Towards realistic predictions for weak bosons as partons of the muon

2nd US μ Collider Meeting – UChicago, USA

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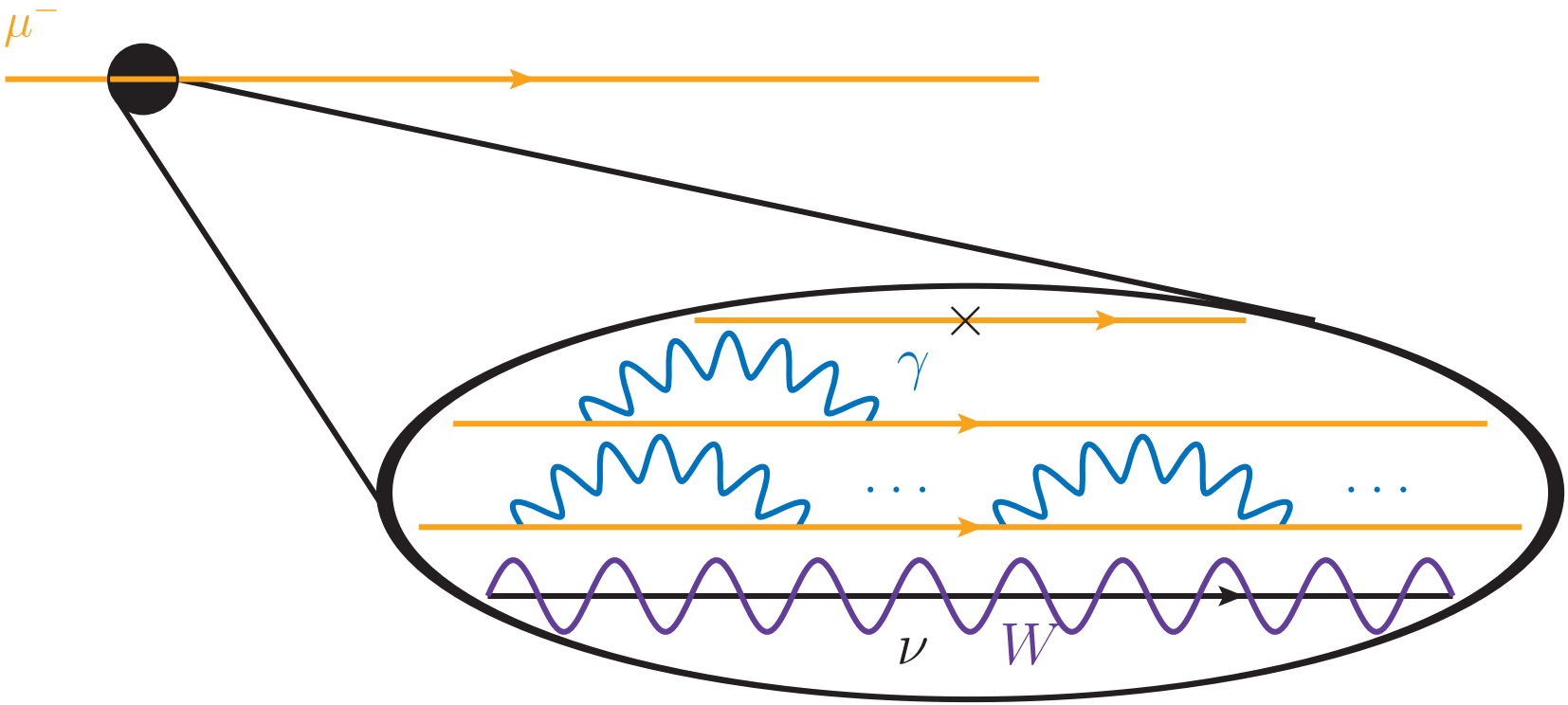
August 7, 2025

big idea

realize a core consequence of QFT+gauge theory



$\mu^\pm$ **carry weak gauge charges, like** $e^\pm$ **and** q

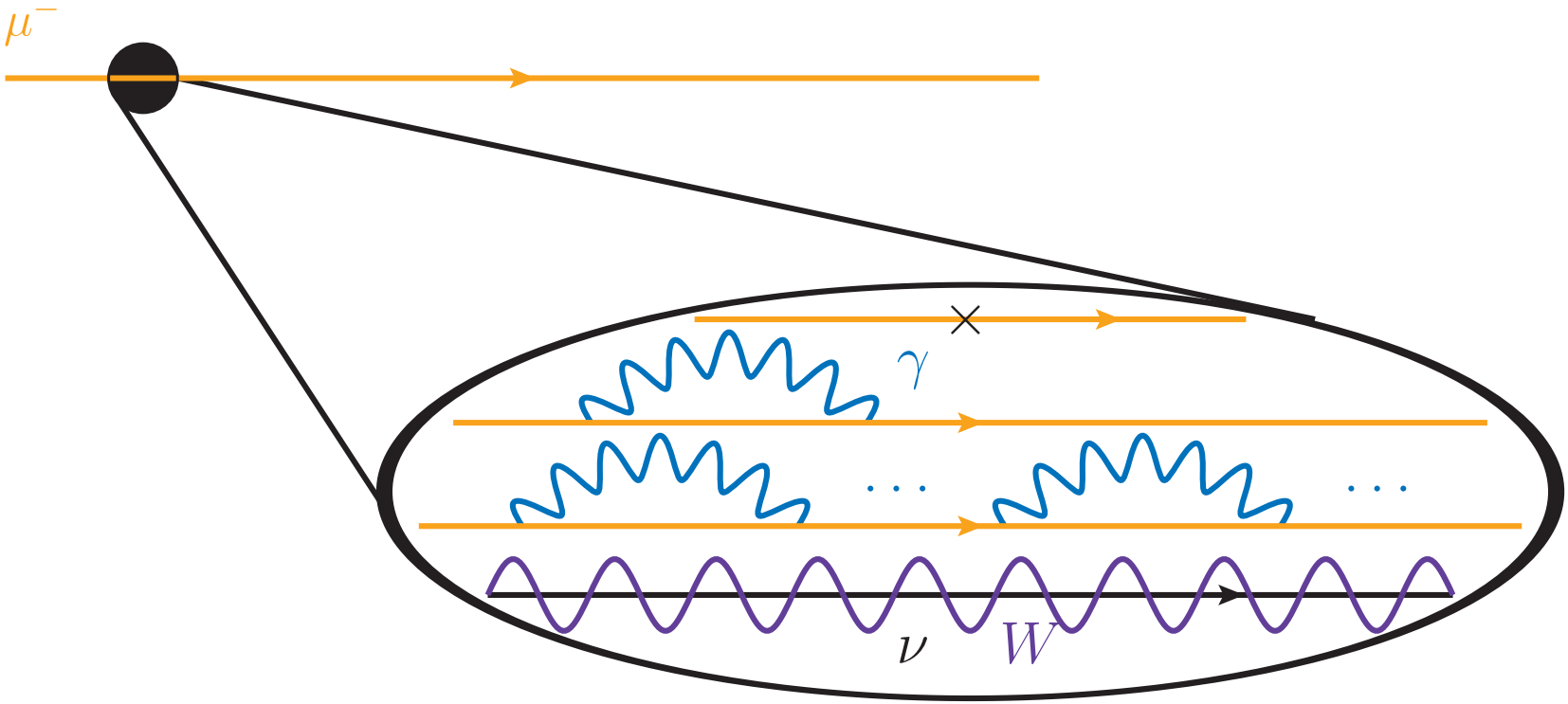
$\Longrightarrow$ **at sufficiently high energies, real** $W\nu$ **pairs are populated**[1]

Dawson('85), Kunszt & Soper ('88)

[1] consequence of quantum mechanics + special relativity

unlike QED and QCD, electroweak sector is spontaneously broken

$$\langle \Phi \rangle \neq 0 \implies M_W, M_Z \propto \langle \Phi \rangle \neq 0$$

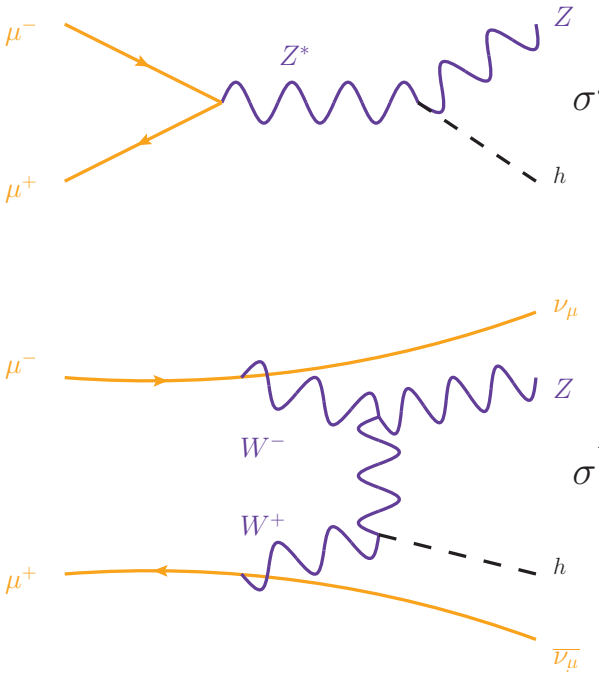


Open Question: is there some hard scattering scale Q above which massive W and Z really behave like massless γ and g?

some[2] accidental evidence for $Q \sim \mathcal{O}(1-2)$ TeV

[2] w/ A. Costantini, Maltoni, Mattelaer, et al [JHEP('20, '21)]

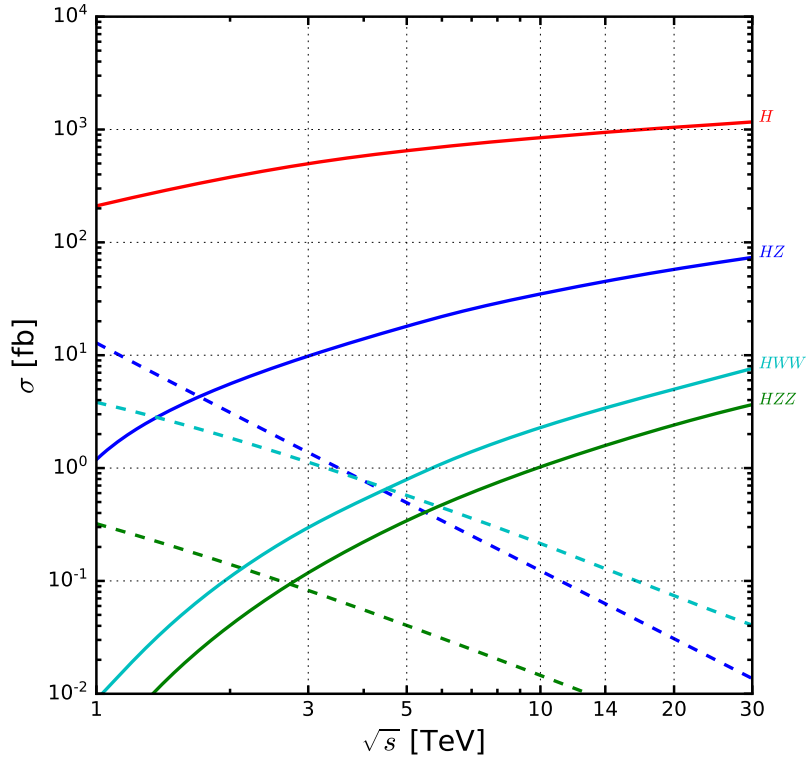
s-channel annihilation vs VBS in $\mu^+\mu^-$ collisions



$$\sigma^{s-channel} \sim \frac{g^4}{s}$$

$$\sigma^{VBF} \sim \frac{g^8}{M_{WW}^2} \log^2\left(\frac{M_{WW}^2}{M_W^2}\right)$$

Higgs Production



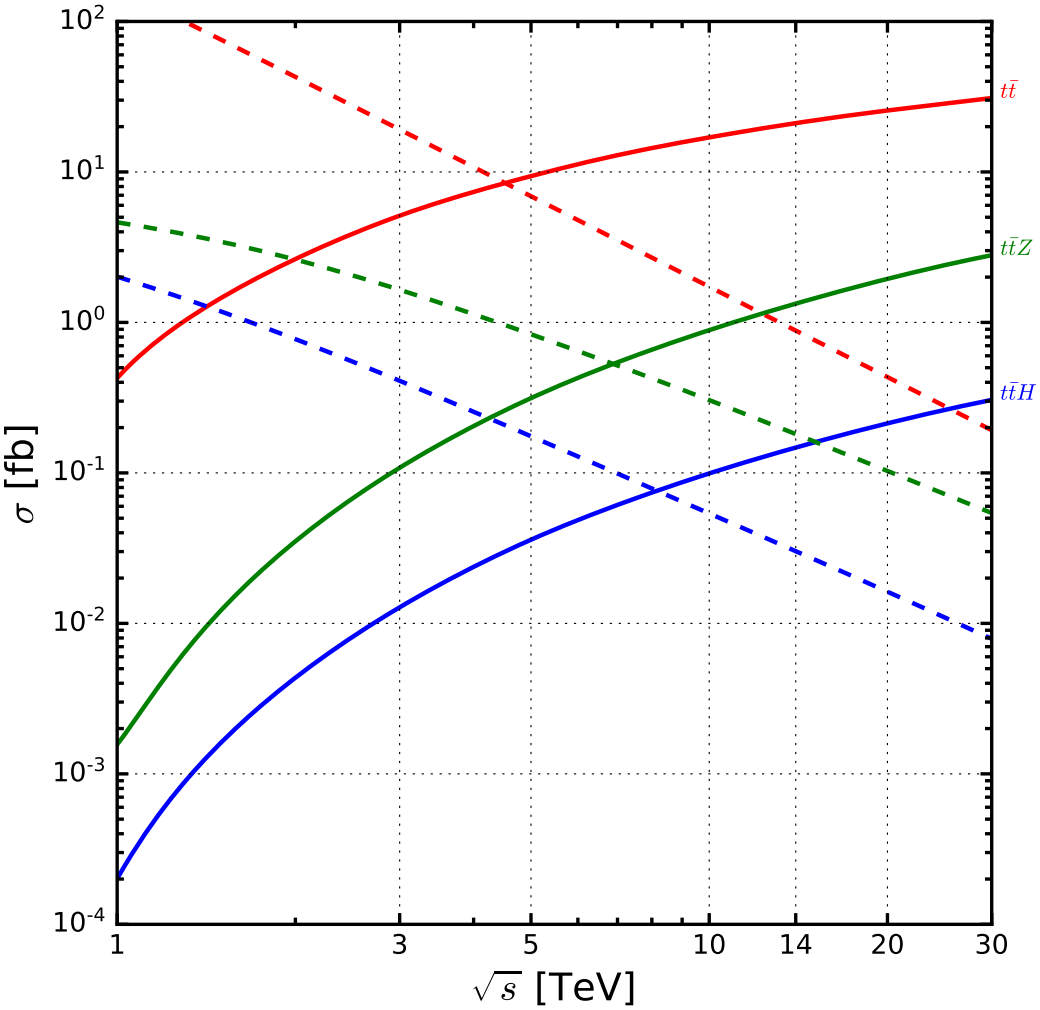
Eventually $\sigma^{VBF} > \sigma^{s-channel}$ since:

more legs $\Longrightarrow$ more propagators $\Longrightarrow$ $\int dk^2/(k^2 - M_W^2) \sim \log(\Lambda^2/M_W^2)$

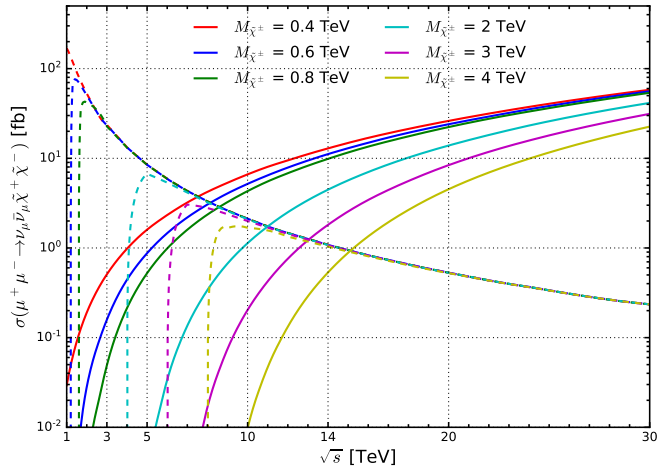
Larger s $\Longrightarrow$ larger (M_{WW}^2/M_W^2) $\Longrightarrow$ collinear V compensate for g

s-channel annihilation vs VBS in $\mu^+\mu^-$ collisions

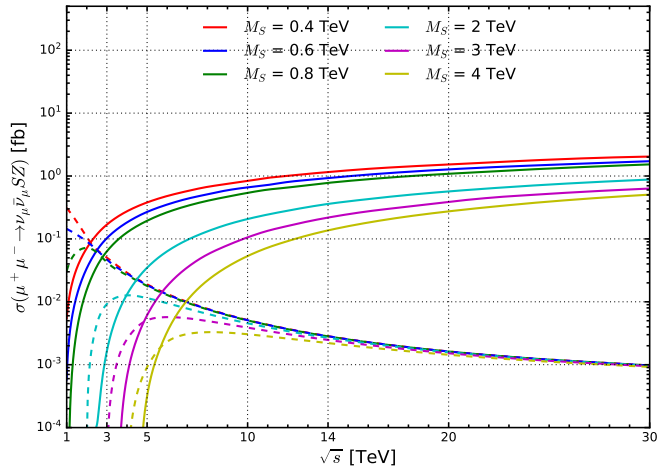
Top Production



Chargino Pair Production



Singlet+Z Production



Evidence for trend that VBS rates will always exceed s-ch. rates

Is this obvious? (yes, obviously, but not me!) **Is there intuition for this?** (yes!)

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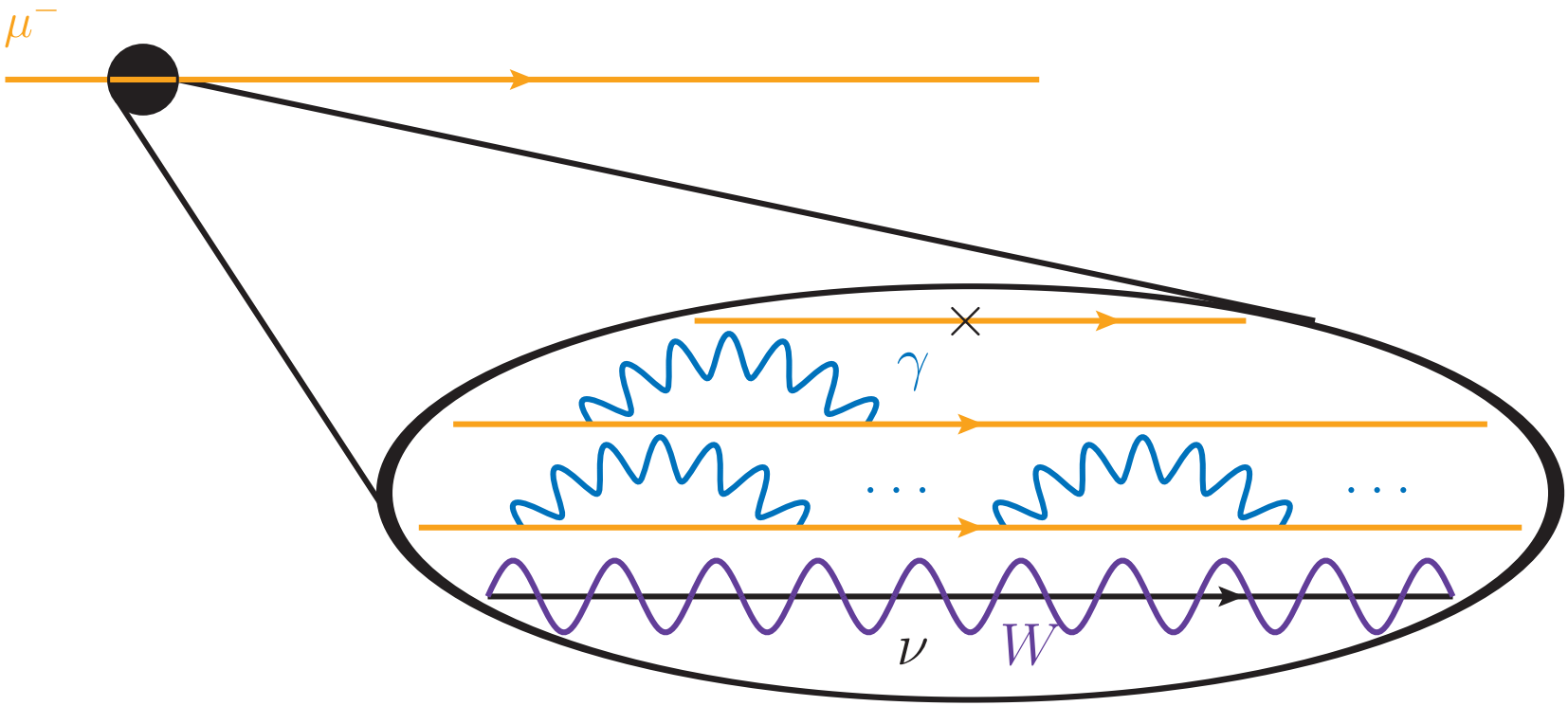

conjecture: increasing $\sigma^{\rm VBS}$ is manifestation of growing partonic content

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$$\sigma^{s-ch.} \sim \frac{(s-M_{\mathcal{F}}^2)}{(s-M_V^2)^2} \sim \frac{(s-M_{\mathcal{F}}^2)}{s^2} \quad \leftarrow \text{assumes } s \gg M_V^2$$

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$$\frac{d\sigma^{VBF}}{dz_1\,dz_2} \sim \underbrace{f_V(z_1)\,f_{V'}(z_2)}_{\text{“}\mu\text{PDFs”}} \underbrace{\frac{(M^2_{VV'} - M^2_{\mathcal{F}})}{(M^2_{VV'} - M^2_V)^2}}_{M^2_{VV'}=z_1 z_2 s \gg M^2_V} \sim f_V(z_1)\,f_{V'}(z_2) \frac{(z_1 z_2 s - M^2_{\mathcal{F}})\ \sigma^{s-ch.}}{(z_1 z_2)^2\ (s-M^2_{\mathcal{F}})}$$

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PDFs are largest when $z = E_V/E_\mu \ll 1$ but $E_V \sim \sqrt{s} \gg M_V$ **like in QCD!**

$$\Longrightarrow f_V(z_i) \sim \frac{g_W^2}{4\pi}\,\frac{1}{z_i}\,\log\left(\frac{s}{M_V^2}\right) \quad \leftarrow \text{crude approximation}$$

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the funny bit: $\sigma^{VBF} = \sigma^{s-ch.} \times \int dz_1\,dz_2 \ldots$ is solvable for $M_{VV'} \gg M_{\mathcal{F}}$

Universal behavior: when production of $\mathcal{F}$ by VBS and annihilation are driven by same physics, VBS **dominates** when $\sqrt{s}$ satisfies

$$\frac{\sigma^{\rm VBS}}{\sigma^{s-ch.}} \sim 4\left(\frac{g_W^2}{4\pi}\right)^2 \left(\frac{s}{M_{\mathcal{F}}^2}\right) \log^2\frac{s}{M_V^2} \log\frac{s}{M_{\mathcal{F}}^2} > 1$$

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Scaling estimate not so bad if $M_{\mathcal{F}} \gg M_V$. Difference is about $\mathcal{O}(10\%)$

mass (M_X) [TeV]	SZ (Singlet)	H_2Z (2HDM)	$t'\bar{t}'$ (VLQ)	$\tilde{t}\bar{\tilde{t}}$ (MSSM)	$\tilde{\chi}^0\tilde{\chi}^0$ (MSSM)	$\tilde{\chi}^+\tilde{\chi}^-$ (MSSM)	Scaling (Eq. 7.7)
400 GeV	2.1 TeV	2.1 TeV	11 TeV	2.9 TeV	3.2 TeV	7.5 TeV	1.0 (1.7) TeV
600 GeV	2.5 TeV	2.5 TeV	16 TeV	3.8 TeV	3.8 TeV	8.1 TeV	1.3 (2.4) TeV
800 GeV	2.8 TeV	2.8 TeV	22 TeV	4.3 TeV	4.3 TeV	8.5 TeV	1.7 (3.1) TeV
2.0 TeV	4.0 TeV	4.0 TeV	>30 TeV	7.8 TeV	6.9 TeV	11 TeV	3.7 (6.8) TeV
3.0 TeV	4.8 TeV	4.8 TeV	>30 TeV	10 TeV	9.0 TeV	13 TeV	5.3 (9.8) TeV
4.0 TeV	5.5 TeV	5.5 TeV	>30 TeV	13 TeV	11 TeV	15 TeV	6.8 (13) TeV

Table 9. For representative processes and inputs, the required muon collider energy $\sqrt{s}$ [TeV] at which the VBF production cross section surpasses the s-channel, annihilation cross section, as shown in figure 17. Also shown are the cross over energies as estimated from the scaling relationship in equation (7.7) assuming a mass scale M_X ($2M_X$).

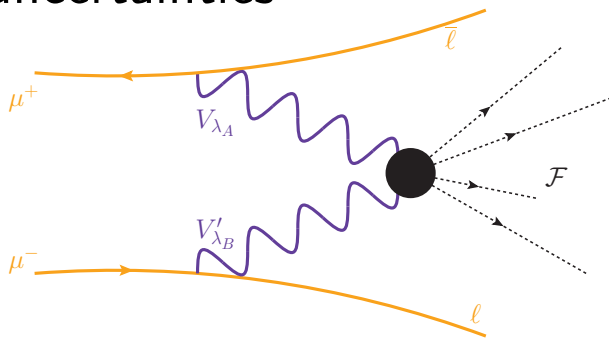
naïvely approximating VBS with PDFs worked too well...

we got curious and decided to be more systematic about things

factorization formula as implemented in mainstream event generators

MadGraph5_aMC@NLO ('20, '21, '25) , WHIZARD ('25)

$$\sigma(\mu^+\mu^- \to \mathcal{F} + \text{ anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$



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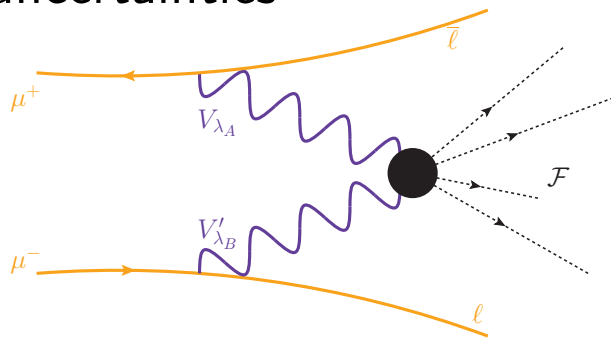
$$= \underbrace{\sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^{1} d\xi_1 \int_{\tau_0/\xi_1}^{1} d\xi_2 \int dPS_{\mathcal{F}}}_{\text{sum over all configurations / phase space integral}}$$



$$\times \left[\underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f)\ f_{V'_{\lambda_B}/\mu^-}(\xi_2, \mu_f)}_{\text{bare } W^+_\lambda/W^-_\lambda/Z_\lambda/\gamma_\lambda \text{ PDFs at LO}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{“hard scattering” at LO}}$$

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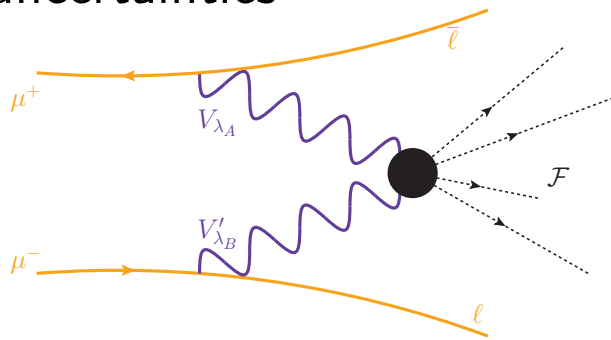

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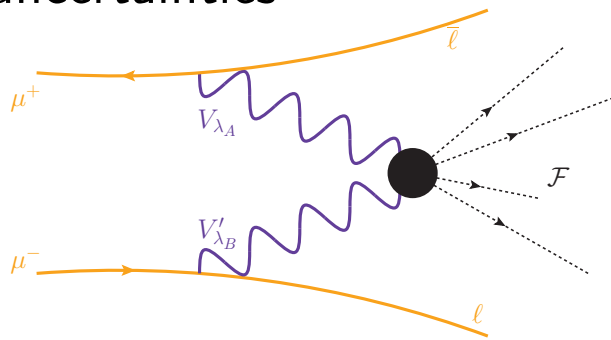

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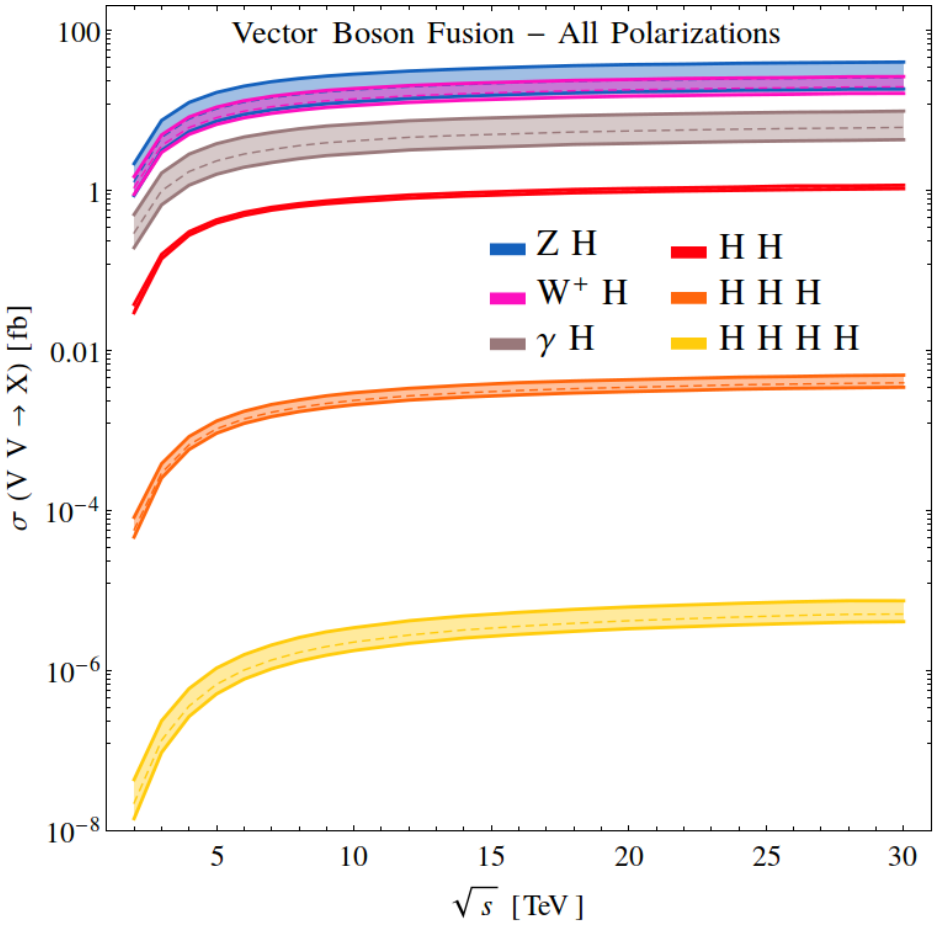
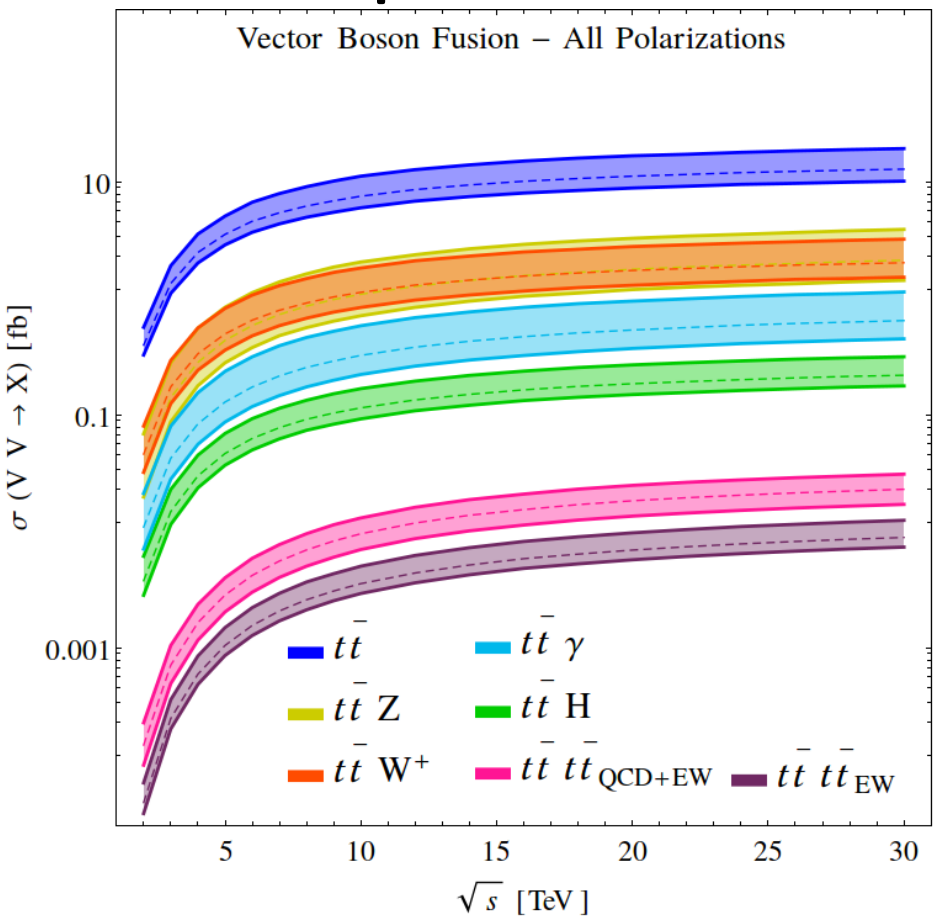
Higgs Production



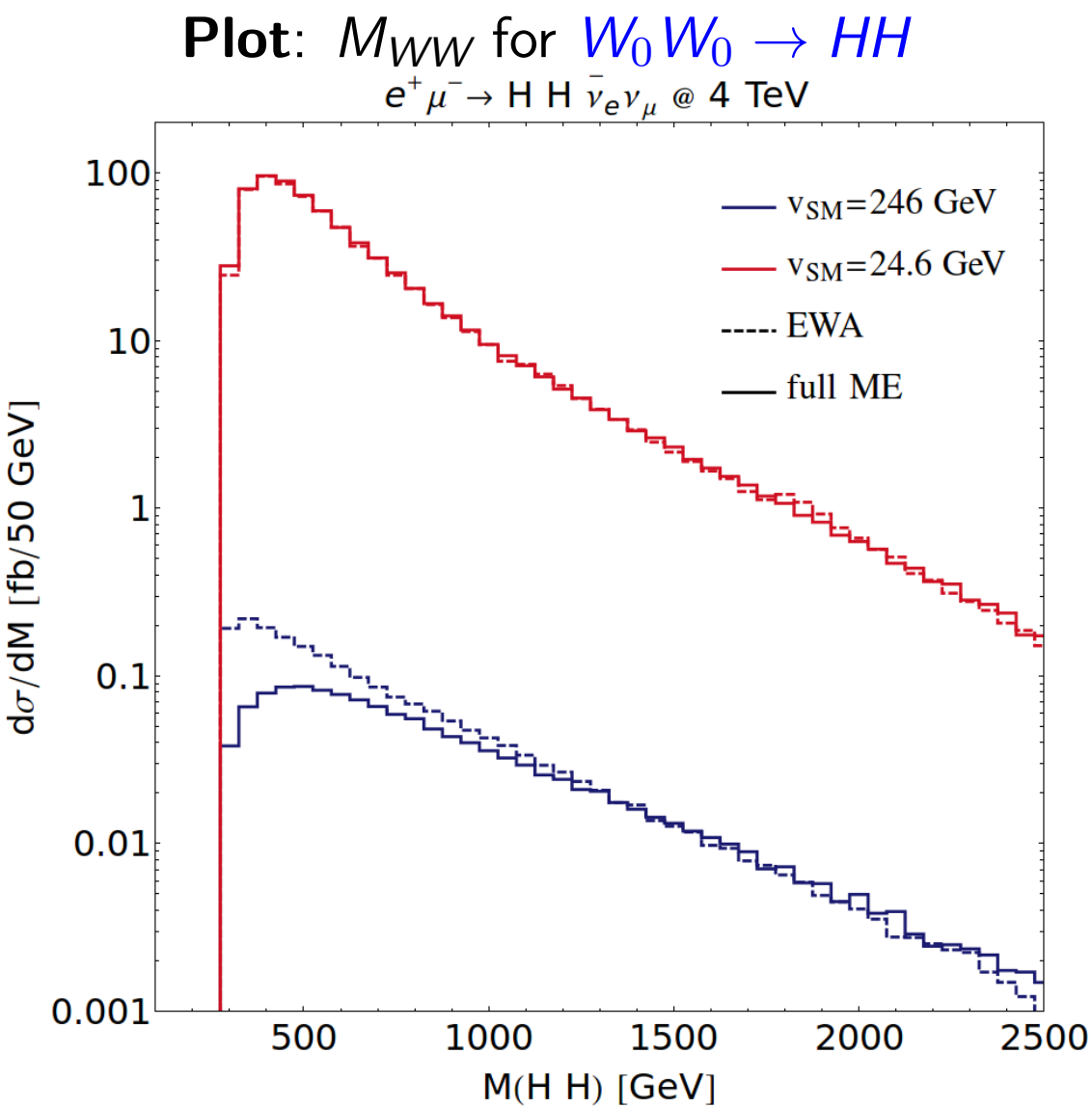
Top Production

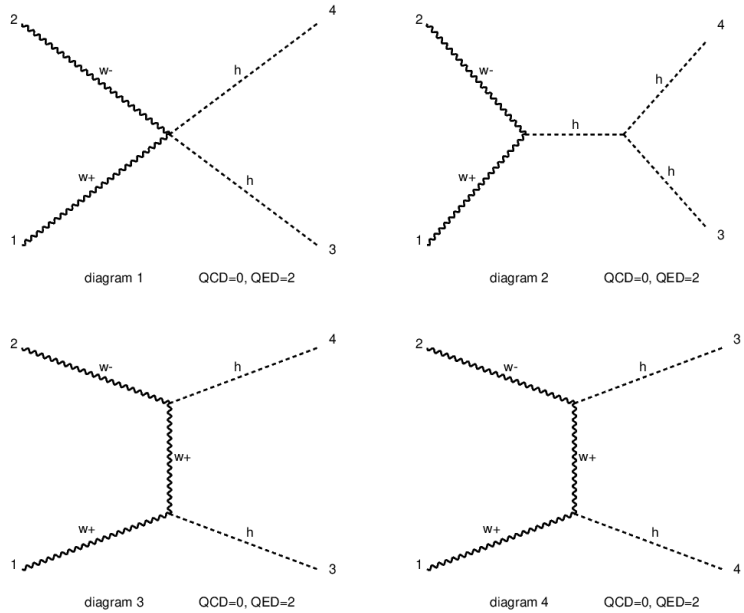


many, many more plots in arXiv:2111.02442

money plot

Plot: M_{WW} for $W_0 W_0 \to HH$





← solid = full ME

← dashed = approximated

lower $= \sqrt{2}\langle\Phi\rangle = v_{\rm EW}$

w/ A. Costantini, et al [2111.02442]

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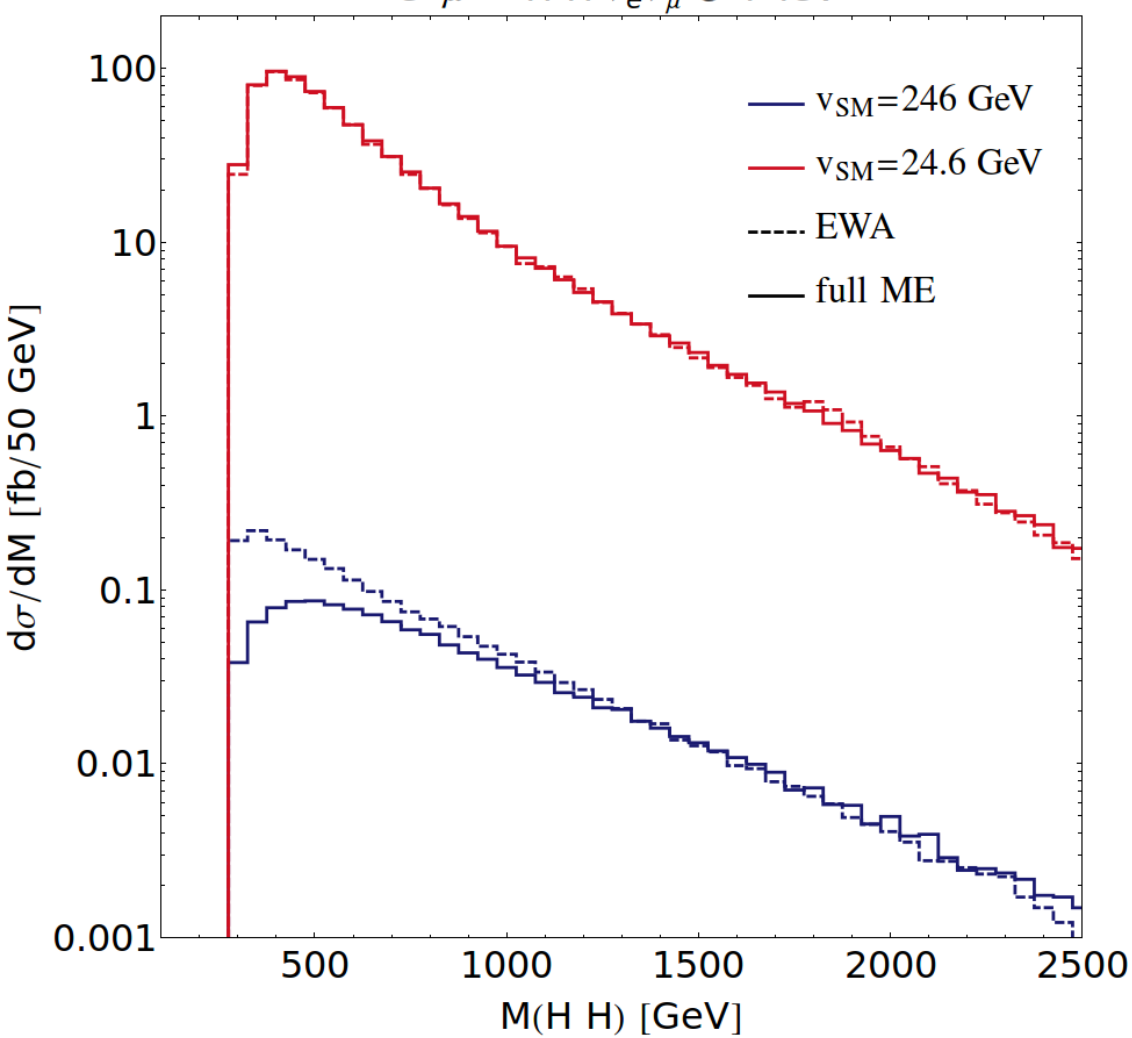

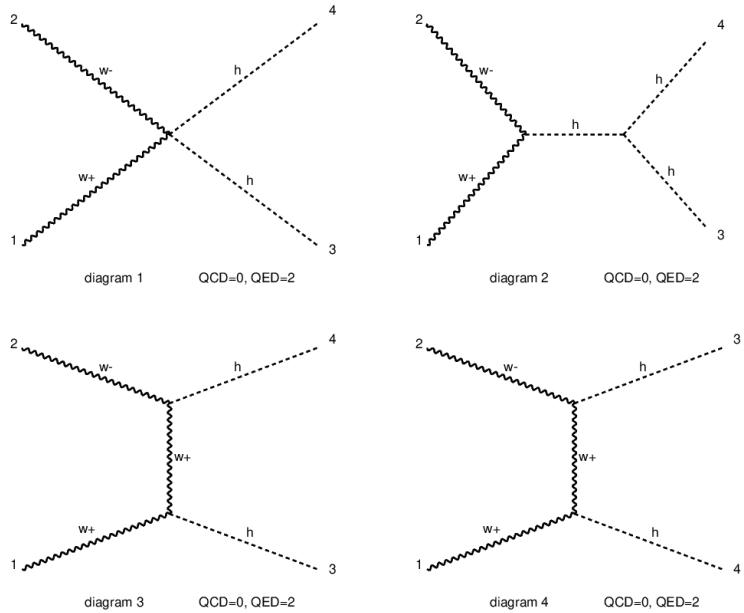

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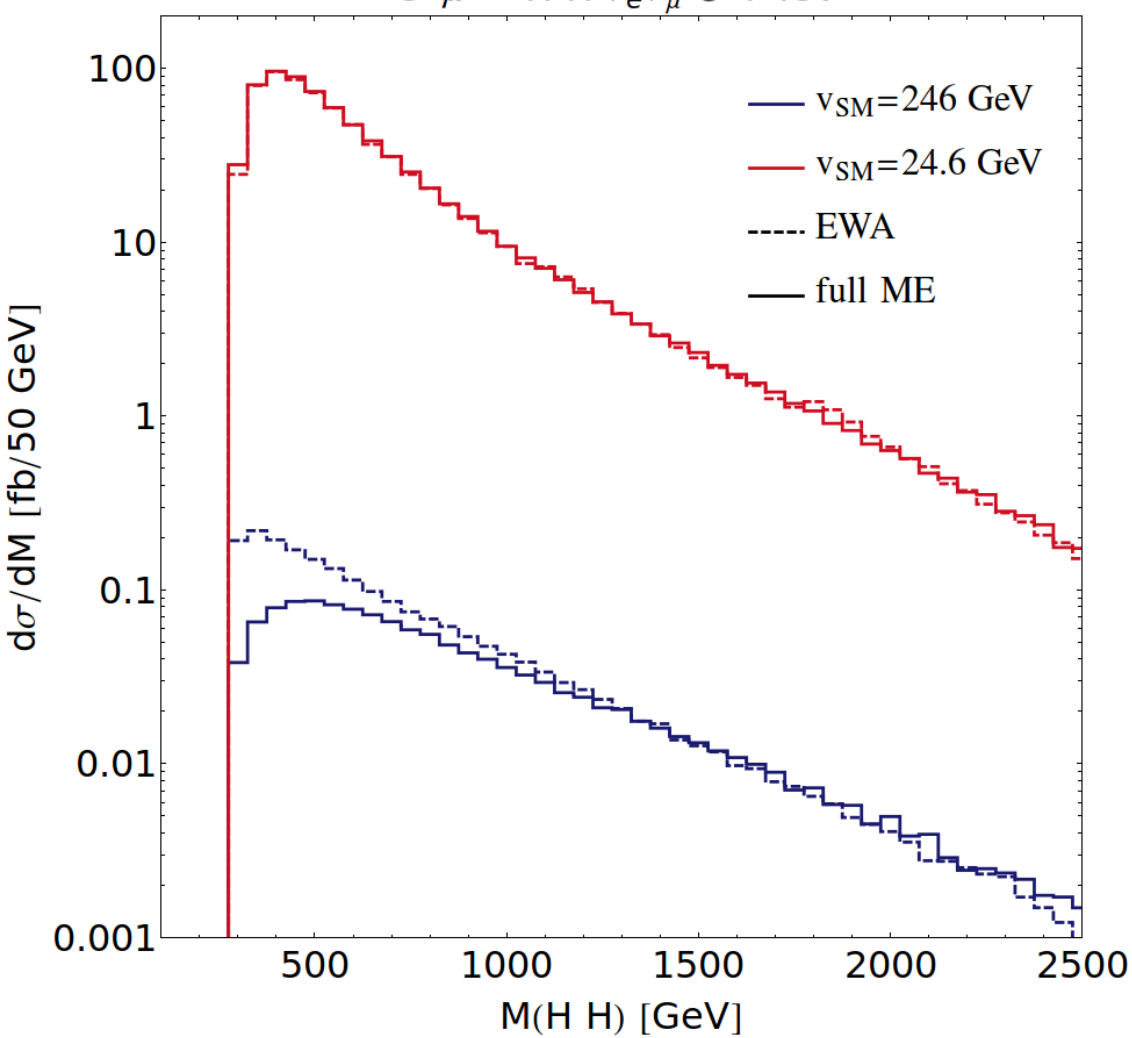

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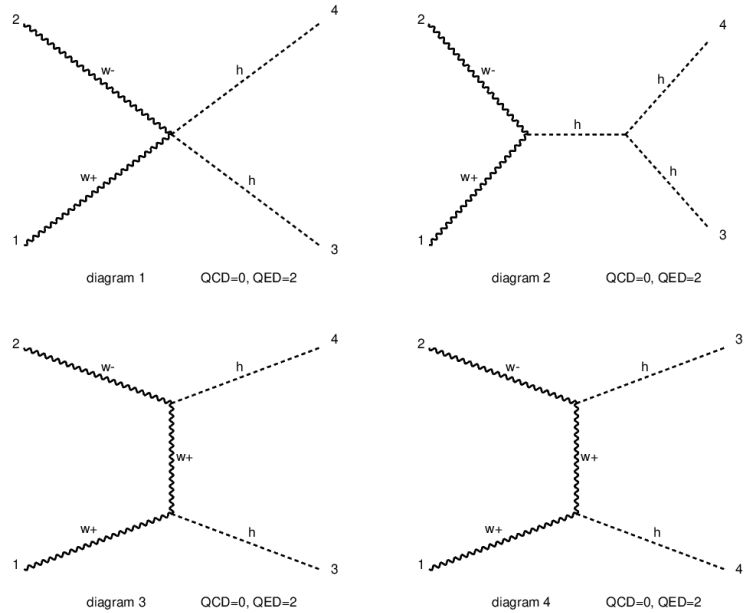

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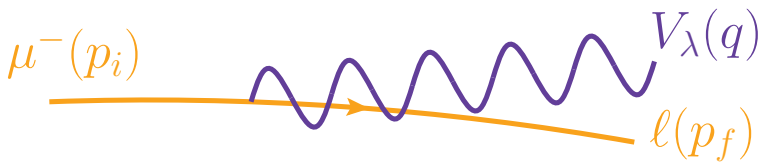
take away: disagreement in "PDF picture" due to M_V

some[3] non-accidental evidence for $Q \sim \mathcal{O}(1-2)$ TeV

[3] w/ I. Bigaran [2502.07878]

power-law corrections originate from taking
collinear expansion of the $\mu^- \to V_\lambda \ell$ matrix element



$$= \underbrace{\text{LLA} + \mathcal{O}\left(\frac{M_V^2}{\mu_f^2}\right)}_{\text{LP in } \theta_V} + \underbrace{\mathcal{O}\left(\frac{\mu_f^2}{E_V^2}, \frac{M_V^2}{E_V^2}\right)}_{\text{NLP in } \theta_V} + \ldots$$

idea: keep track of higher order terms in the collinear expansion

similarities to case of $e^- \to \gamma e^-$ splitting by Frixione, et al (PLB'93) [hep-ph/9310350]

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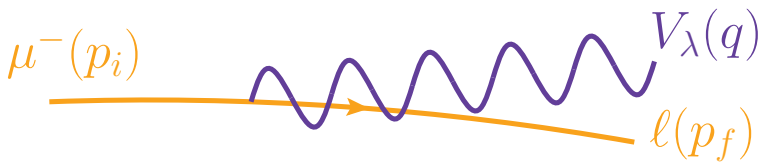

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similarities to case of $e^- \to \gamma e^-$ splitting by Frixione, et al (PLB'93) [hep-ph/9310350]

unexpected result: NLP corrections introduce pathologies!

- PDFs go **negative** for too small momentum fractions $z = E_V/E_\mu$

  (violating assumption of V being on shell if $E_V = zE_\mu < M_V$)

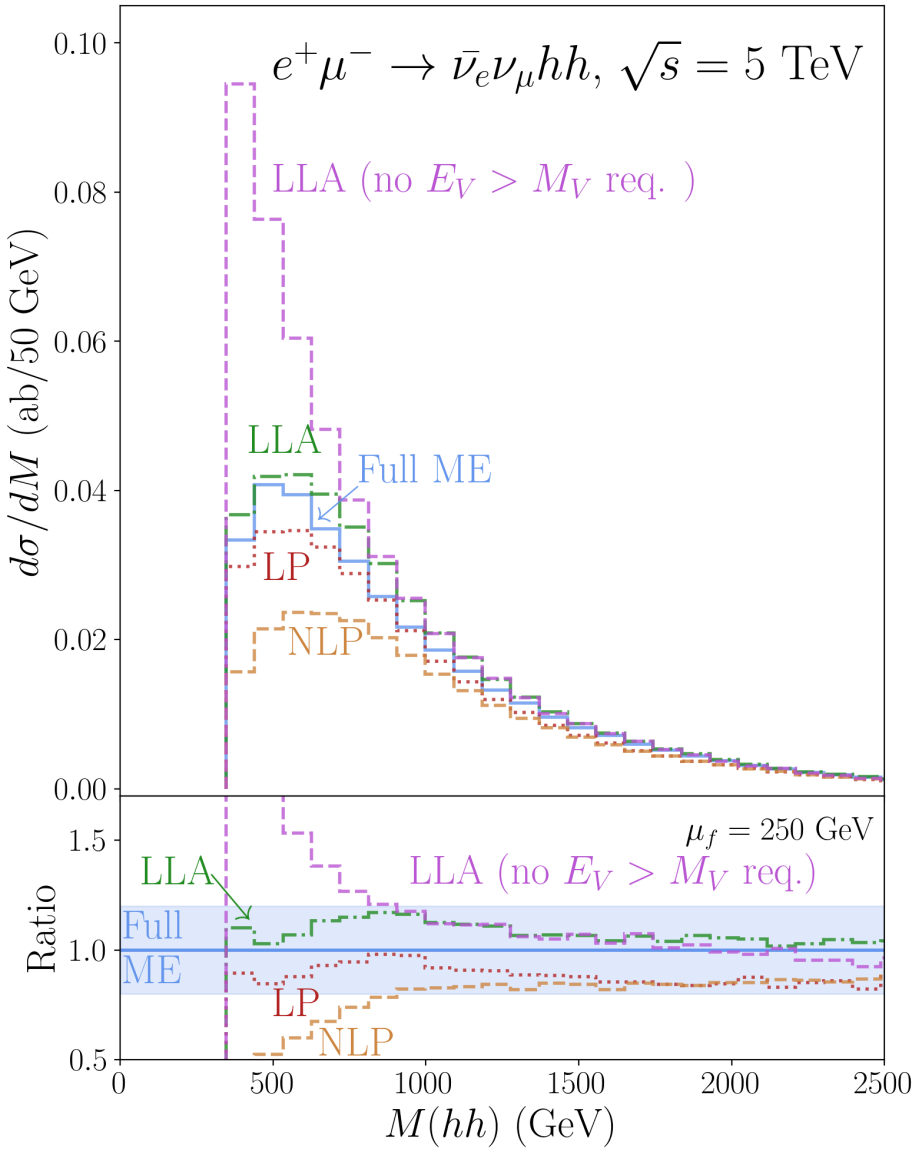
- PDFs go **negative** for too large cutoff scale choices μ_f

  (violating collinear approximation if $\mu_f \sim p_T^V \gtrsim E_\mu$)

  - demanding negligible NLP corrections $\implies$ well-defined kinematics

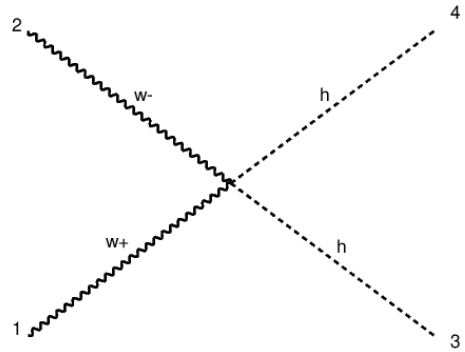
money plot (v2.0)

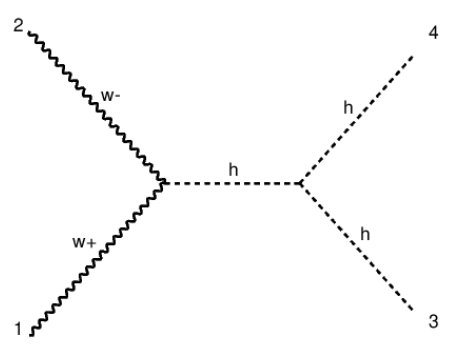
Plot: M_{WW} for $W_0 W_0 \to HH$

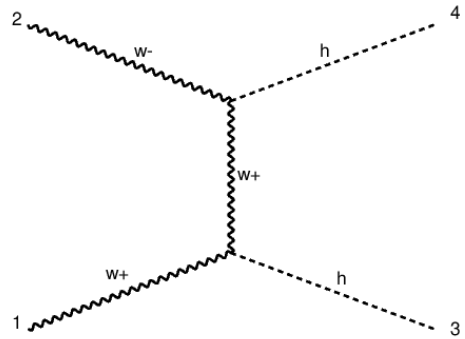


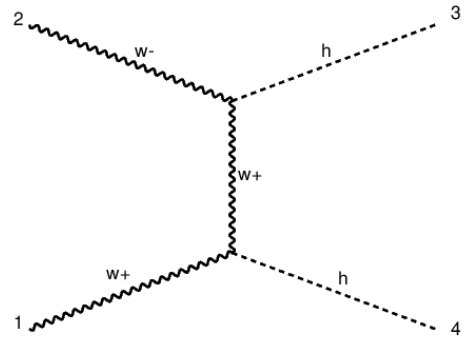
w/ I. Bigaran [2502.07878] `Mg5aMC v3.7.0` stable dev at

github.com/mg5amcnlo/mg5amcnlo/tree/3_x _EVA _NLP







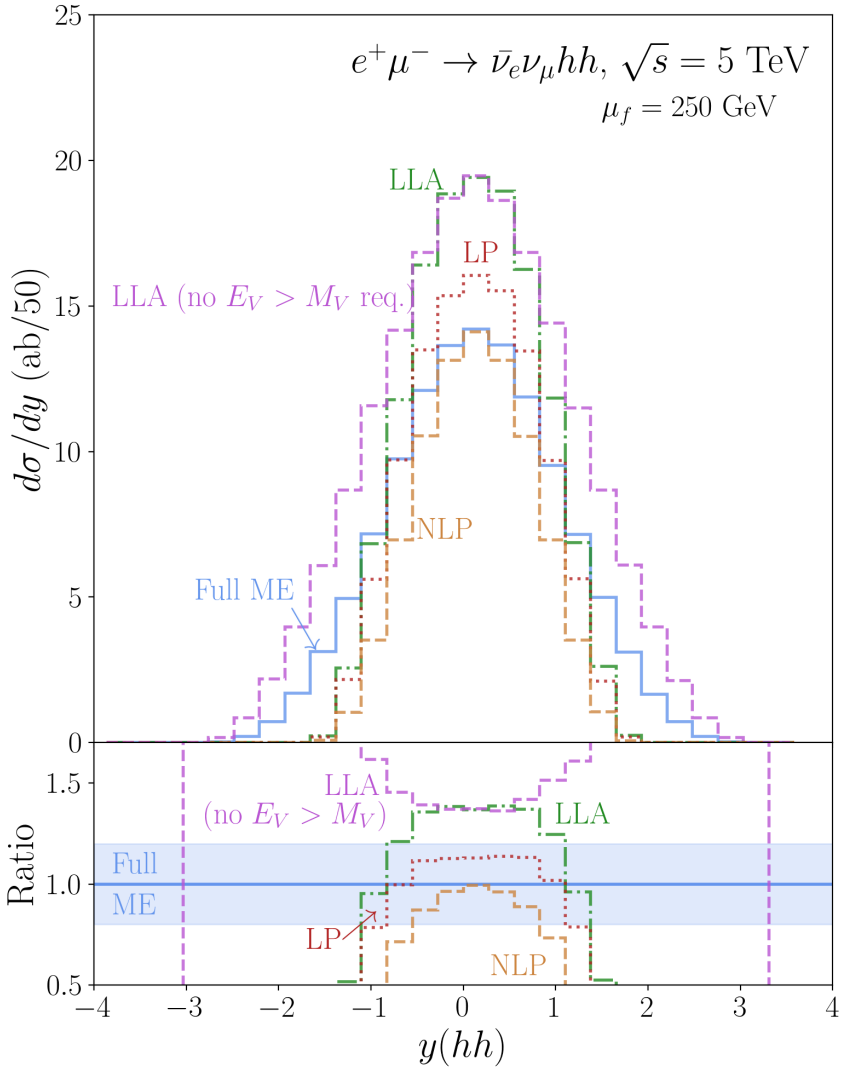


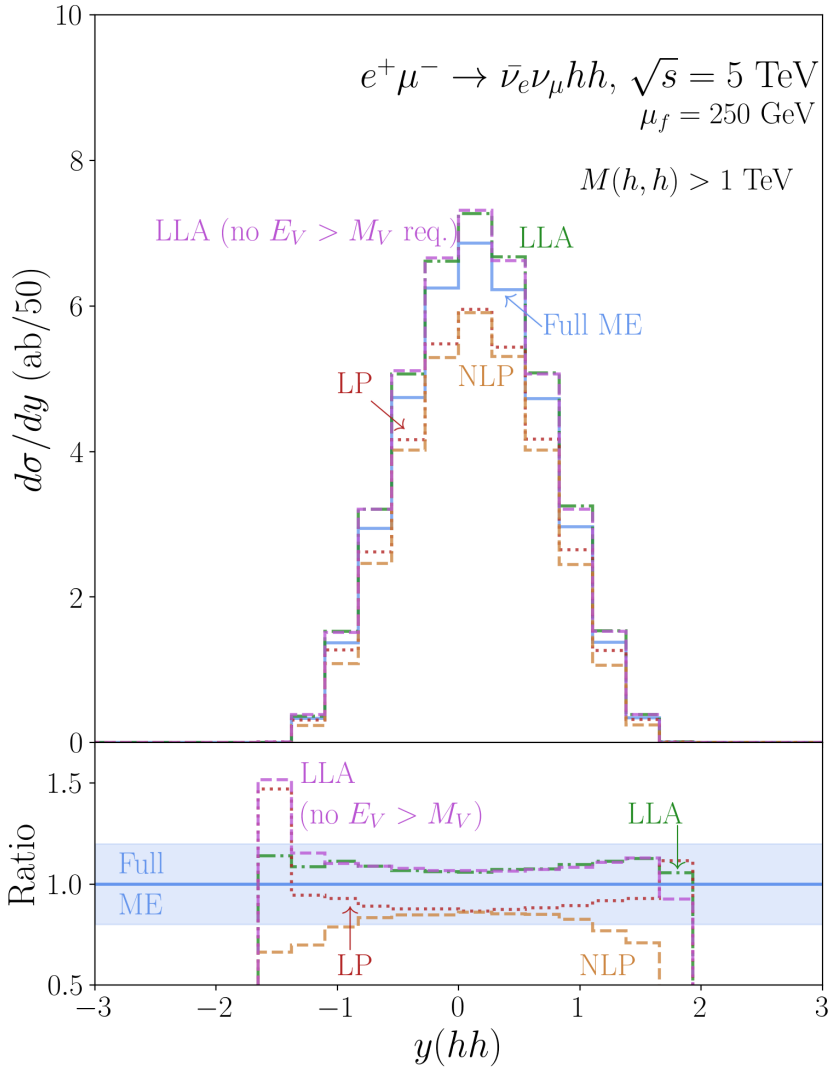
← **pink** = old LLA

← **green** = new LLA

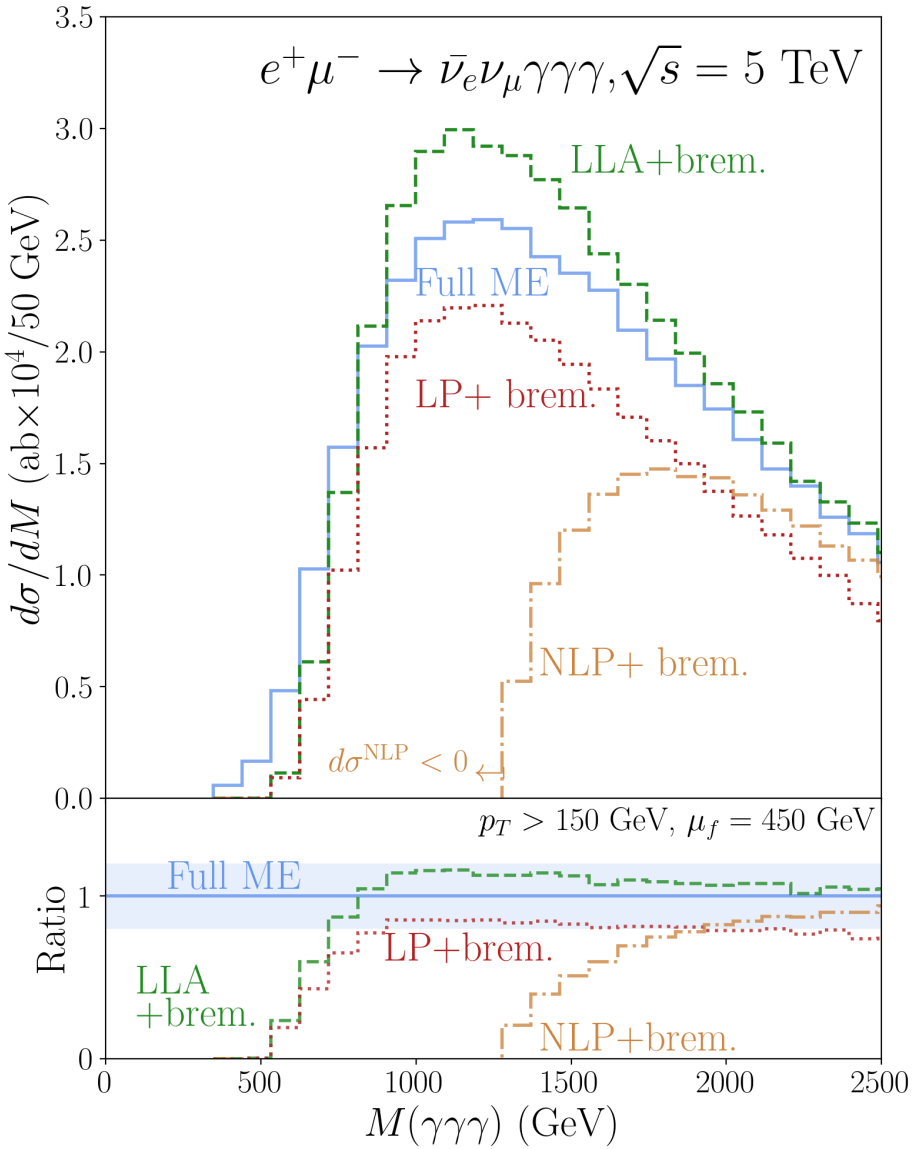
take away: "PDF picture" seems under control for incoming $E_W \gtrsim 10 \times M_V$ (pCM)

Plotted: $y(hh)$ (L) without and (R) $M(WW) > 1$ TeV

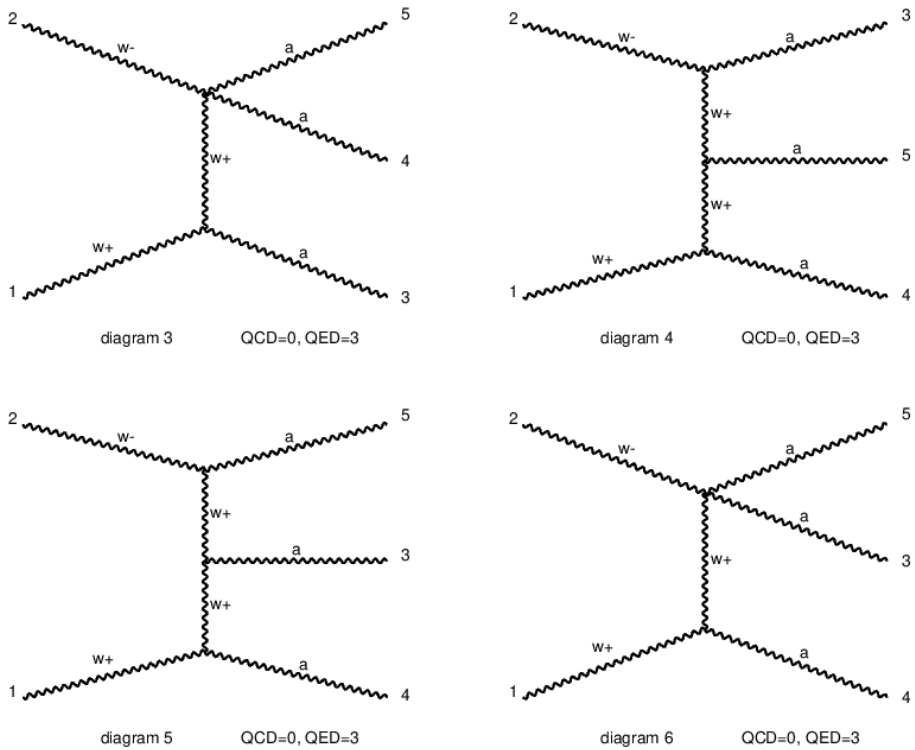




Plot: M_{WW} for $W_T W_T \to \gamma\gamma\gamma$



w/ I. Bigaran [2502.07878]



sum of processes at $|\mathcal{M}|^2$ level:

– VBS (EWA) $\to \gamma\gamma\gamma$

– ($\gamma\gamma$ full ME)$\times$ Sudakov

agreement at $\mathcal{O}(10\%)$ level with naïve scale choices! **(i.e., w/o parton shower)**

results suggest the following conjecture:[4]

- **when V carries large energy in pCM:** $(M_V^2/E_V^2) \ll 1$
- **when V carries small p_T relative to parent in pCM:** $(\mu_f^2/E_V^2) \ll 1$
- **when s_{ij} and $|t_{ij}|$ are comparble in hard subproccesses Q**

full, tree-level processes $\approx \sum_{\text{PDFs, FF}}$ **(factorized subprocesses)**

(kinematic regime for tree-level factorization below 10 TeV pCM)

[4] hints already with specific processes: Accomando, Ballestrero, et al ('06); Borel, Franceschini, et al ('12); others

while many challenging tasks remain,
clear roadmap for achieving fully differential / exclusive
Monte Carlo simulations for hard scattering procs. at lowest order

- **PDFs with "partonic" W/Z well known** ✓
- **hard matrix elements with "partonic" W/Z now under control** ✓

this is the new stuff!

- **complete parton shower with soft/collinear W/Z missing**
- **applicable and extendable to protons, particularly the LHC**

thank you!

2:00 PM → 3:30 PM — **Parallels: Theory** — MCP 201

Convener: Matheus Hostert (Harvard University)

2:00 PM	Towards realistic predictions for weak bosons as partons of the muon (15'+3') **Speaker**: Richard Ruiz	18m
2:18 PM	Guaranteed Deliverables at the Muon Collider (15'+3') **Speaker**: Rodolfo Capdevilla	18m
2:36 PM	Precision Physics at the Muon Collider **Speaker**: Yang Ma (UCLouvain)	18m
2:54 PM	Colored Particle Production at High-Energy Muon Colliders **Speaker**: Arthur Wu (University of Pittsburgh)	18m
3:12 PM	Probing Anomalous Quartic Gauge Couplings at the Muon Collider (15'+3') **Speaker**: Yu-Chen Guo (University of Pittsburgh (Liaoning Normal University))	18m

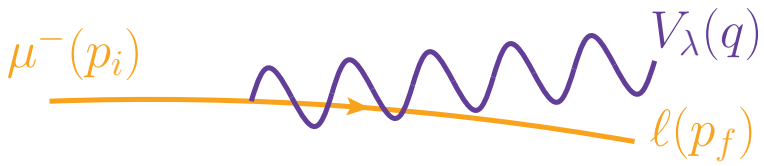
+ Samuel Homiller (Fri.)

(we have been thinking about high-energy $\mu^{\pm}$ for a while...)

backup

Non-expert: W/Z PDFs can be obtained from Taylor expanding the $\mu^- \to V_\lambda \ell$ matrix element in powers of opening angle collinear limit!

Build matrix element: $\mathcal{M}\left(\mu^- \to V_\lambda \ell\right) \propto J^{\mu}_{\ell \leftarrow \mu} \cdot \varepsilon^*_\mu(q)$



$$= \underbrace{\mathrm{LLA} + \mathcal{O}\left(\frac{M_V^2}{\mu_f^2}\right)}_{\text{LP in } \theta_V} + \underbrace{\mathcal{O}\left(\frac{\mu_f^2}{E_V^2}, \frac{M_V^2}{E_V^2}\right)}_{\text{NLP in } \theta_V} + \ldots$$

Next:

- evaluate amplitudes for individual helicity, e.g., for $\mu^- \to V_0 \ell$ use

$$\varepsilon^\mu(q, \lambda = 0) = \frac{q^\mu}{M_V} + \frac{M_V}{(E_V + |\vec{q}|)|\vec{q}|}\left(-|\vec{q}|, q_x, q_y, q_z\right)^\mu ,$$

- square matrix element
- integrate over 1-body phase space (kinematics of spectator lepton)
- plug into "factorization formula"

PDFs at leading logarithmic accuracy (LLA)

bare, leading log (LL) PDFs for $\ell^{\pm} \to W_\lambda/Z_\lambda/\gamma_\lambda + \ell$ depend on helicities (λ)

Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

$$f_{V_+/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_-/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_0/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_-/f_L}(z,\mu_f^2)$$

$$f_{V_-/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_+/f_L}(z,\mu_f^2)$$

$$f_{V_0/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_0/f_L}(z,\mu_f^2)$$

PDFs at leading power (LP)

$$f^{\rm LP}_{V_+/\ell_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\,\frac{g_L^2(1-z)^2}{2z}\left[\log\left(\frac{\mu_f^2+M_V^2}{M_V^2}\right) - \left(\frac{\mu_f^2}{\mu_f^2+M_V^2}\right)\right]$$

$$f^{\rm LP}_{V_-/\ell_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\,\frac{g_L^2}{2z}\left[\log\left(\frac{\mu_f^2+M_V^2}{M_V^2}\right) - \left(\frac{\mu_f^2}{\mu_f^2+M_V^2}\right)\right]$$

$$f^{\rm LP}_{V_0/\ell_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\,\frac{g_L^2(1-z)}{z}\left(\frac{\mu_f^2}{\mu_f^2+M_V^2}\right)$$

$$f^{\rm LP}_{V_+/\ell_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f^{\rm LP}_{V_-/\ell_L}(z,\mu_f^2)$$

$$f^{\rm LP}_{V_-/\ell_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f^{\rm LP}_{V_+/\ell_L}(z,\mu_f^2)$$

$$f^{\rm LP}_{V_0/\ell_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f^{\rm LP}_{V_0/\ell_L}(z,\mu_f^2)$$

leading $(M_V^2/\mu_f^2) \to 0$ term recovers usual leading log approx(LLA)

Dawson('85), Kunszt & Soper ('88)

PDFs at next-to-leading power (NLP)

results for the R_ξ / **unitary gauge**

$$f^{\rm NLP}_{V_+/\ell_L}(z,\mu_f^2) = f^{\rm LP}_{V_+/\ell_L}(z,\mu_f^2)\left[1+\frac{(2-z)}{(1-z)}\frac{M_V^2}{E_V^2}\right] - \frac{\mu_f^2}{4E_V^2}(2-z)\, f^{\rm LP}_{V_0/\ell_L}(z,\mu_f^2)$$

$$f^{\rm NLP}_{V_-/\ell_L}(z,\mu_f^2) = f^{\rm LP}_{V_-/\ell_L}(z,\mu_f^2)\left[1+(2-z)\frac{M_V^2}{E_V^2}\right] - \frac{\mu_f^2}{4E_V^2}\frac{(2-z)}{(1-z)} f^{\rm LP}_{V_0/\ell_L}(z,\mu_f^2)$$

$$f^{\rm NLP}_{V_0/\ell_L}(z,\mu_f^2) = f^{\rm LP}_{V_0/\ell_L}(z,\mu_f^2) - \frac{M_V^2}{2E_V^2}\left[f^{\rm LP}_{V_+/\ell_L}(z,\mu_f^2) + f^{\rm LP}_{V_-/\ell_L}(z,\mu_f^2)\right]$$

lots of interesting behavior:

– $\delta f_\lambda^{\rm NLP} \sim f_{\lambda'}^{\rm LP} \implies$ **helicity flipping!**

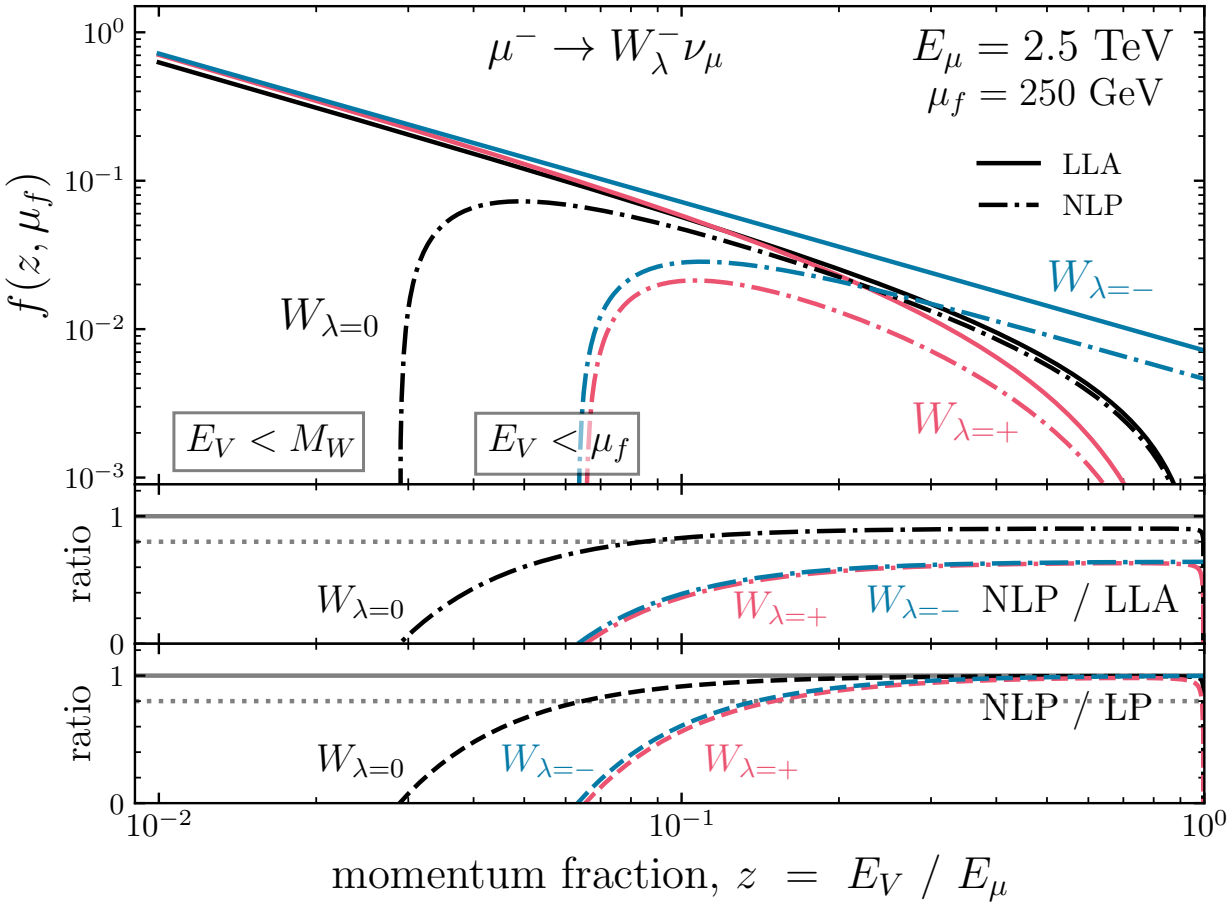
– NLP corrections are **negative**

– $\mathcal{O}(\mu_f^2/E_V^2)$ and $\mathcal{O}(M_V^2/E_V^2)$ terms vanish in DIS limit

– subtle **gauge dependence** in **axial gauges** **(backup!)**

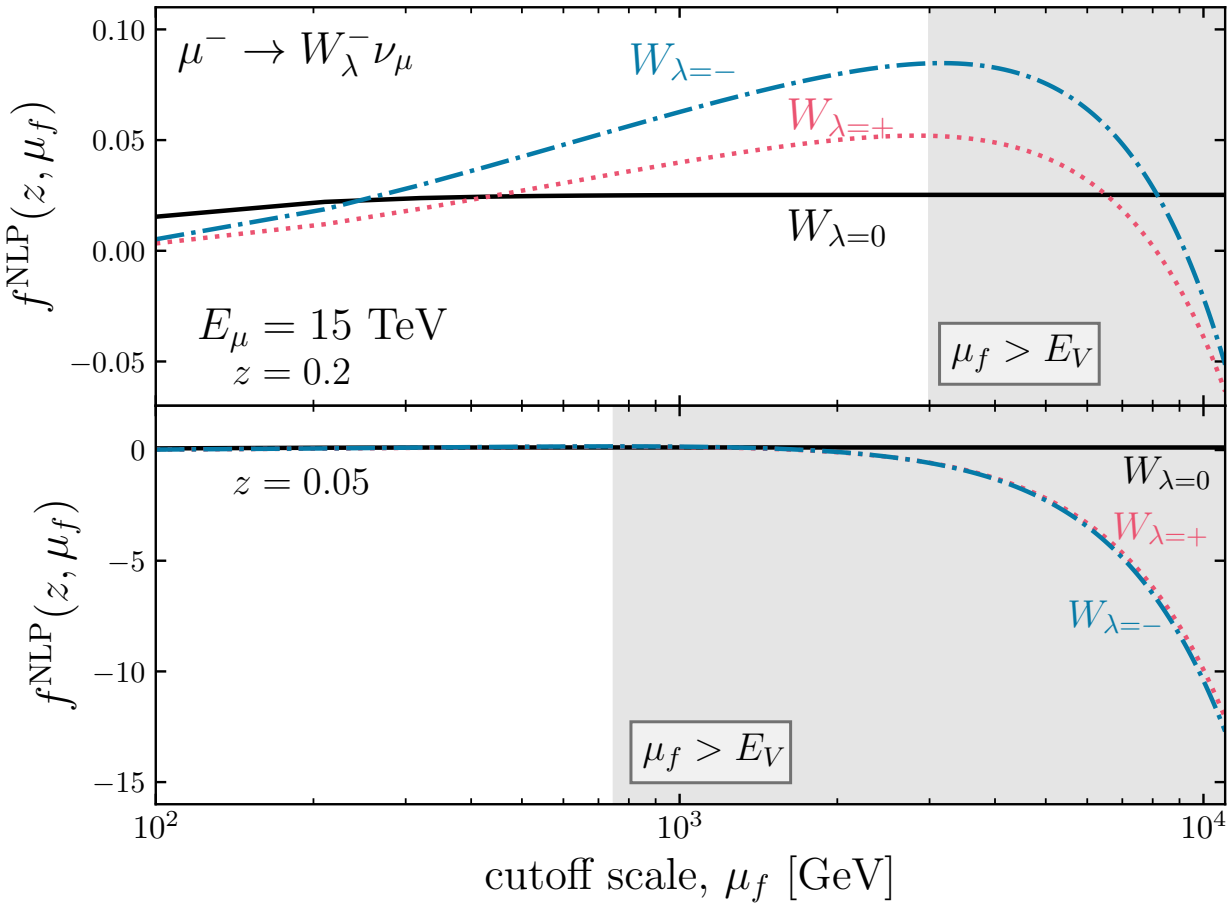
numerical impact of NLP corrections

Plotted: PDFs vs $z = E_V/E_\mu$ for $E_\mu = 2.5$ TeV



NLP corrections go negative when $E_V < M_V$

Plotted: PDFs vs μ_f

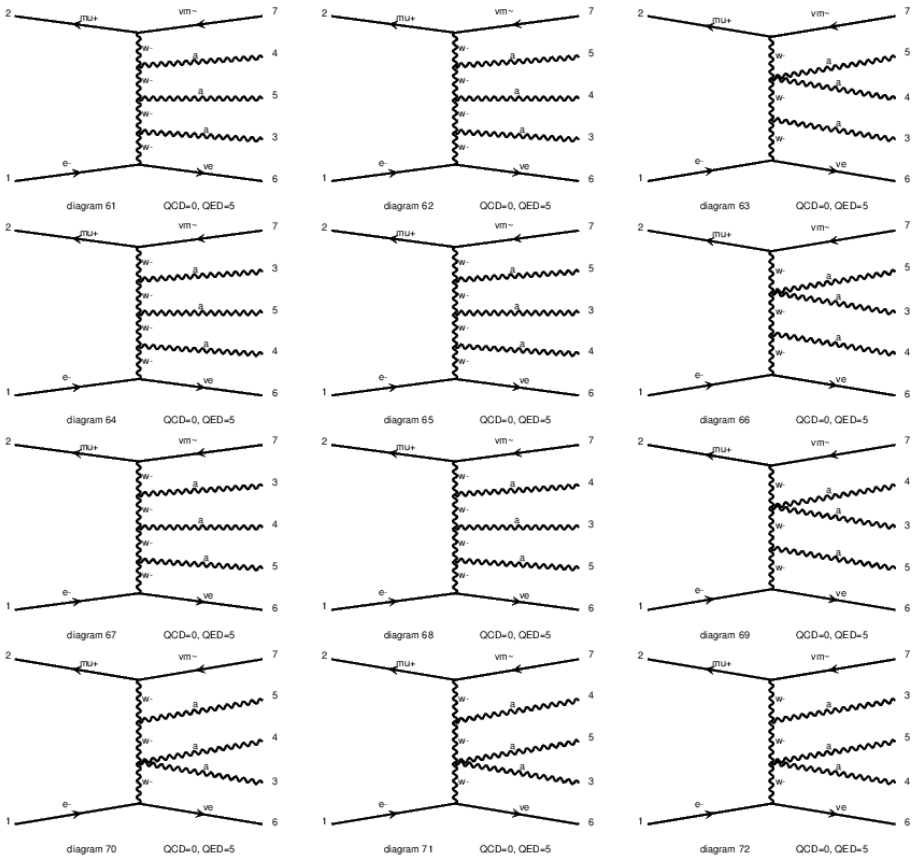


since $\mu_f \sim p_T^V$, **NLP corrections go negative** when $\mu_f > E_V$

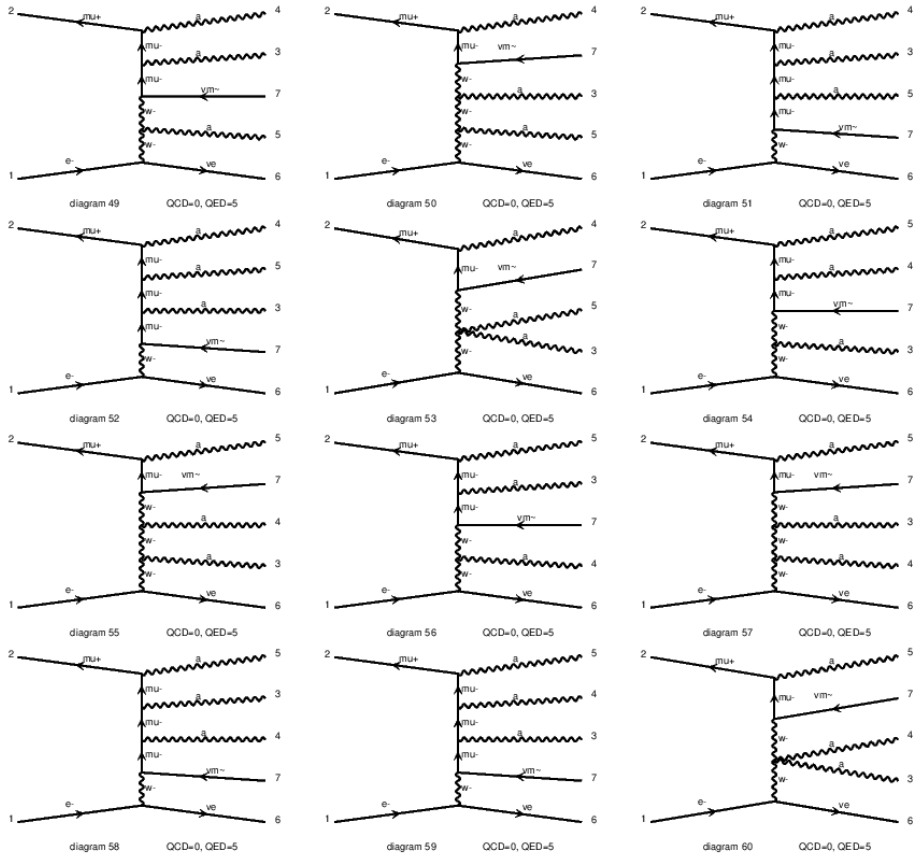
(breakdown of collinear approximation)

$e^+\mu^- \to \overline{\nu}_e\nu_\mu\gamma\gamma\gamma$ is more than just VBS

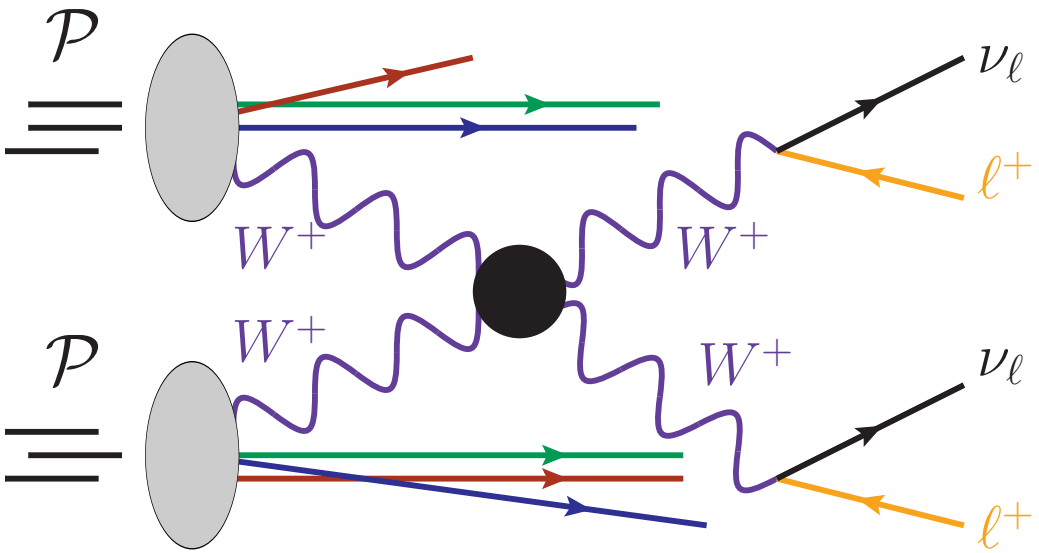
VBS topologies

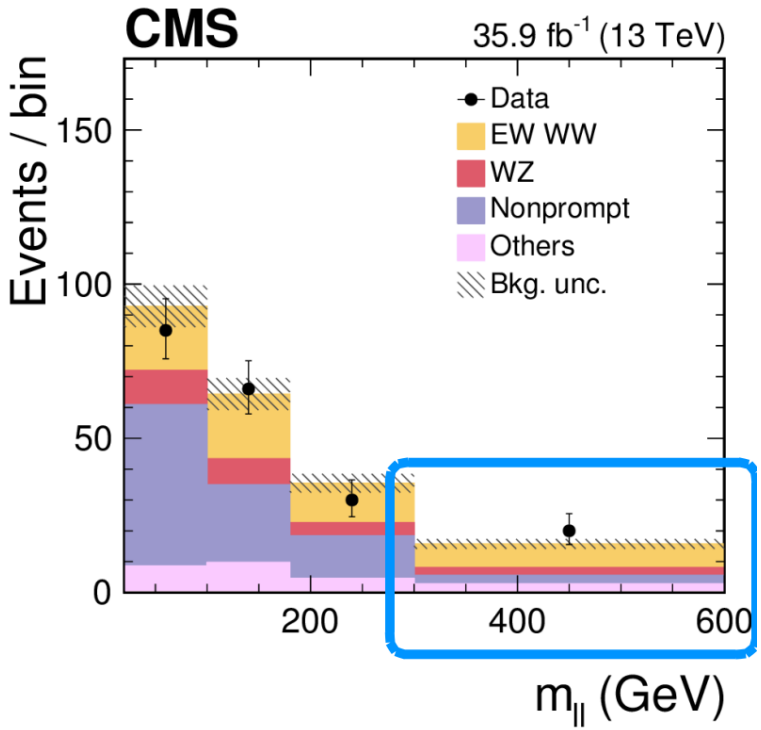


W-exchange + i-iii γ ISR



$M_{WW} \gtrsim 1 - 2$ TeV is already accessible at LHC





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much need on the Monte Carlo side,

but may already be possible to probe proton's W/Z content at LHC