US MCC 2025 School (UChicago)
Homework Day 1 Assignments

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Group 1: High Energy Collider Design

1.1: Consider a 14 TeV cme collider in the Large Hadron Collider tunnel (total circumference 26.7km, 18.5km of dipole, average dipole bending radius is 2.8 km). Evaluate synchrotron radiation effects colliding particles are protons, muons, or electrons/positrons:
(a) Calculate the critical energy of photons in the three cases.
(b) Calculate the energy loss per turn per proton, muon, e+/e-.
(c) Calculate the total power radiated by synchrotron radiation with an average current of 1 A in proton, muon, or e+/e- beams.

1.2: Suppose 2km of LHC tunnel were set aside to accelerate the beam at a rate of 30 MV/m. What is the maximum electron beam energy that could be stored, where the radiation energy loss exactly matches the energy restored by RF acceleration? Finally, what acceleration gradient (MV/m) would be needed to linearly accelerate an electron beam energy from rest to that same energy over the LHC tunnel radius (i.e 4.25 km).

Group 2: Particle Production, Solenoidal Capture

2.1: Consider a narrow pion production target housed within a solenoid with a field strength $B_{||}$=20 T, an inner diameter of d=1.2 m, and a length L=2 m (i.e. forward from end of target).
(a) What is the Larmor radius for an 8 GeV proton with a 1 mrad divergence? What is the change in transverse position of the proton at the end of the solenoid length?
(b) For a 140 MeV (longitudinal kinetic energy) pion centered in the solenoid, what is the maximum transverse momentum that can be transported to the end of the 2 m solenoid without striking the solenoid aperture?

2.2: Consider the 20 T solenoid adiabatically tapering down to 2 T.
(a) Estimate the length and s-dependence of the magnetic field necessary to taper to adiabatically transport the 140 MeV pion.
(b) Keeping the inner diameter at 1.2 m, what is the maximum initial transverse momentum of the 140 MeV pion that can be transported to the end of the taper without striking the solenoid aperture?

Group 3: Muon Acceleration

3.1: Consider a bunch of muons accelerated from 50 GeV to 7 TeV.
a) If accelerated in a linear accelerator with a gradient of 100 MV/m - What would be the linac length and how many muons would survive?
b) If accelerated in a circular accelerator with a circumference of 30 km, what energy gain per turn is needed for 50% of muons to survive?

3.2: Calculate the required magnetic field and magnet ramp rate (db/dt) for the for 30 km ring, 50% decay loss scenario described above. Discuss various factors limiting maximum attainable field B and ramping rate dB/dt in fast-ramping electromagnets for particle acceleration to high energies.

Group 4: Muon Collider Optics

4.1: Prove that for a triplet-focusing low-beta insertion, the phase advance from the max beta function location on one side to the other side is close to π. Please assume the distance from either end of the insertion to the minimum beta location (s^*) is long than the minimum beta value (β^*).

4.2: Consider a 14 TeV cme muon collider.
(a) Assume a quadrupole with a 7 cm inner diameter, 12 T pole tip field, 5 m long. What is the quadrupole gradient (T/m) and what is the focal length f.
(b) Consider a FODO cell with phase advance per cell 60 degrees, what is the distance between quadrupoles? How many of such cells can constitute collider ring in the LHC tunnel (C=26.7 km)? For an LHC tunnel lattice composed entirely of such cells, what will the tune be?
(c) Estimate the natural chromaticity for the LHC FODO ring? For muon collider with an rms energy variation of 70 MeV, what would the RMS chromatic tune spread be if uncompensated?
(d) Assume the space between quadrupoles is filled with dipoles, what is the bending radius in the dipoles of the ring? Average dispersion and average beta function? Estimate the number of 12 T pole fip field, 5 m long sextupoles that would be needed to compensate the natural chromaticity.

Group 5: Muon Collider Luminosity

5.1: Calculate the beam-beam tuneshift parameter ξ for a 10 TeV muon collider. What is the maximum transverse angular kick experienced by a muon from the opposite bunch passing through one IP? Compare with the rms angular spread at the IP in the muon bunch.

5.2: Consider a 1 TeV cme muon-proton collider at BNL: https://arxiv.org/pdf/2107.02073.
(a) How should we think about the beam-beam parameter and the luminosity at a beam-beam limit in this case?
(b) The muon beam is injected and refreshed repeatedly while the proton beam stays stored. Estimate the transverse emittance growth rate in 275 GeV proton beam due to the inconsistent focusing environment between beam-beam effect for muon bunches, versus the absence of the beam-beam effect.

Group 6: Advanced Topics

6.1: Consider three scenarios for an interaction between a particle beam and particles in a solid target.
(a) What's the minimum energy of the proton beam to hit a solid target and produce antiprotons via reaction (p+ + p+) → (p+ + p+) + (p+ + p-).
(b) Same for pion pair production (p+ + p+) → (p+ + p+) + (π+ + π-).
(c) What's the minimum energy of the positron beam to hit a solid target and produce a pair of muons via (e+ + e-) → (μ+ + μ-)

6.2: Consider an electron lens - a 2m long, 1 mm diameter line object that consists of electrons (1A current, 10 kV) - estimate a) max transverse kick such e-lens could produce on a 7 TeV muon in a circular muon collider in the LHC tunnel, b) how quickly such a muon can be driven onto a collimator, placed at 5 beam sigma, if the e-lens kick are random turn-to-turn? if applied resonantly?