Muon Capture and Transport

- How many 10 GeV protons per second are in a 4 MW beam? With 5 Hz bunches, how many protons/bunch?
- If beam of 10 GeV protons produces, on average, one pion pair per proton with mean momentum of 250 MeV/c (per pion), what percentage of the proton beam kinetic energy is converted to pions?
- If the target is surrounded by a 20T solenoid with a 5 cm radius aperture, what maximum transverse momentum of pions is accepted?
- If B is adiabatically reduced to 2.5T what is the resulting transverse momentum and beam orbit size?
- What is the Betatron function for a 300 MeV/c muon in a 2T solenoid? Hint: $K = (B/(2B\rho))^2$
- What is the rms beam sigma if the normalized emittance is 0.02m?

US MCC 2025 School (UChicago)
Homework Day 2 Assignments

August 5, 2025

Instructor: Jeffrey Eldred (FNAL).

Proton Driver Bunch Rotation

1: Consider a 10 GeV proton bunch with 2 ns bunch length with a Laslett tuneshift parameter of -0.4.
a) What will the Laslett tuneshift parameter be if (all other parameters the same) the bunch length was at 20 ns and the beam energy was 5 GeV?
b) What would the Laslett tuneshift parameter be if the bunches were split into four and the beam emittances were also a factor of four smaller?
c) What would the impact on the Laslett tuneshift parameter if the longitudinal beam profile were changed from a Gaussian distribution to the following bimodel distribution:

$$\frac{1}{4.58\sigma_z}\exp\left[\left(\frac{z}{2\sigma_z}\right)^2-\left(\frac{z}{2\sigma_z}\right)^4\right]$$

2: Let's consider the Hamiltonian for synchrotron (pendulum) motion. Particles near the center of the RF bucket (i.e. small angles where the Hamiltonian approximates a harmonic oscillator) follow a phase-space ellipse in ϕ and δ. Let $\hat{\phi}$ and $\hat{\delta}$ be the semi-major and semi-minor axises of that ellipse, such that $H(\hat{\phi},0)=H(0,\hat{\delta})=\hat{H}$.

Now let's imagine the voltage is suddenly jumped up from V_1 to V_2. After a $\pi/2$ synchroton oscillation, show that new coordinate positions are

$$(\hat{\phi},0)\longrightarrow(0,\sqrt{V_2/V_1}\hat{\delta})$$
$$(0,\hat{\delta})\longrightarrow(-\sqrt{V_1/V_2}\hat{\phi},0)$$

for counterclockwise motion (without loss of generality)

Fermilab

August 5, 2025

Ionization Cooling for Muon Beams Q & A

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U.S. DEPARTMENT of ENERGY
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FERMI NATIONAL ACCELERATOR LABORATORY

Question 1

1. Estimate the RMS beam size at the collision point to reach the design luminosity 10^{34} cm^{-2} s^{-1} using the luminosity formula, $\mathscr{L} = \dfrac{N_{\mu^+} \bullet N_{\mu^-} \bullet f \bullet n_b}{4\pi \bullet \sigma_x \bullet \sigma_y}$

   - Number of muons per single spill: $N_\mu = 10^{12}$
   - Bunch revolution: $f = 1000$ per spill
   - Repetition rate: $n_b = 5$ Hz

2. Estimate the beta star (β^*) at the collision point for the Center-of-Mass energy 10 TeV if the normalized muon beam transverse emittance is 20 mm mrad (= 20 μm rad)

β^*: beta function at the collision point

Emittance evolution

Break Liouville' theorem

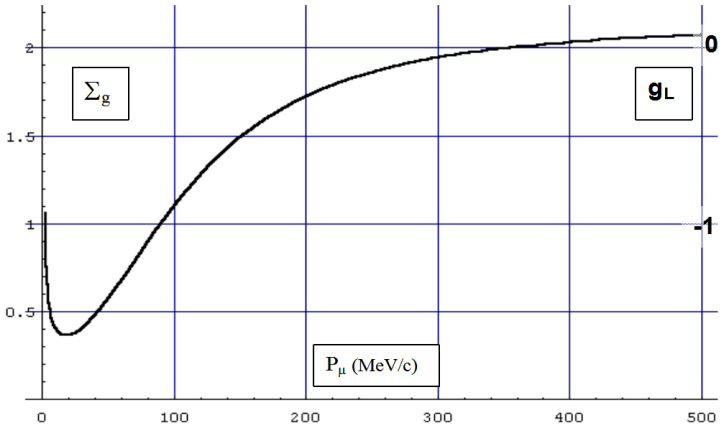
Question 2

1. Establish the emittance evolution formula
   - Mean muon momentum 200 MeV/c
   - Liquid H2 absorber, no dispersion ($D = 0$)
     - Radiation length of LH2: $L_R =$ 890.4 cm
   - Beta function $\beta_{x,y}$ are 40, 20, and 5 cm and initial normalized emittance are 5.1, 2.4 and 0.7 mm, respectively
2. If the absorber is wedge shape
   - If $w = 10$ cm and dispersion is 5.2 cm, what is $g_{x,y}$?
   - Right plot shows Σ_g~1.7 at p = 200 MeV/c, what is g_L?

Muons in hydrogen liquid (H_2)

Z	A [g/mol]	ρ [g/cm^3]	I [eV]	a	$k = m_s$	x_0	x_1	$\overline{C}$	δ_0
1 (H)	1.008(7)	7.080×10^{-2}	21.8	0.13483	5.6249	0.4400	1.8856	3.0977	0.00

T	p [MeV/c]	Ionization	Brems	Pair prod	Photonucl	Total	CSDA range [g/cm^2]
		[MeV cm^2/g]					
10.0 MeV	4.704×10^1	16.508				16.508	3.316×10^{-1}
14.0 MeV	5.616×10^1	12.812				12.812	6.097×10^{-1}
20.0 MeV	6.802×10^1	9.956				9.956	1.147×10^0
30.0 MeV	8.509×10^1	7.684				7.684	2.307×10^0
40.0 MeV	1.003×10^2	6.539				6.539	3.727×10^0
80.0 MeV	1.527×10^2	4.870				4.870	1.105×10^1
100. MeV	1.764×10^2	4.568				4.568	1.530×10^1
140. MeV	2.218×10^2	4.267				4.267	2.440×10^1
200. MeV	2.868×10^2	4.104			0.000	4.104	3.879×10^1
300. MeV	3.917×10^2	4.037			0.000	4.038	6.342×10^1
344. MeV	4.372×10^2	4.034			0.000	4.034	*Minimum ionization*
400. MeV	4.945×10^2	4.038			0.000	4.038	8.820×10^1
800. MeV	8.995×10^2	4.134			0.000	4.135	1.862×10^2



Solenoid base beam optics

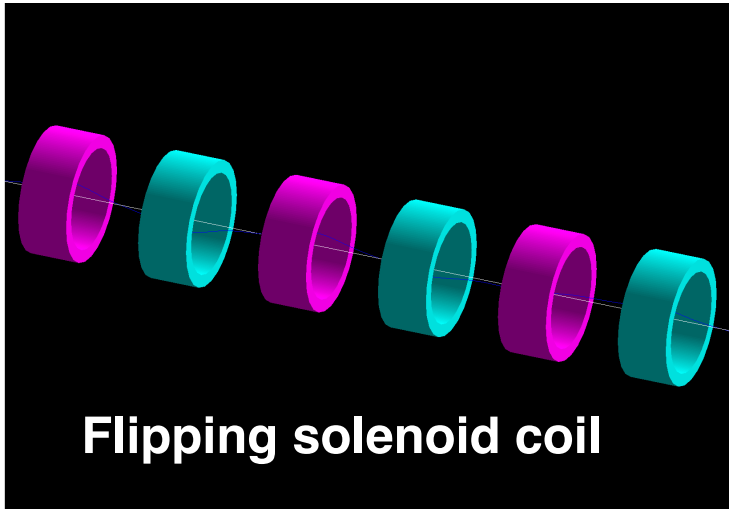
X and Y motions and their beta functions are coupled

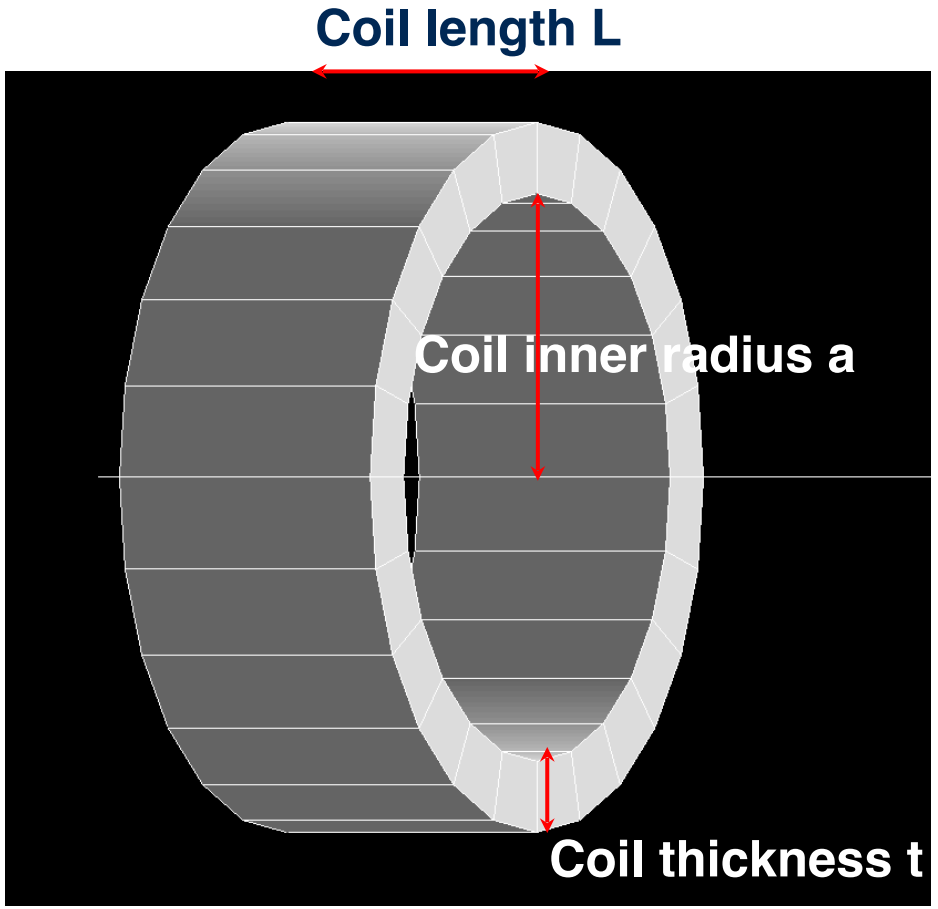
Question 3 (Optional)

- Evaluate FOFO channel (use my input file if you have g4beamline)

Fernow's coil (referred from PRSTAB 10, 064001 (2007)

- **Coil length = 400 mm**
- **Coil inner radius = 400 mm**
- **Coil thickness = 100 mm**
- **Current = ±100 Amp/mm2**
- **Period = 1000 mm**





Question 3

1. Demonstrate stop band
2. Demonstrate maximum and minimum beta functions for Fernow's four different channels (see slide 55)