded into Cooper pairs. The density of unpaired, “normal” electrons is given by the Boltzmann factor

$$n_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

  where 2Δ is the energy gap around Fermi level between the ground state and the excited state.

- Cooper pairs move without resistance and thus dissipate no power. In DC case the lossless Cooper pairs short out the field, hence the normal electrons are not accelerated, and the superconductor is lossless even for $T > 0$ K.
- The Cooper pairs do nonetheless have an inertial mass, and thus they cannot follow an AC electromagnetic fields instantly and do not shield it perfectly. A residual EM field remains and acts on the unpaired electrons as well, therefore causing power dissipation.
- We expect the surface resistance to drop exponentially below T_c.

Surface resistance of superconductors

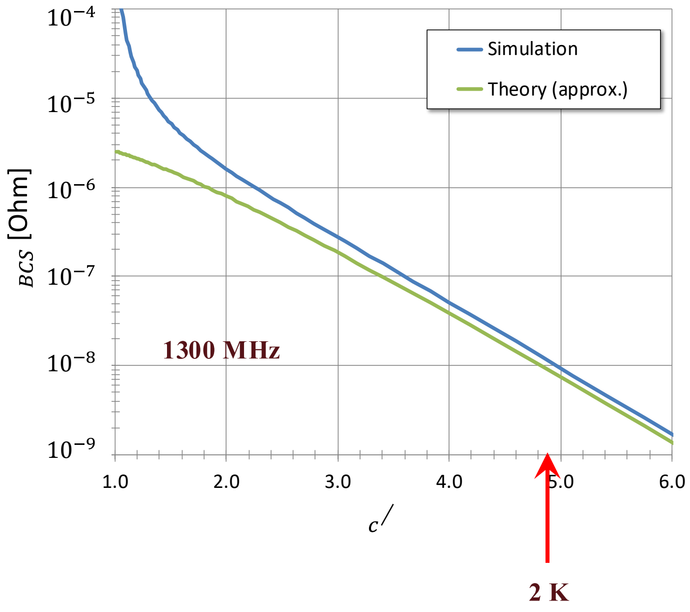
- Combining the two-fluid model with Maxwell's equations, one can show that for superconductors $R_s \propto \omega^2$.
- Calculation of surface resistance must take into account numerous parameters. Mattis and Bardeen developed a theory based on BCS, which predicts the following temperature dependence

$$R_{BCS} = A\frac{\omega^2}{T}e^{-\left(\frac{\Delta}{k_B T_c}\right)\frac{T_c}{T}}$$ (A is the material constant)

- While for low frequencies (≤ 500 MHz) it may be efficient to operate at 4.2 K (liquid helium at atmospheric pressure), higher frequency structures favor lower operating temperatures (typically superfluid liquid He at 2 K, below the lambda point of helium, 2.172 K).
- Approximate expression for Nb (the material of choice for superconducting cavities):

$$R_{BCS,\,Nb} \approx 2 \times 10^{-4}\left(\frac{f[\mathrm{MHz}]}{1500}\right)^2 \frac{1}{T} e^{\frac{-17.67}{T}}\ [\Omega]$$

- Above $\sim T_c/2$, this formula is not valid, and one needs to perform more complicated calculations.



But R_s also depends on the RF field!

NC vs. SC RF cavities

$$P = \frac{V_c^2}{R_{sh}} = \frac{V_c^2}{Q_0 \cdot R/Q} = \frac{V_c^2}{G \cdot R/Q} \cdot R_s$$

NC cavities favor high frequencies, SC cavities favor low frequencies.

Normal conducting RF cavities

For normal conductors, $R_s \propto \omega^{1/2}$

$$\frac{P}{L} \propto \omega^{-1/2}$$

$$\frac{P}{A} \propto \omega^{1/2}$$

Superconducting RF (SRF) cavities

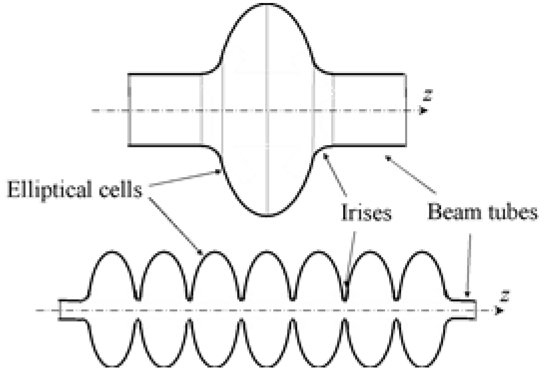
For superconductors, $R_s \propto \omega^2$

$$\frac{P_0}{L} \propto \omega$$

$$\frac{P_0}{A} \propto \omega^2$$

- SC cavities excel in applications requiring CW or long-pulse accelerating fields above a few MV/m.
- Best $1.3\ \mathrm{GHz}$ cavities reached $\sim 50\ \mathrm{MV/m}$.
- For NC cavities (usually made of copper) power dissipation in cavity walls is a huge constraint in these cases → cavity design is driven by this fact, optimized for lowest possible wall dissipation → small beam aperture.
- The surface resistivity of SC cavities is many orders (up to 6) of magnitude less than that of copper → in many cases the SC accelerating system is more economical: less wall plug power, fewer cavities required,…
- Additional benefit: the cavity design decouples from the dynamic losses (wall losses associated with RF fields) → free to adapt design to a specific application.
- The presence of accelerating structures has a disruptive effect on the beam and may cause various instabilities, dilute beam emittance and produce other undesirable effects. Fewer SC cavities → less disruption. Also, SC cavities can trade off some of wall losses to a larger beam pipe → reduce disruption more.

Single-cell elliptical cavity



Multi-cell elliptical cavity

Is SRF more efficient?

- NC cavities: $P_{total\_NC} = \eta_{RF} P_{RF} = \eta_{RF}(P_{beam} + P_{loss\_NC})$
- SRF cavities: $P_{total\_SRF} = \eta_{RF} P_{RF} + P_{cryo} = \eta_{RF}\left(P_{beam} + P_{loss\_SRF}\right) + P_{cryo}$ ($P_{loss\_SRF} \approx 0$)
- We need to compare $\eta_{RF} P_{loss\_NC}$ and P_{cryo}

Cryogenic efficiency

- The cryogenic plant efficiency is characterized by its overall coefficient of performance COP (the ratio of the heat removed at low temperature to the work at high temperature):

  $\mathrm{COP} = \frac{\dot{Q}_{cold}}{\dot{W}} = COP_C \cdot \eta_t$, where the Carnot cycle $\mathrm{COP}_C = \frac{T_{cold}}{(T_{hot} - T_{cold})}$

  and η_t is the plant technical efficiency.
- For different temperatures we have

Refrigeration temperature	COP_C^{-1} (ideal world)	XFEL specification (real world)	% Carnot
2 K	149	870	17
5 K	59	220	27
40 K	6.5	20	33

- Then, knowing static and dynamic heat loads at particular temperatures, we calculate

$$P_{AC} = \Sigma_T\left(P_{static} + P_{dynamic}\right)_T / \mathrm{COP}_T$$

Very often SRF is more efficient than normal conducting RF!

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Why Nb?

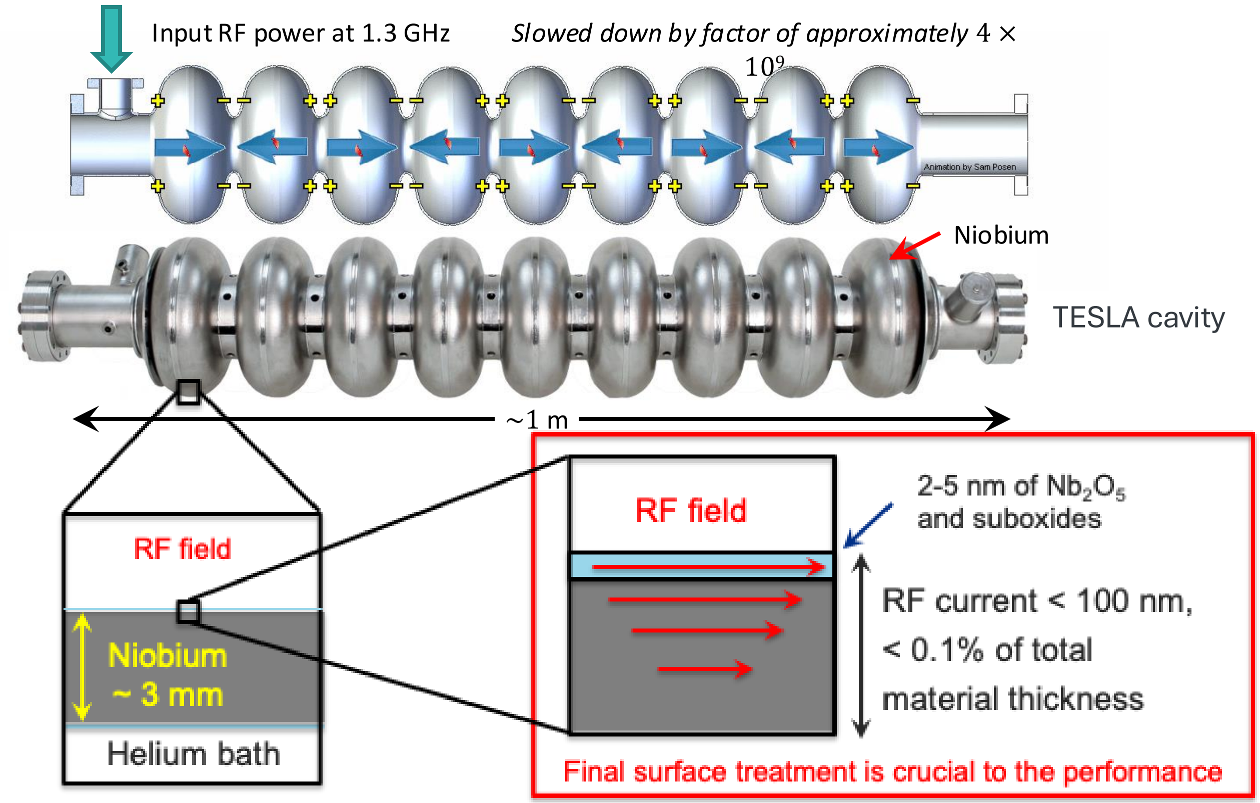
- Pure niobium has the highest $T_c = 9.25\ \mathrm{K}$ among single elements, and H_{c1} and H_{c2} are both high.

	Type	T_c	H_{c1}	H_c	H_{c2}	Fabrication
	-	K	Oe	Oe	Oe	-
Nb	**II**	**9.25**	**1700**	**2060**	**4000**	**bulk, film**
Pb	I	7.20	-	803	-	electroplating
Nb_3Sn*	**II**	**18.1**	**380**	**5200**	**240000**	**film**
MgB_2	II	39.0	300	4290		film
Hg	I	4.15	-	411/339	-	-
Ta	I	4.47	-	829	-	-
In	I	3.41	-	281.5	-	-

- Low residual resistances are needed for operation in superfluid helium at (typical accelerator operation domain)
- High theoretical Meissner state breakup field (H_{sh}~$240\ \mathrm{mT}$) for an ideal surface, which scales with H_c
- Good formability is desirable for ease of cavity fabrication.
- Pure intermetallic compounds, like Nb_3Sn with a critical temperature of $18.1\ \mathrm{K}$, look attractive for possible $4.2\ \mathrm{K}$ operation at first sight as they are "clean" superconductors
- However so far, the gradients achieved in Nb_3Sn-coated niobium cavities have been limited to below $25\ \mathrm{MV/m}$, probably due to field enhancement caused by surface roughness.
- Alloys are "dirty" superconductors due to their small mean free path and consequently large BCS surface resistivity and poor thermal conductivity.
- High temperature superconductors have been tried in the past and showed very high surface resistances, problems arise from very low coherence length = sensitivity to defects, gap anisotropy etc.

Some difficulties with SRF cavities

- Cavity performance is determined by nanometer-scale structure of inner surface.



- Final surface treatment is critical → particulate-free assembly in cleanroom
- Ideally, if the external magnetic fields is less than H_{c1}, the DC flux will be expelled due to Meissner effect. In reality, there are lattice defects and other inhomogeneities, where the flux lines may be "pinned" and trapped during cooldown.
- For high purity Nb prepared by chemical etching the rough estimate is

$$R_{mag} = 0.3[\mathrm{n\Omega}] \cdot H_{ext}[\mathrm{mOe}]\sqrt{f[\mathrm{GHz}]}$$

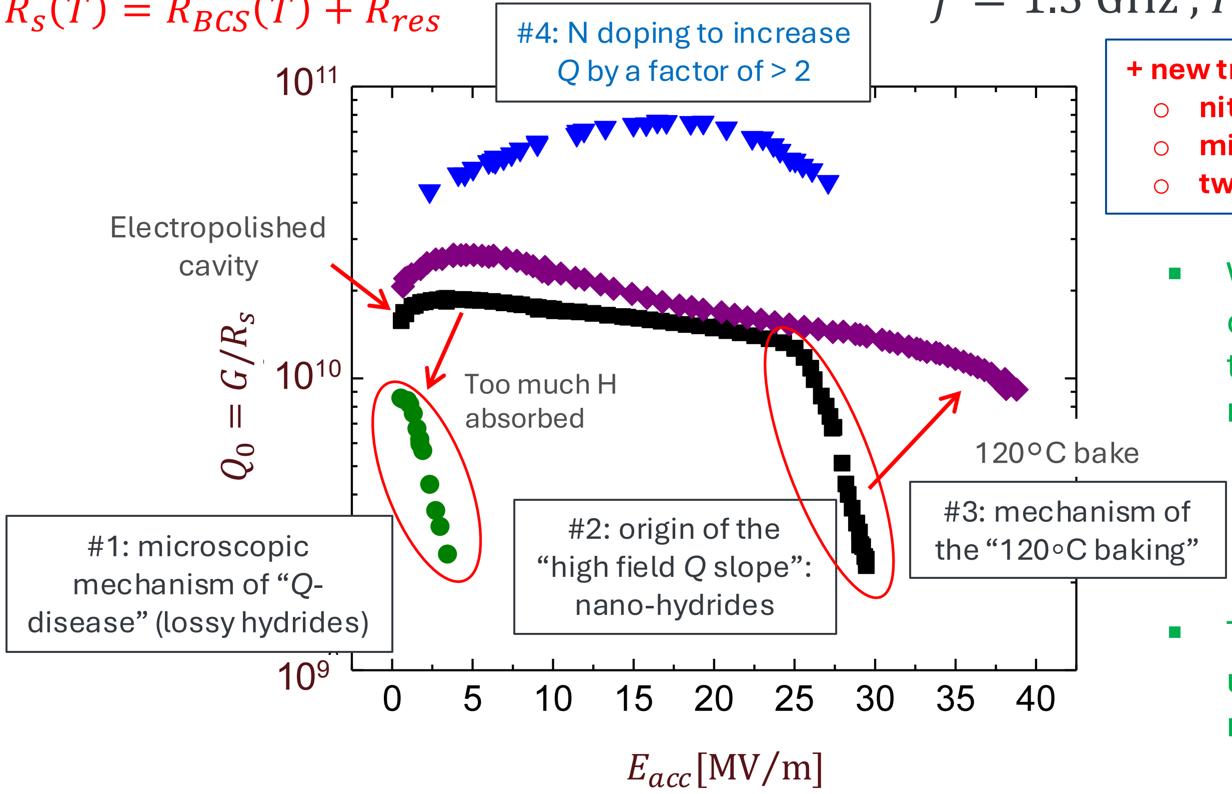
- The Earth's magnetic field is $0.5\ \mathrm{G}$, which if trapped would produce residual resistivity of $\sim 150\ \mathrm{n\Omega}$ at $1\ \mathrm{GHz}$ and $Q_0 < 2 \cdot 10^9$

- Magnetic shielding around the cavity is needed to reach 10^{10} range.
- Usually, the goal is to have residual magnetic field of less than 10 mG.

SRF cavity performance vs. RF field

$R_s(T) = R_{BCS}(T) + R_{res}$

$f = 1.3$ GHz , $T = 2$ K



+ new treatments:
- **nitrogen infusion**
- **mid-T baking**
- **two-step low-T baking**

- While the field has better understanding of mechanisms behind different surface treatments nowadays, **developing new recipes is still mostly empirical work**.
- The goal is to **reach the level of understanding when we can "engineer" recipes** easily to project specifications.

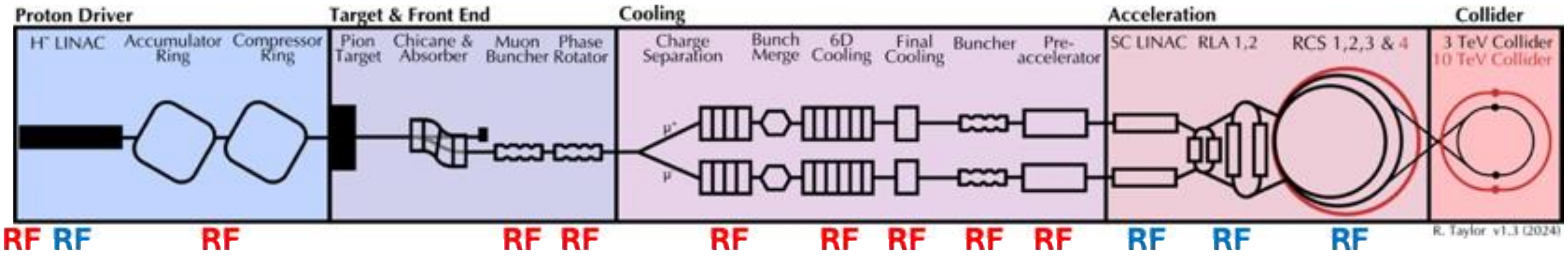
It is customary to represent performance of an SRF cavity using Q vs. E or $Q_0\ (E_{acc})$ plot

5

RF for Muon Collider

Some materials from ***Muon collider RF R&D: 5+5 years plan***, reported at IAC Review of the IMCC R&D Plan, June 2025

Muon Collider layout and RF



NC RF for capture and cooling

- High-gradient cavities in high magnetic field
- High charge, Huge beam size, Important beam losses
- Peak RF power
- Little synergy with other projects

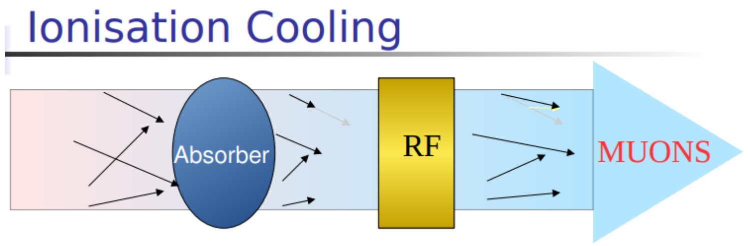
SRF for acceleration

- High charge, short bunch
- High efficiency at high gradient
- Maintain beam quality
- Longitudinal and transverse stability

RF system for muon cooling (ESPPU, 2025)

Require high gradient normal conducting RF cavity operation in a strong magnetic field

Region	Length [m]	N of RF cav.	Frequencies [MHz]	Gradient [MV/m]	Magnetic field [T]	Peak RF power [MW/cav.]
Buncher	21	54	490 - 366	0 - 15		1.3
Rotator	24	64	366 – 326	**20**		2.4
Rectilinear cooler A-type	**363**	**1605 x2**	**352, 704**	**26, 32**	**2-4, 5-7**	**4, 2**
Bunch merge	130	26 x2	108 - 1950	**~ 10**		
Rectilinear cooler B-type	**487**	**2057 x2**	**352, 704**	**21, 30**	**3-7, 9-17**	**4, 2**
Final Cooling	140	96 x2	~100 - 10			
Reacceleration			~100 - 10			
Total	1165	**7686**				**=> ~20GW**



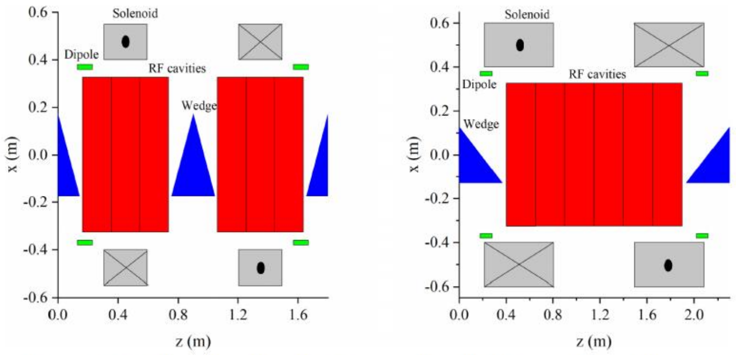


Figure 5.2.4: Schematic of the (left) A-type and (right) B-type cooling cell layout.

State-of-the-Art: RF cavities for muon cooling

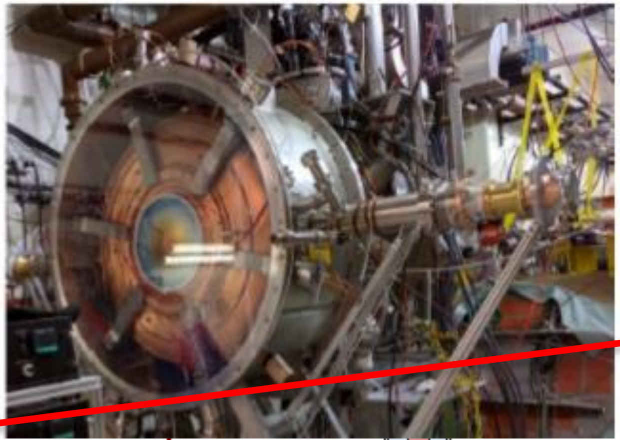
Challenge: **overcoming RF breakdown in magnetic field**

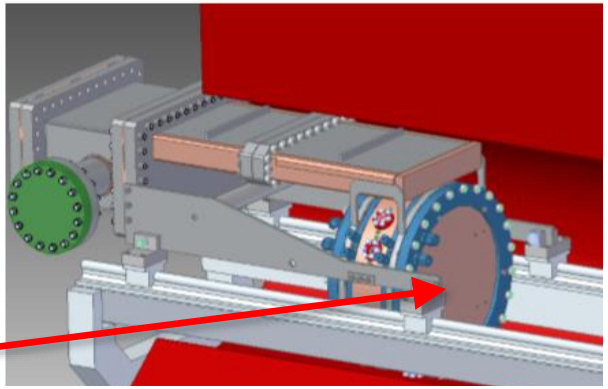
Model: beamlets focused by magnetic field cause heating. If $\Delta T > \Delta T_{plastic}$, breakdown occurs.

$\Delta T_{plastic}$:
38°C for Cu
129°C for Be
224°C for Al

- MICE 200 MHz RF module prototype (with Be windows):

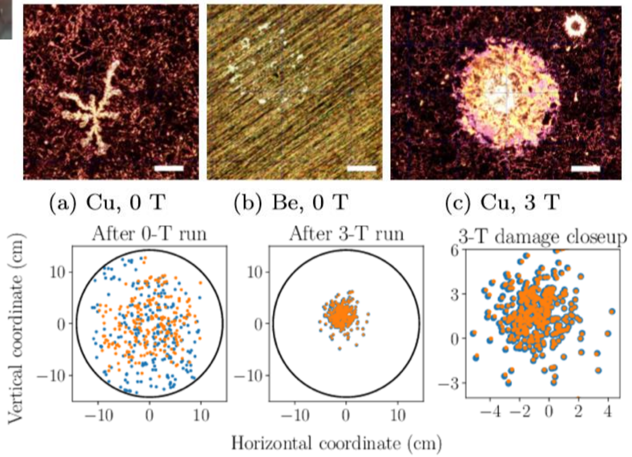
  4 T, 10 MV/m, 1 ms@1 Hz





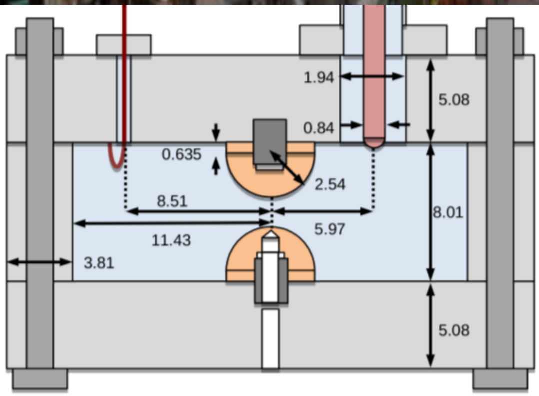
- 800 MHz cavity with beryllium end plate:

  3 T, **50 MV/m**, 30 μs @10 Hz



- Gas filled RF cavity:

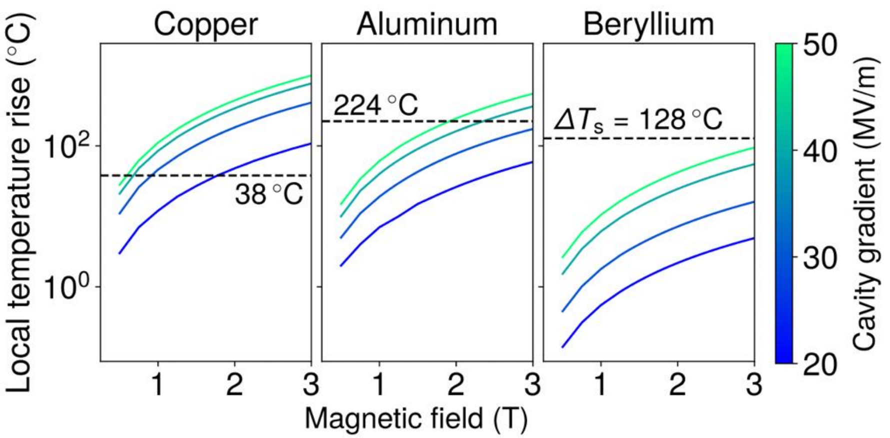
  small gap, 800 MHz, > 50 MV/m



MAP 805 MHz cavity with modular plates

Strong indication that Al could be a good middle ground between safety of Cu and performance of Be.

More experiments are needed, in particular: 1) to identify the best material for the cavity window; 2) test cavities in 10 – 15 T magnetic field to find the limit on the achievable operating gradient.

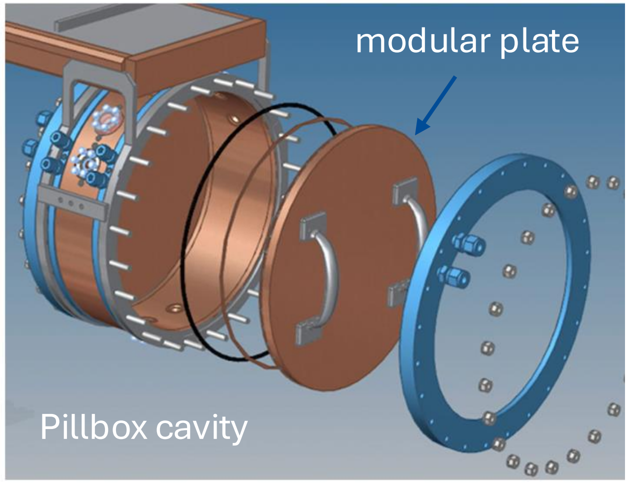


Operation of normal-conducting rf cavities in multi-Tesla magnetic fields for muon ionization cooling: A feasibility demonstration

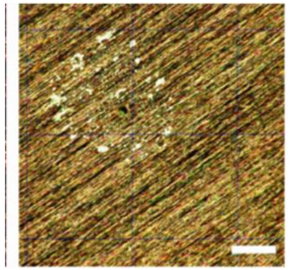
D. Bowring, A. Bross, P. Lane, M. Leonova, A. Moretti, D. Neuffer, R. Pasquinelli, D. Peterson, M. Popovic, D. Stratakis, K. Yonehara, A. Kochemirovskiy, Y. Torun, C. Adolphsen, L. Ge, A. Haase, Z. Li, D. Martin, M. Chung, D. Li, T. Luo, B. Freemire, A. Liu, and M. Palmer

Phys. Rev. Accel. Beams **23**, 072001 – Published 2 July 2020

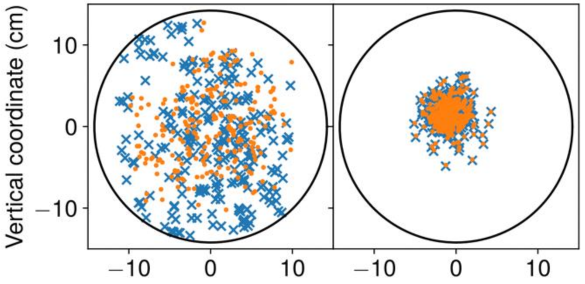
FIG. 2. Semi-log plot of local ΔT for Cu, Al, and Be cavities at various gradients and across a range of solenoidal magnetic field strengths. ΔT_s [Eq. (4)] is indicated in each plot by a horizontal, dashed line. Note that for Be, the local temperature rise is lower than ΔT_s for a broad range of gradients and magnetic fields.



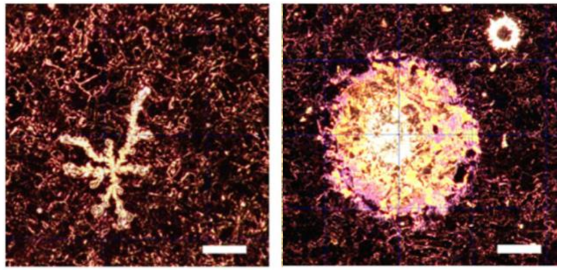
Material	B-field (T)	SOG (MV/m)
Cu	0	24.4 ± 0.7
Cu	3	12.9 ± 0.4
Be	0	41.1 ± 2.1
Be	3	$> 49.8 \pm 2.5$
Be/Cu	0	43.9 ± 0.5
Be/Cu	3	10.1 ± 0.1



Be: 0 & 3 T



Cu: 0 T **Cu: 3 T**



SRF for accelerators

	RLA2			RCS				
	Acc. Cav.	Lin. cav.	Total	1	2	3	4	Total
RF frequency [MHz]	**352**	1056		1300				
Number of cells per cavity	4	6		9				
Synchronous phase [deg]	95	275		135				
Nominal Gradient: E_{acc} [MV/m]	**15**	**25**		30				
Combined beam current (μ+, μ-) [mA]	**134**			**43.3**	**39**	**19.8**	**5.5**	
Q_{ext} of the input coupler	0.4E6	0.2E6		0.7E6	0.8E6	1.5E6	5.5E6	
Total RF voltage [GV]	**15.2**	**1.7**	**16.9**	**20.9**	**11.2**	**16.1**	**90**	**138.2**
Number of cavities	600	80	680	683	366	524	2933	4506
Number of cryomodules	200	16	216	76	41	59	326	502
Total RF section length [m]	**1110.6**	80.8	1191.4	962	519	746	**4125**	**6351**
RF duty factor [%]	0.19	0.05		0.19	0.57	1.22	3.36	
Peak RF power [kW/cavity]	**3425**	**2965**		**1128**	**1017**	516	144	
Total average RF power [MW]	5.16	0.16	5.32	1.9	2.8	4.4	18.9	28

Initial LINAC and RLA1 are not included, no design yet

State of the art

352 MHz LEP2 cavity (Nb/Cu)

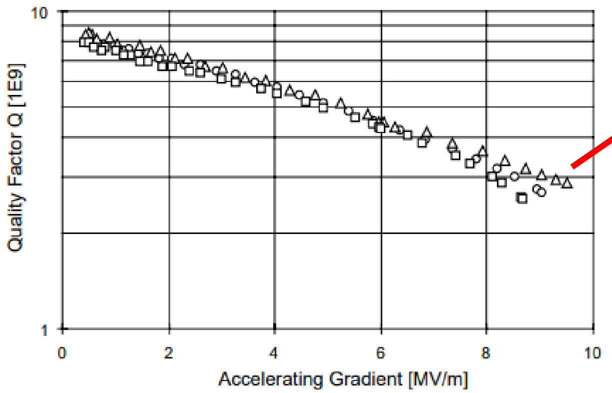
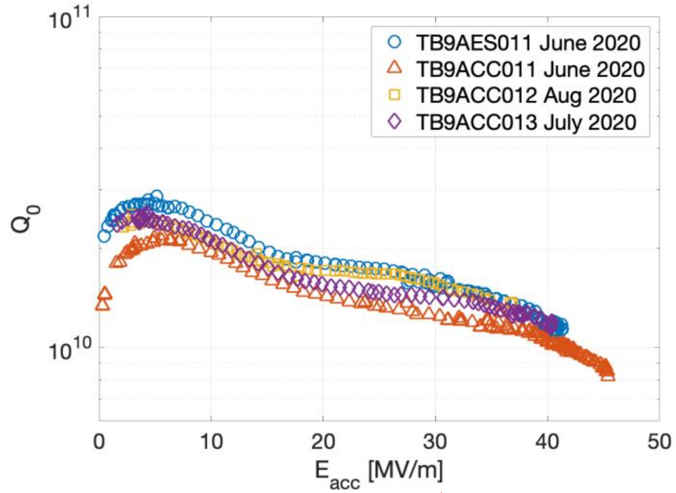


Fig. 2 Best Q vs E curves from three manufacturers

Need performance improvement

1300 MHz TESLA cavity for ILC

9-cell cavities



OK, but there are other challenges

SRF for MuC: Challenges

There are **significant challenges** that need to be addressed. Among these challenges are:

- a very high muon bunch intensity would favour larger cavity apertures and hence lower RF frequencies;
- a relatively small aperture of 1300 MHz cavity may not be sufficient and might require switching to lower frequency;
- high gradient operation of multi-cell lower frequency cavities must be demonstrated;
- the lower frequency SRF cavities of 352 MHz most likely will utilize Nb/Cu SRF technology – requires significant R&D efforts;
- stray magnetic fields from high-field magnets may significantly degrade performance of SRF cavities – there is a need to developing efficient magnetic shielding and/or develop SRF cavities based on alternative superconductors;
- effect of high-intensity radiation from muon decays on the performance of SRF cavities is unknown and must be studied.

6

Further reading

Further reading

- **Textbooks on superconductivity**

  W. Buckel and R. Kleiner, *Superconductivity: Fundamentals and Applications*, Wiley-VCH, 2004

  M. Tinkham, *Introduction to Superconductivity,* Second Edition, Dover, 2004

- **Textbooks on electrodynamics and microwaves**

  S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, John Wiley & Sons, 1994

  J. D. Jackson, *Classical Electrodynamics,* John Wiley & Sons, 1999

  R. E. Collins, *Foundations for Microwave Engineering*, John Wiley & Sons, 2001

  D. M. Pozar, *Microwave Engineering,* Wiley, 2005

- **Books on SRF**

  H. Padamsee, J. Knobloch and T. Hays, *RF Superconductivity for Accelerators,* Wiley, 1998

  H. Padamsee, *RF Superconductivity: Science, Technology, and Applications,* Wiley-VCH, 2009

  H. Padamsee, *Superconducting Radiofrequency Technology for Accelerators: State of the Art and Emerging Trends,* Wiley-VCH, 2023

  M. Hein, *High-Temperature-Superconductor Thin Films at Microwave Frequencies*, Springer, 1999

- **Tutorials** from International Conferences on RF Superconductivity, SRF2003 – SRF2023

- **Websites**

  Superconducting RF Cavities: A Primer by J. Graber from http://w4.lns.cornell.edu/public/CESR/SRF/BasicSRF/SRFBas1.html

  Superconducting Radio Frequency from Wikipedia http://en.wikipedia.org/wiki/Superconducting_RF

- **Review articles in journals**

  Vol. 5 (2012) of Reviews of Accelerator Science and Technology dedicated to applications of superconductivity to accelerators https://www.worldscientific.com/toc/rast/05

  Special issue of Superconducting Science and Technology (SIST) focused on superconducting RF for accelerators https://iopscience.iop.org/journal/0953-2048/page/Focus-SRF

- **Courses** at various US Particle Accelerator Schools, CERN Accelerator Schools, etc.

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