Aspects of Electroweak Symmetry Restoration at High-Energy Muon Collider

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Based on work in progress with Matthew Forslund, Zhen Liu, Patrick Meade.

Outline

- At a high-energy muon collider, we are probing energy far above the electroweak symmetry breaking scale $E \gg v \simeq 246\,\mathrm{GeV}$.
  - We start probing the limit $m_W, m_Z, m_h \to 0$. Massive amplitudes start converging to the massless amplitude predicted by SM
  - Goldstone Boson Equivalence predicts $W_L, Z_L \to$ Goldstones up to $\mathcal{O}(m_V/E)$.

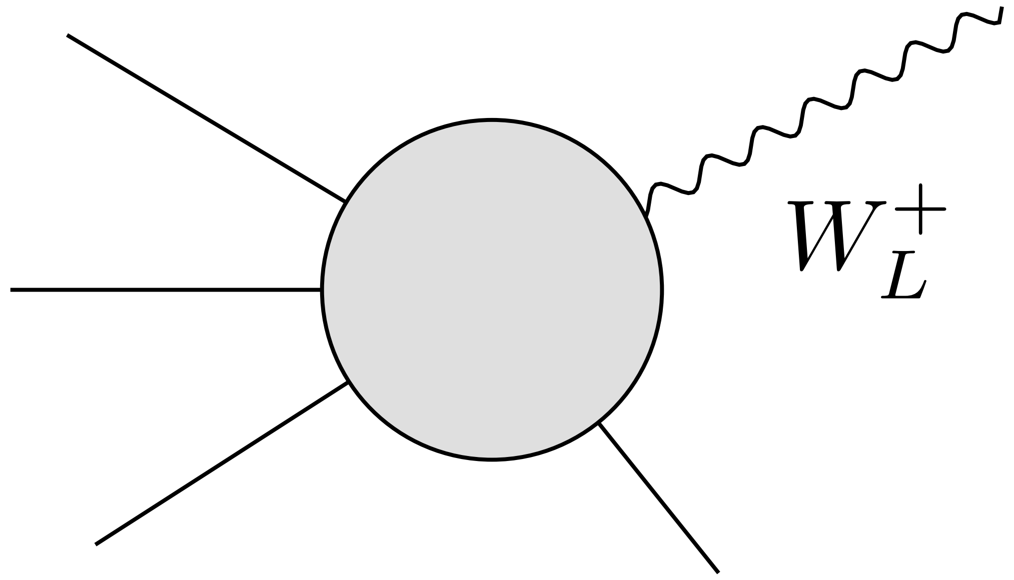
- How do we ***define*** and ***quantify*** restoration operationally (amplitude level)?

- Can we ***observe*** restoration at a high-energy muon collider?

Goldstone Boson Equivalence Theorem

- Long history of studying goldstone equivalence

Cornwall, Levin, Tiktopoulos (1974), Lee, Quigg, Thacker (1977), Chanowitz, Gaillard (1985), Bagger, Schmidt (1990), Brehmer Jaeckel, Plehn (2014), Cuomo, Vecchi, Wulzer (2020) …



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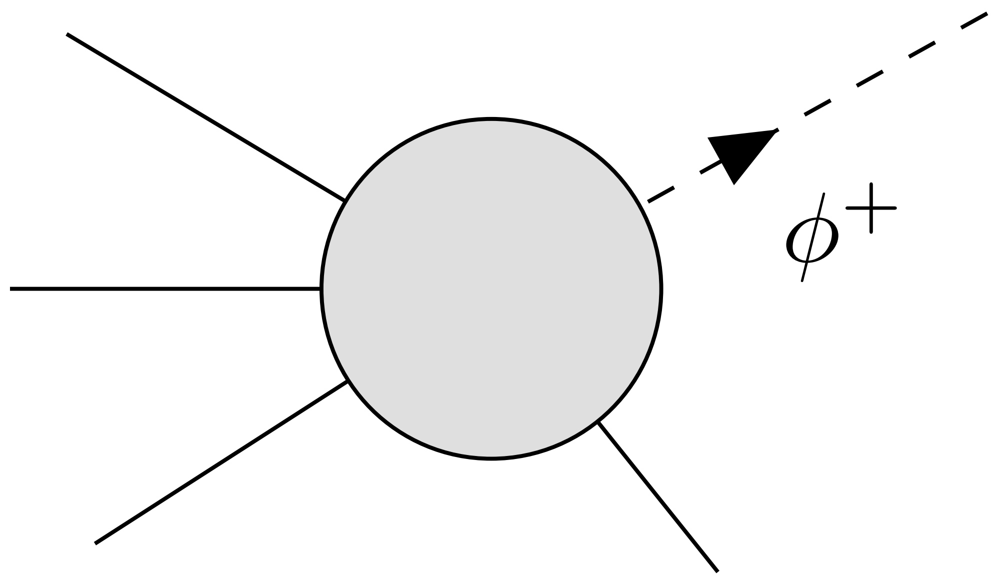


$\times \left(1+\mathcal{O}(m_W^2/E^2)\right)$

$$\lim_{E_V^2 >> m_V^2} \mathcal{A}(\ldots \to V_L^a \ldots) \sim \mathcal{A}(\ldots \to \phi^a \ldots)$$

For on-shell external states in a *spontaneously broken gauge theory*, amplitudes with longitudinally polarized massive vectors equal those with the corresponding Goldstone bosons, up to corrections suppressed by $\mathcal{O}(m_V/E)$

Equivalence Theorem

Generalized Ward identity:

$$i\, k_\mu\, \mathcal{A}\{W^\mu[k]\} = -\, m_W \mathcal{A}\{\pi[k]\} \qquad \text{(Tree-level)}$$

Cuomo, Vecchi, Wulzer (2020)

The three-point on-shell amplitude between three massive vector

$$\mathcal{M}(A_1^a, A_2^b, A_3^c) = g f^{abc}\left[(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot p_1) + (\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot p_2) + (\epsilon_3 \cdot \epsilon_1)(\epsilon_2 \cdot p_3)\right]$$

$$p_1^\mu\, \mathcal{M}^{abc}_{\mu\nu\rho}(p_1, p_2, p_3) = m_1\, \mathcal{M}^{abc}_{\nu\rho}\big(\phi^a(p_1),\, A_\nu^b(p_2),\, A_\rho^c(p_3)\big).$$

Standard Yang-Mills structure on-shell

Three point amplitude satisfy generalized Ward identity at three-point level when on-shell

Equivalence Theorem

Since n-point tree-level amplitude can be constructed from 3-pt on-shell amplitude by analytic continuation

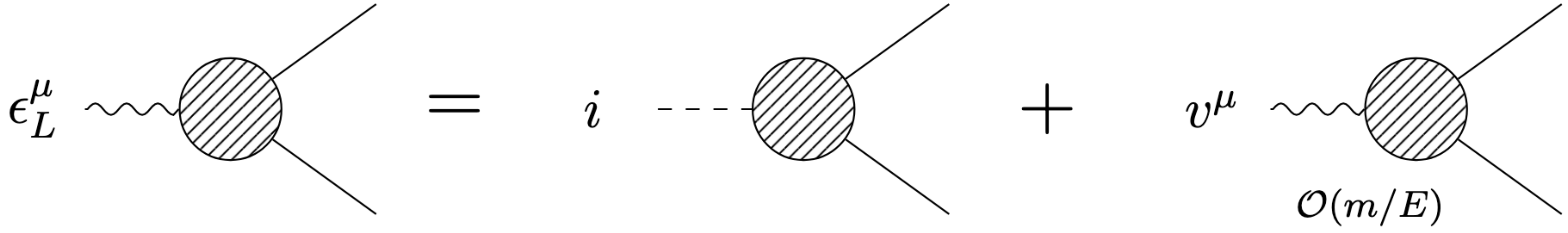
$$\mathcal{A}_n = \mathcal{A}_3 \times \mathcal{A}_3 \times \ldots \times \mathcal{A}_3$$

Ema, Gao, Ke, Liu, Lyu, IM (2024)

Properties of 3-pt amplitude is extended to n-point amplitude

Decomposition of longitudinal polarization

$$\epsilon_L^\mu = \frac{p^\mu}{m} + \underbrace{v^\mu(p)}_{\sim m/E}$$



What do we need to test?

1. Test the convergence to the massless amplitude ($v \to 0$) predicted by SM via Goldstone Boson Equivalence.

2. Test the symmetry between the scalars at the high-energy limit. Show convergence for amplitudes involving Higgs.

3. Test for restoration of gauge invariance for transverse modes at the high-energy limit.

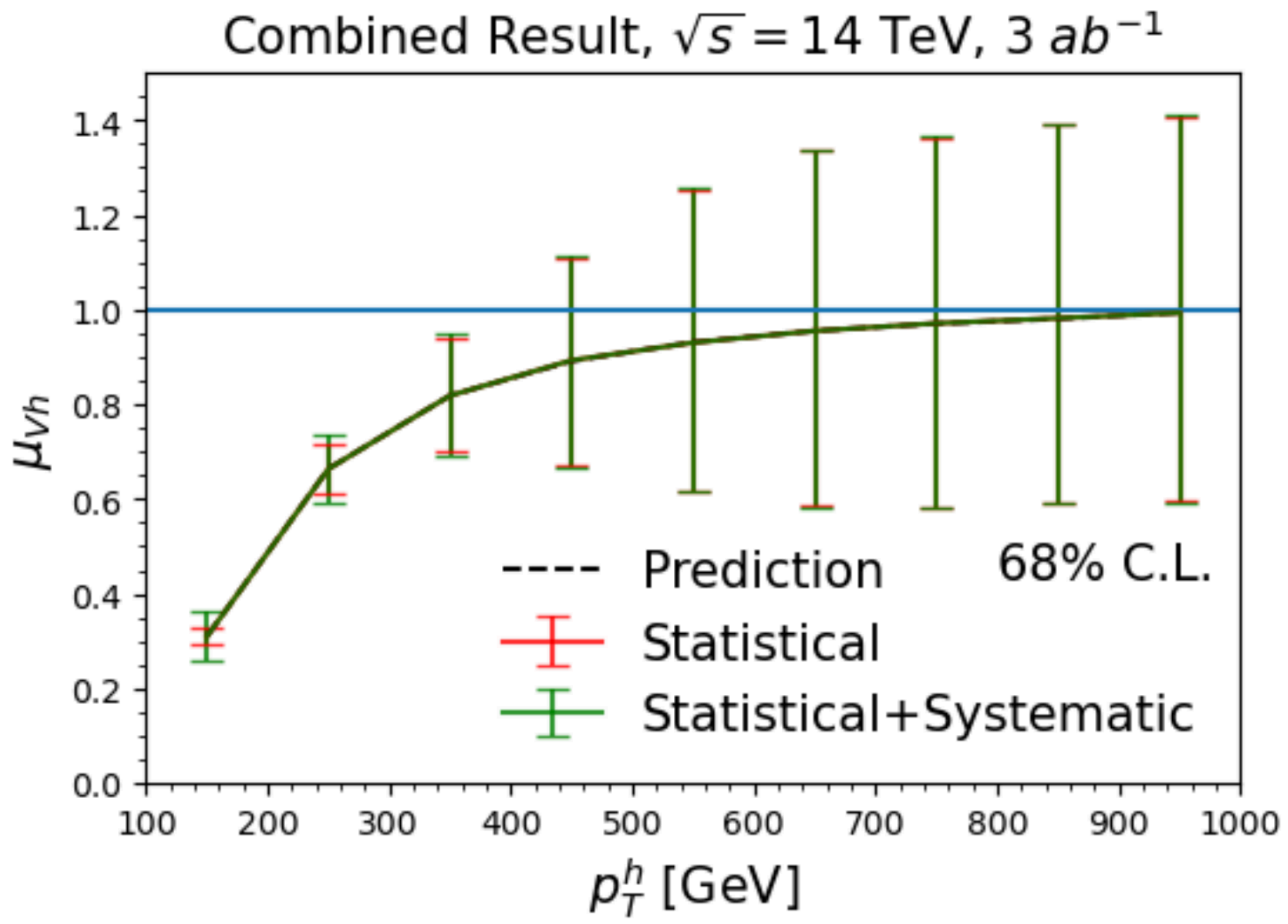
Define Electroweak Restoration

Restoration via test of **Goldstone Boson Equivalence** Huang, Lane, Lewis and Liu (2021)

Operational definition: Take the zero-vev theory and compare to the SM via

$$\mu_{Vh} = \frac{d\sigma/dp_T^h(Gh)\Big|_{v=0}}{d\sigma/dp_T^h(Vh)\Big|_{\rm SM}}\,.$$

For $p_T^h \gg v, m_V, m_h$, electroweak symmetry is restored and $\mu_{Vh} \to 1$; deviations from 1 quantify BSM effects.



Plot μ_{Vh} vs p_T^h to test the restoration and its rate of convergence

Comments on Vh channel

Why Vh final state?

Huang, Lane, Lewis and Liu (2021)

At high energies, the amplitude is dominated entirely by longitudinal-mode contributions.

$$
\begin{aligned}
\mathcal{A}(q_+\bar{q}_- \to Z_L h) &= \pm i\,\frac{e^2\, g_R^{qZ}}{2\, c_W^2\, s_W^2}\sin\theta + \mathcal{O}(\hat{s}^{-1}),\\
\mathcal{A}(q_-\bar{q}_+ \to Z_L h) &= \pm i\,\frac{e^2\, g_L^{qZ}}{2\, c_W^2\, s_W^2}\sin\theta + \mathcal{O}(\hat{s}^{-1}),\\
\mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm h) &= -i\,\frac{e^2}{2\sqrt{2}\, s_W^2}\sin\theta + \mathcal{O}(\hat{s}^{-1}),\\
\mathcal{A}(q_\pm\bar{q}_\mp \to Z_\pm h) &\sim \mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm h) \sim \mathcal{O}(\hat{s}^{-1/2}),\\
\mathcal{A}(q_+\bar{q}'_- \to W_\pm^\pm h) &= \mathcal{A}(q_+\bar{q}'_- \to W_\mp^\pm h) = 0.
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\end{aligned}
$$

Zh channel in MuC

- $\mu^+\mu^- \to Zh \to \ell^+\ell^- h$: loses resolving power at 1 TeV and large ZZ background
- $\mu^+\mu^- \to Zh \to 4j$: low precision

WW channel in MuC

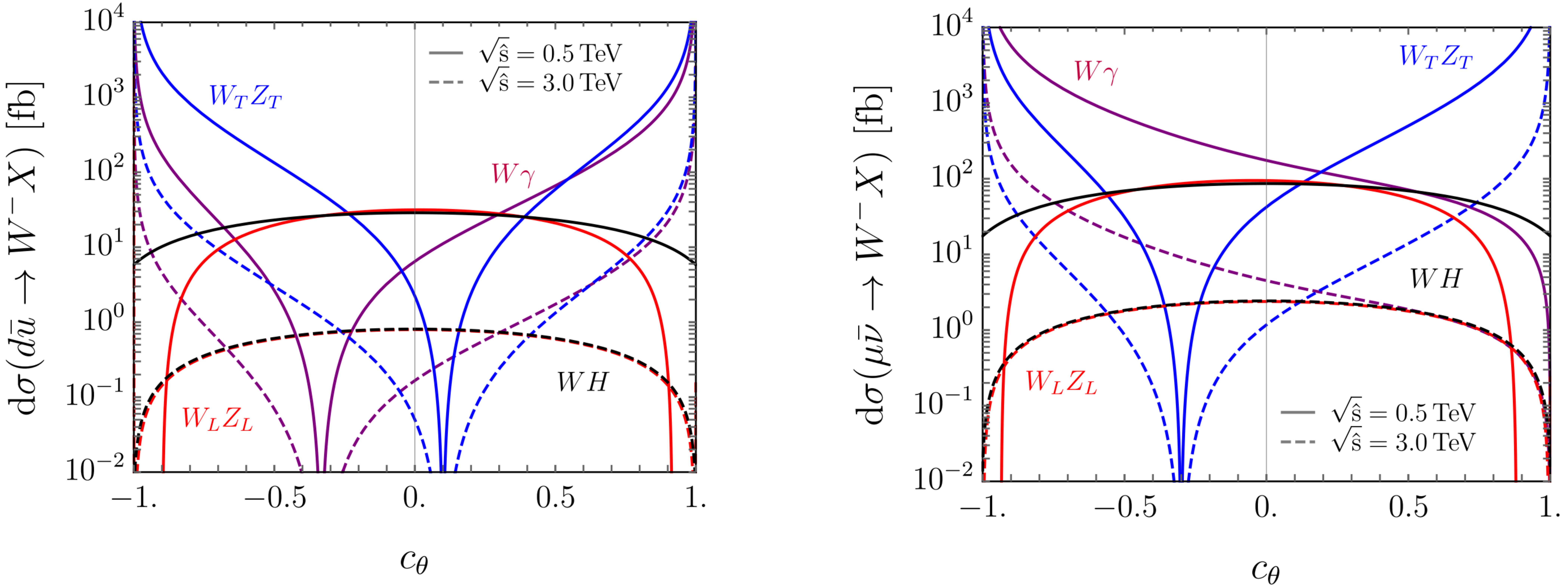
Precision (%)

Energy	$\ell\nu jj$
1 TeV	4
3 TeV	3.5
10 TeV	3.7

We will focus on collider analysis for WW case

Radiation Zero

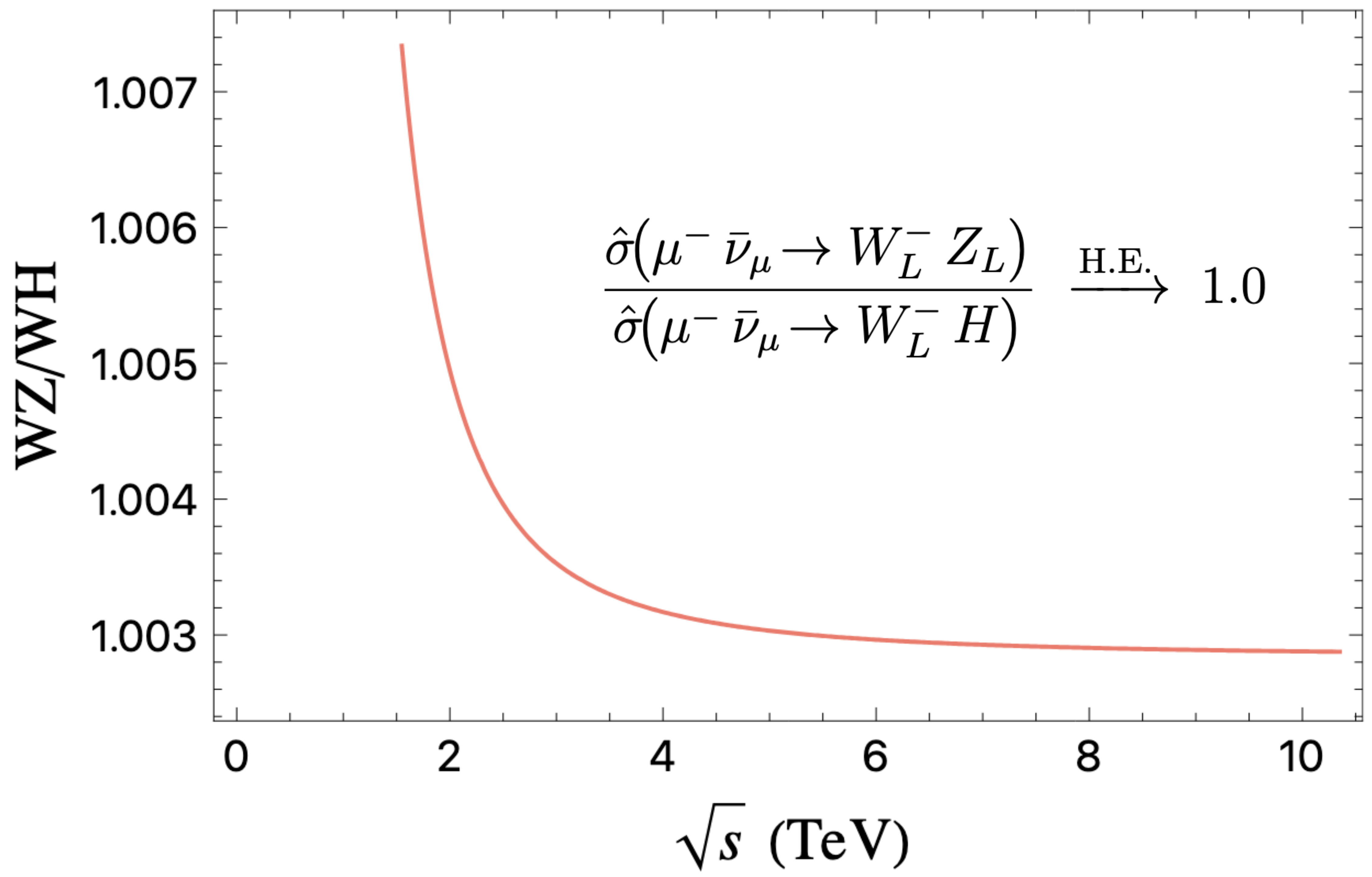
Capdevilla, Han (2024)

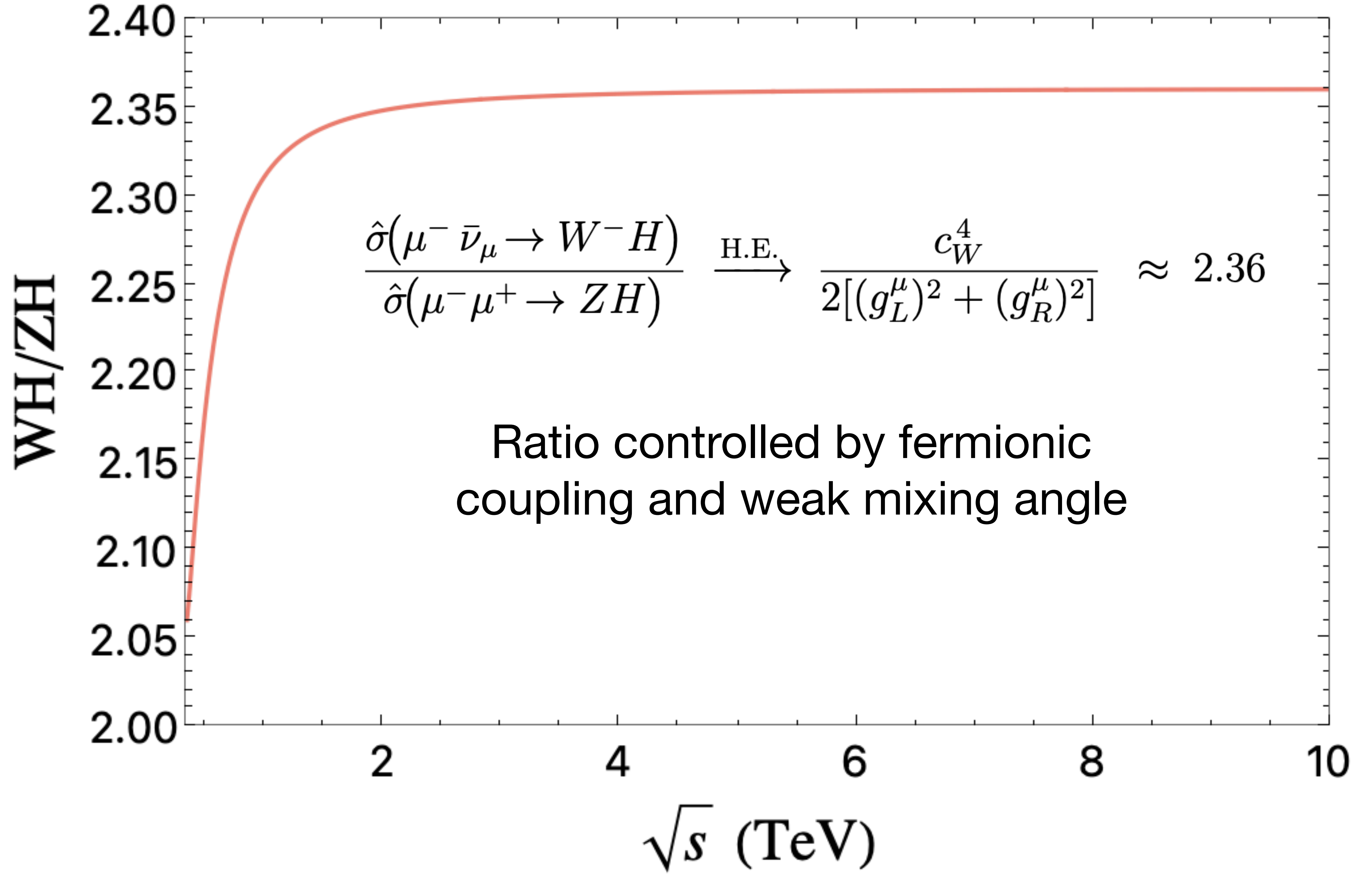


- Transverse sector → restoration of gauge symmetry. EW restoration shows up as a characteristic **radiation-zero** in the amplitude.
- Longitudinal $W_L, Z_L \approx$ Goldstone; with h they reconstruct the **full** $\mathcal{O}(4)$ **scalar multiplet**, signaling symmetry restoration.

Symmetry restoration in the scalar sector

Test: Ratio of amplitude





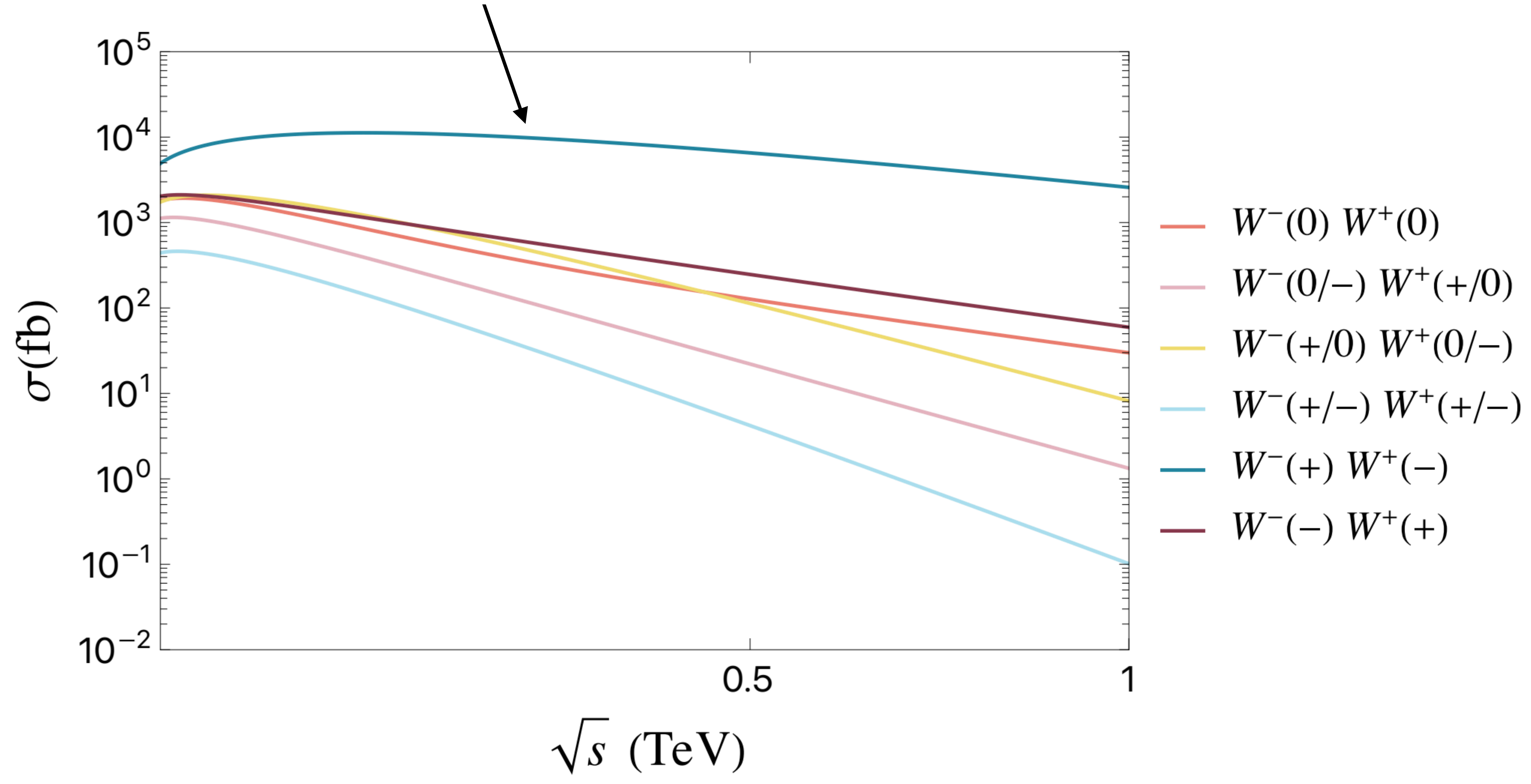
- **Test:** O(4) multiplet ⇔ Higgs + 3 Goldstones
- EFT operators ⇒ shift these SM-fixed numbers

Collider Analysis

$\mu^-\mu^+ \to W^-W^+$

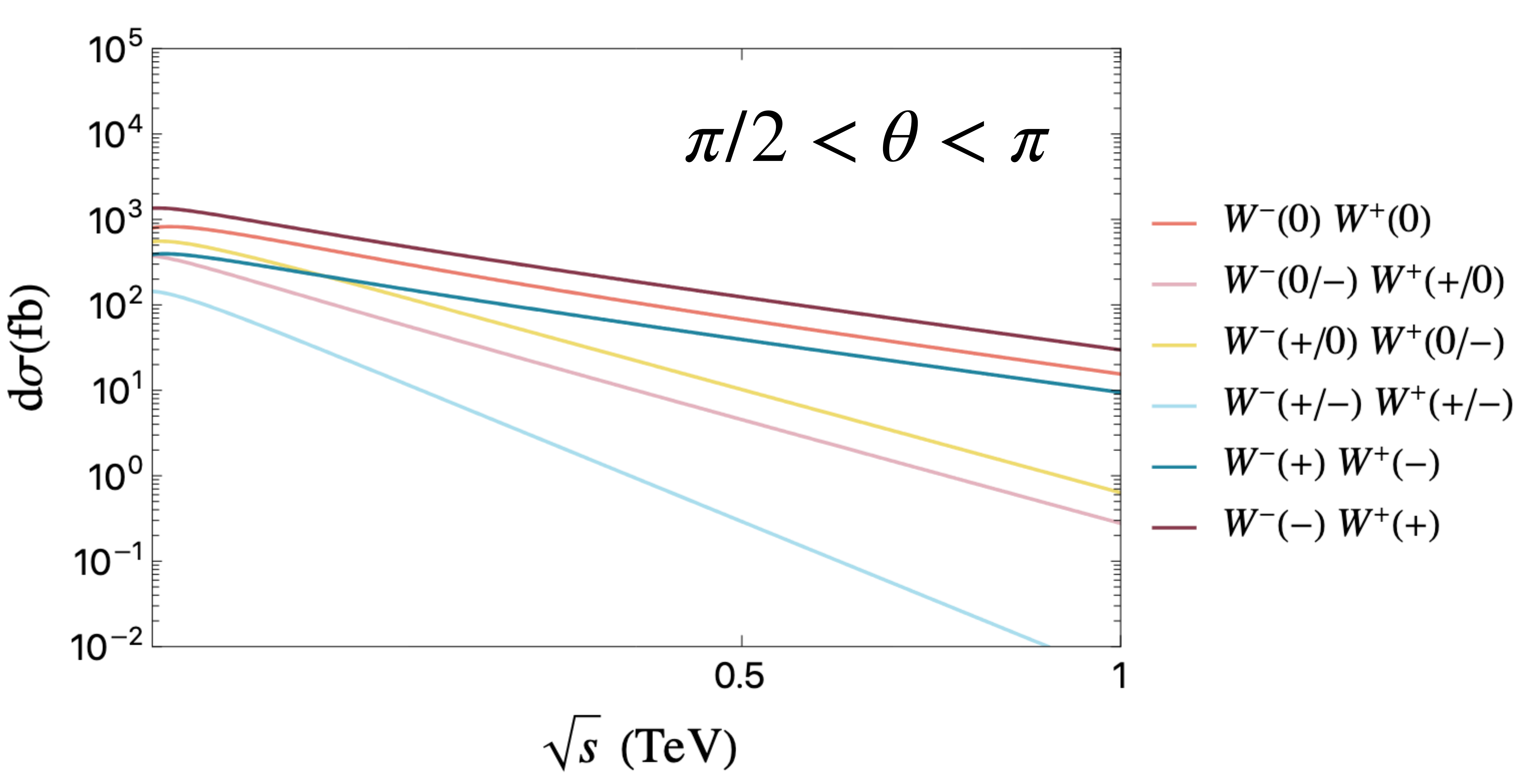
Forward $(\cos\theta \to 1)$: differential cross-section dominated by transverse mode $W_T W_T$

$$\lim_{\sqrt{\hat{s}}>>m_W^2} \frac{d\sigma(\mu^-\mu^+ \to W_+^- W_-^+)}{d\sigma(\mu^-\mu^+ \to W_L^- W_L^+)} \approx 100$$



Backward $(\cos\theta \to -1)$: differential cross-section for $W_T W_T$ and $W_L W_L$ are comparable

$$\lim_{\sqrt{\hat{s}}>>m_W^2} \frac{d\sigma(\mu^-\mu^+ \to W_+^- W_-^+)}{d\sigma(\mu^-\mu^+ \to W_L^- W_L^+)} \approx 0.1$$



Forward-angle cuts can isolate the Goldstone-equivalence signal

Collider Analysis

$$\mu^-\mu^+ \to W^-W^+\gamma$$

- To see restoration explicitly at a single energy muon collider, we need a mechanism to scan over different energies

- We do that by considering the radiation of an additional photon.

Event topology- semileptonic: $W^-W^+\gamma \to l^\pm \nu jj\gamma$

- Baseline cuts: $P_{T,j}, P_{T,\ell} > 40GeV,\ |\eta_j|, |\eta_{ell}| < 2.5,\ m_{jj} > 30GeV$
- Extra photon: $P_{T,\gamma} > 25GeV,\ |\eta_\gamma| < 2.5$
- Forward tag: $q_l\ \eta_{jj}^{CM} > 0$

- **Backward-selection viable only in the semileptonic channel ($\ell\nu jj$), fully hadronic $4j$ lacks the charge ID needed.**
- **Photon predominantly from initial state, lowering effective $\sqrt{s}$ of the s-channel process.**

Collider Analysis

$\mu^-\mu^+ \to W^-W^+\gamma$

Background- (≪ signal)

- $\nu\nu WW\gamma, \ell\ell WW\gamma$
- $\nu\ell WZ\gamma, \nu\nu ZZ\gamma$
- $t\bar{t}\gamma$(s-channel, VBF), $ZZ\gamma$

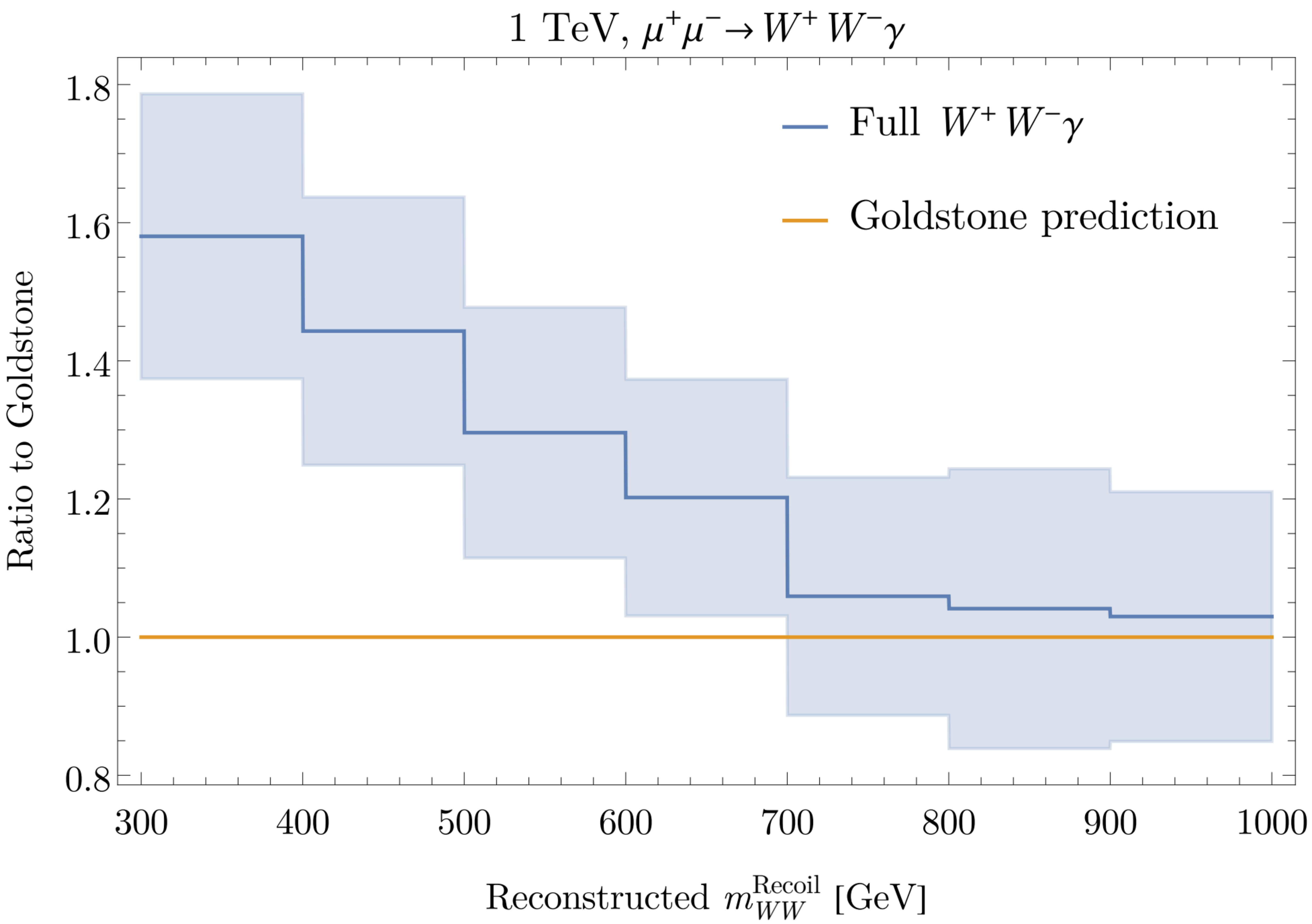
Unfolding to Truth Level-

- Build forward/backward efficiency matrices: $\epsilon_{ij}^{F/B} = n_{ij}/N_j^{\mathrm{T}}$
- Number of events in i-th bin: $n_i = \sum_j \left(\varepsilon_{ij}^F N_j^{F,truth} + \varepsilon_{ij}^B N_j^{B,truth}\right)$
- Perform unfolding algorithm.
- Compare **unfolded spectrum in the backward region** with Goldstone-equivalence prediction

Collider Analysis

$\mu^-\mu^+ \to W^-W^+\gamma$

PRELIMINARY



Outlook

- Detailed phenomenological test of convergence at 3 TeV and 10 TeV muon colliders.
- How is the cross-section ratio test modified in SMEFT and HEFT?
- Goldstone Boson Equivalence and on-shell description.

THANK YOU!

BACKUP

Goldstone Boson Equivalence

$$\int \mathcal{D}(\text{fields})\; O\; e^{i\int d^4x\left[\mathcal{L}_0+\mathcal{L}_{\text{ghosts}}-\frac{1}{2}(\mathcal{F}_a+J_a)^2\right]}$$

Cuomo, Vecchi, Wulzer (2020)

Slavnov-Taylor Identities

$$\langle 0|\, T\{F_a(x)\, F_b(y)\}\, |0\rangle \;=\; -\,i\,\delta_{ab}\,\delta^{(4)}(x-y)$$

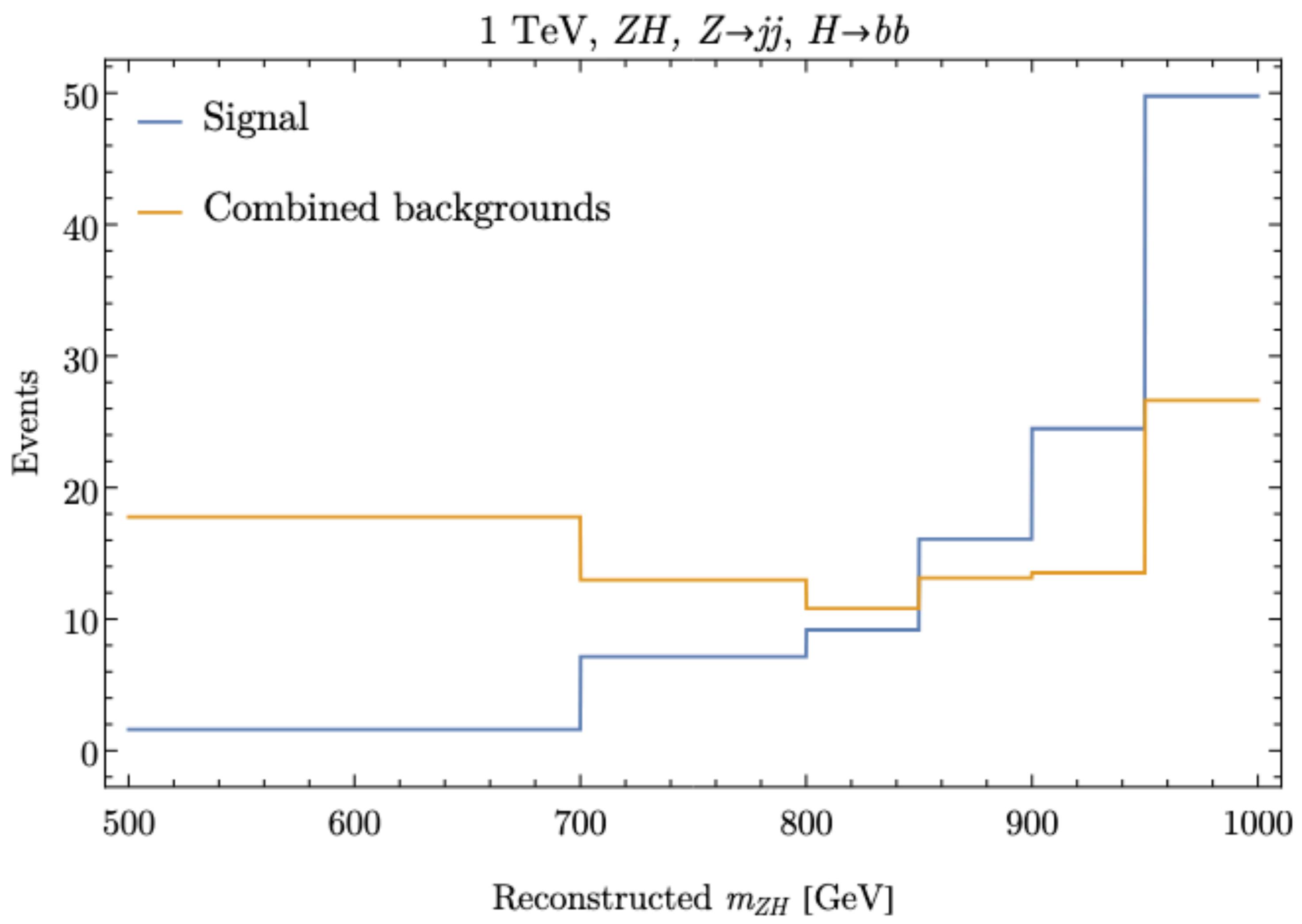
$$F_a(x) = \begin{cases} \partial^\mu W^\pm_\mu - \xi m_W \phi^\pm \\ \partial^\mu Z_\mu - \xi m_Z \phi^0 \\ \partial^\mu A_\mu \end{cases}$$

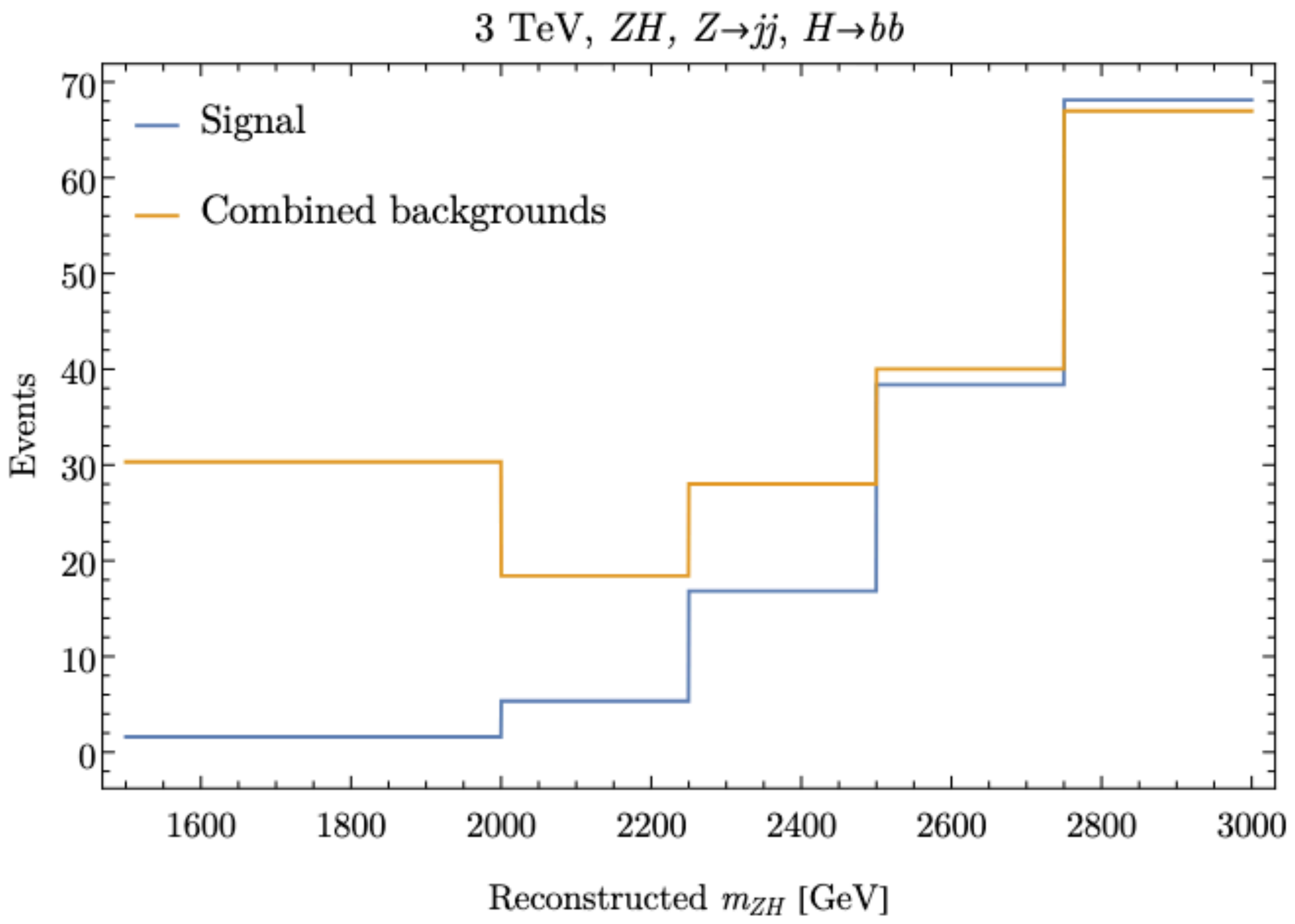
These identities guarantee that any connected Green function containing at least one insertion of F_a vanishes

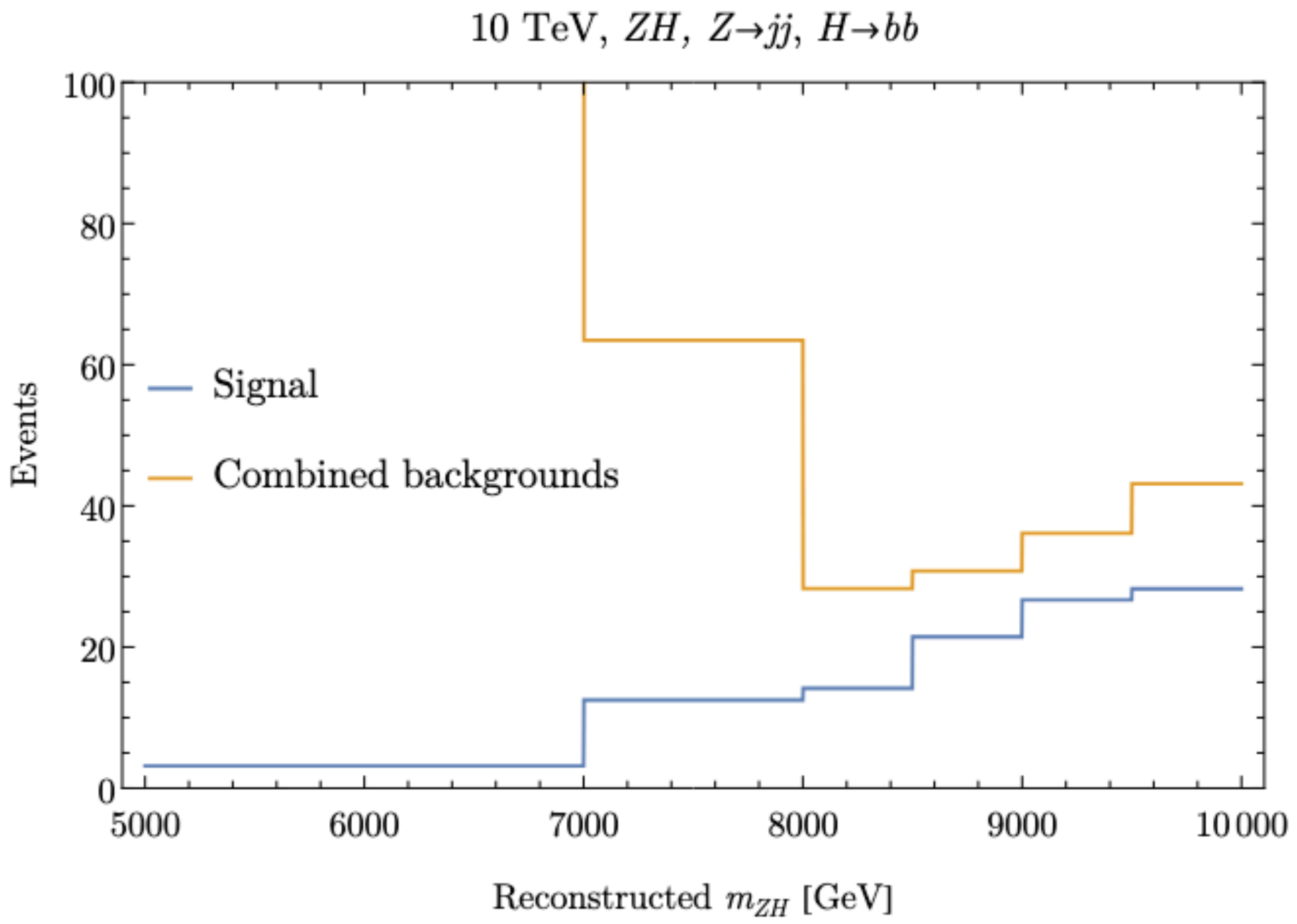
Precision for Zh case in MuC

		Precision (%)		
	Energy	Tight	Medium	Loose
Any one b-tag	1 TeV	12	12	16
	3 TeV	12	12	16
	10 TeV	15	16	26
Any two b-tags	1 TeV	18	14	13
	3 TeV	21	15	14
	10 TeV	22	16	21
$H \to bb$ tagged	1 TeV	17	13	11
	3 TeV	19	13	13
	10 TeV	23	17	22

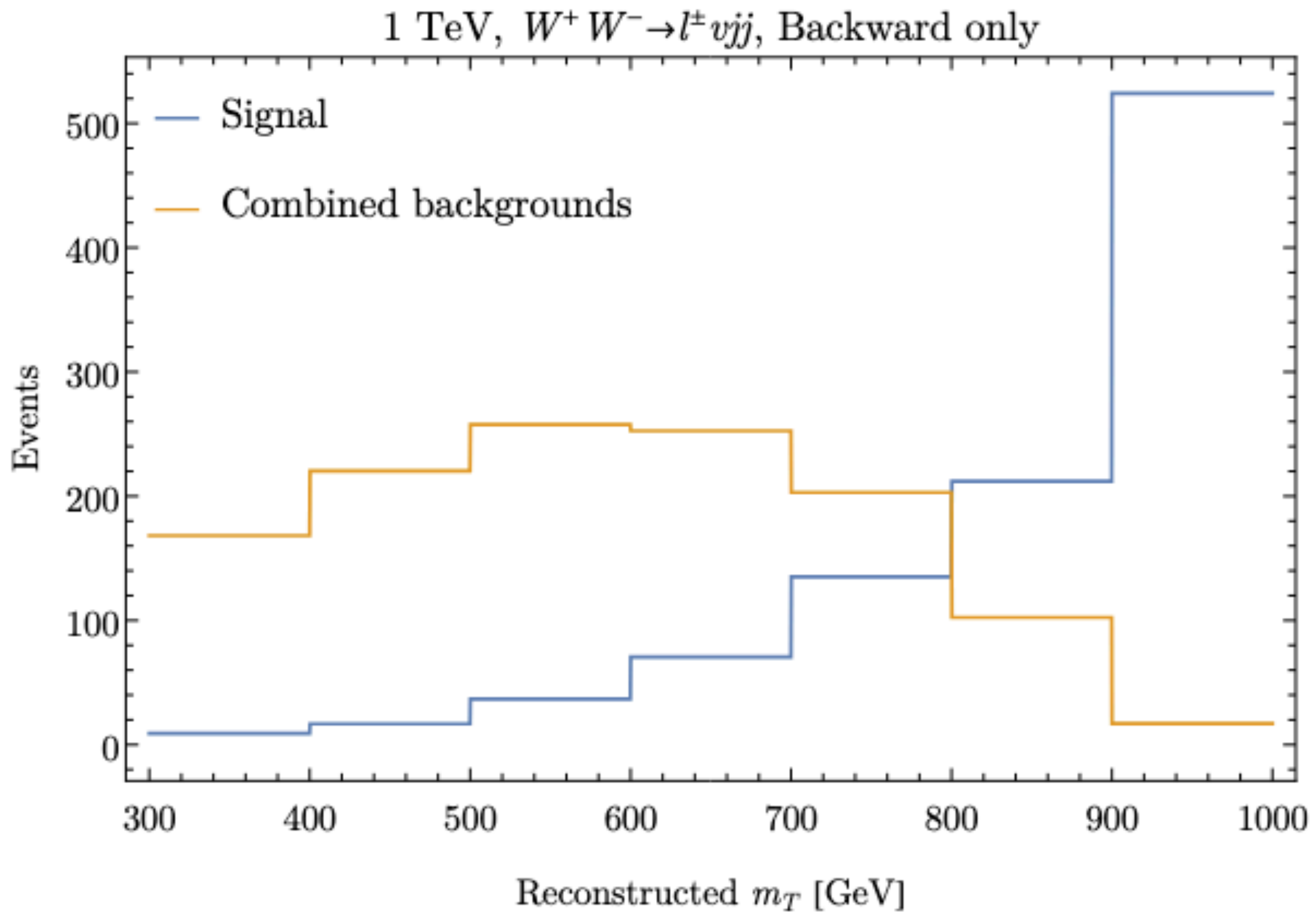
Precision for Zh case in MuC

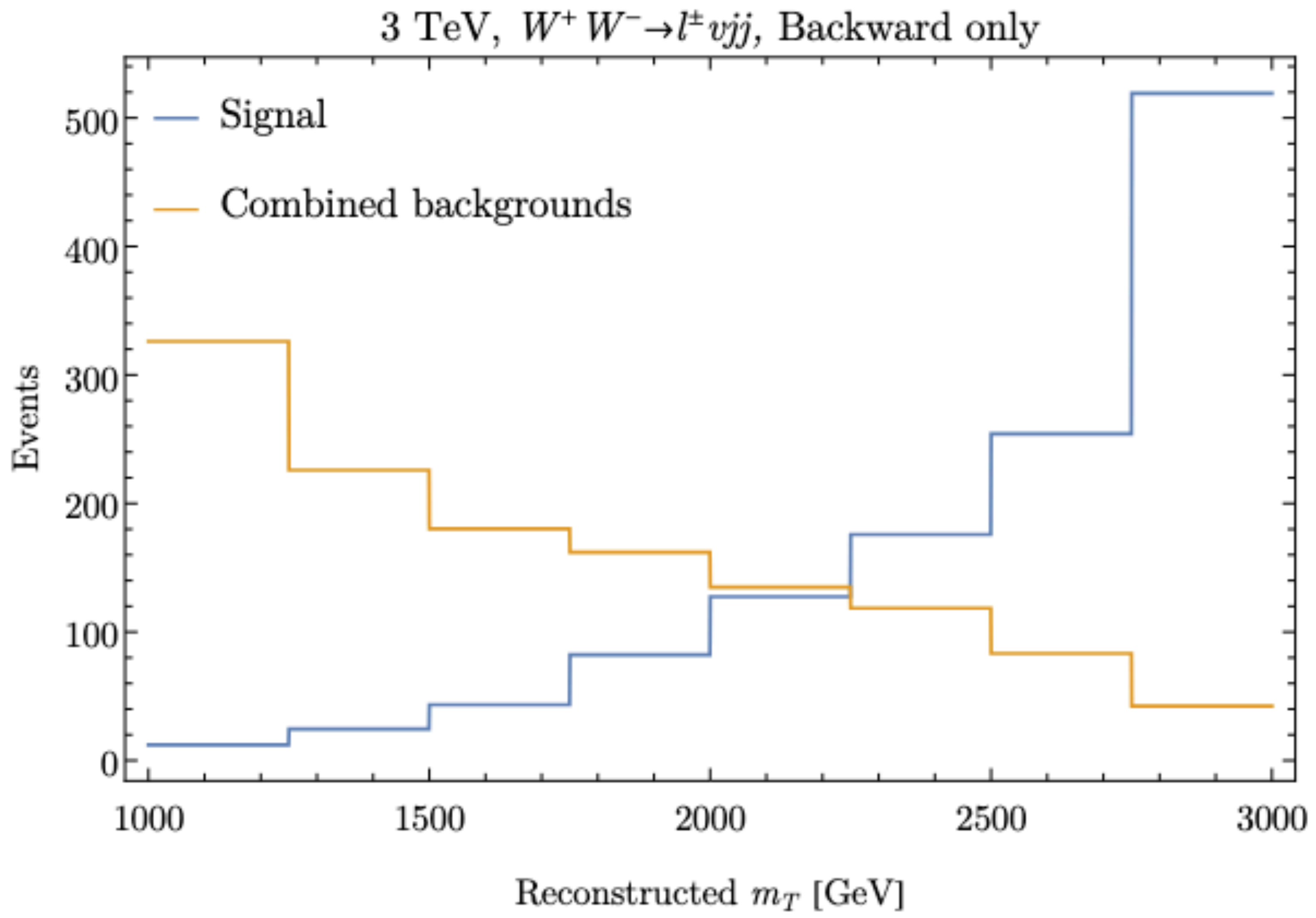


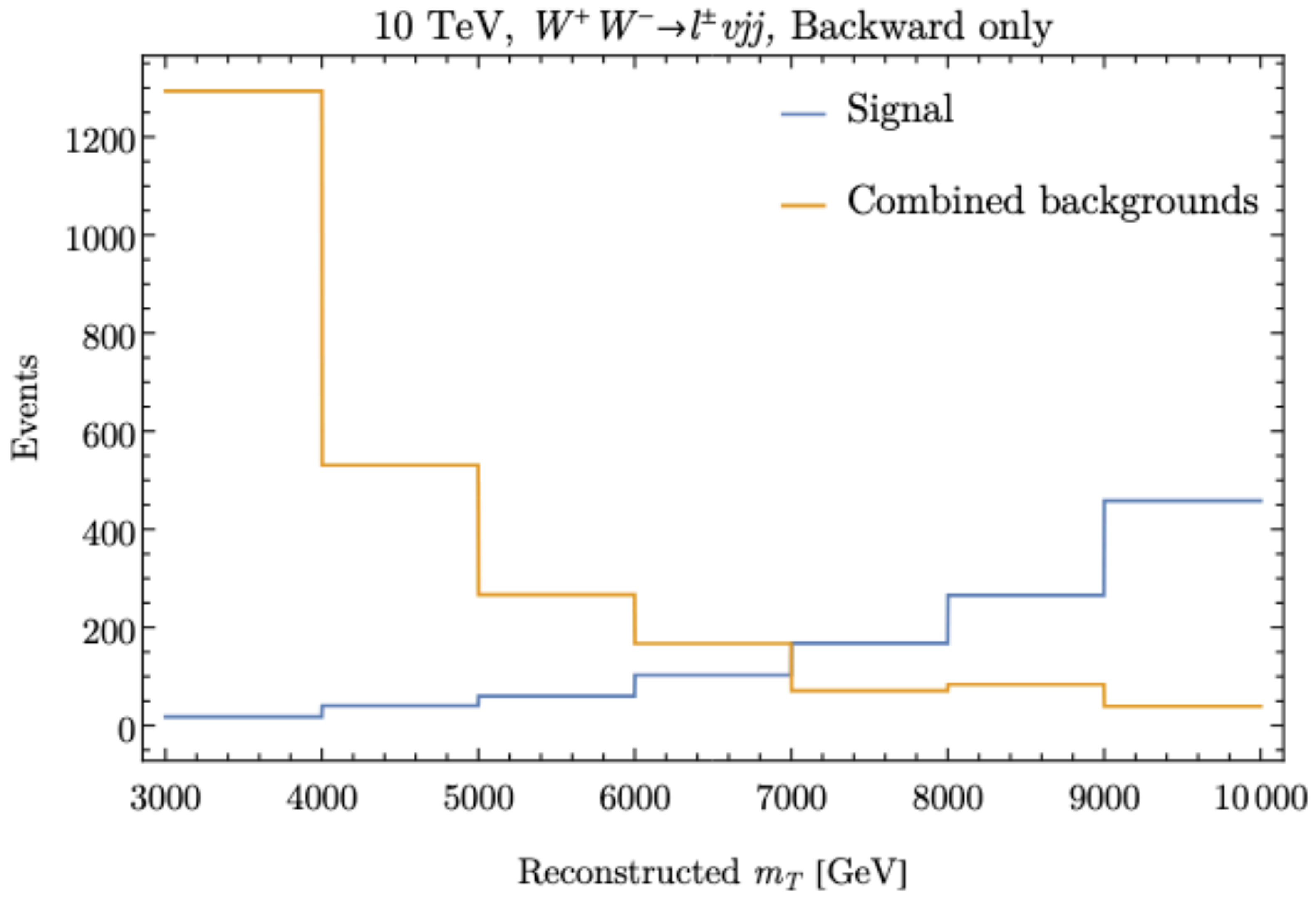




Precision for WW case in MuC

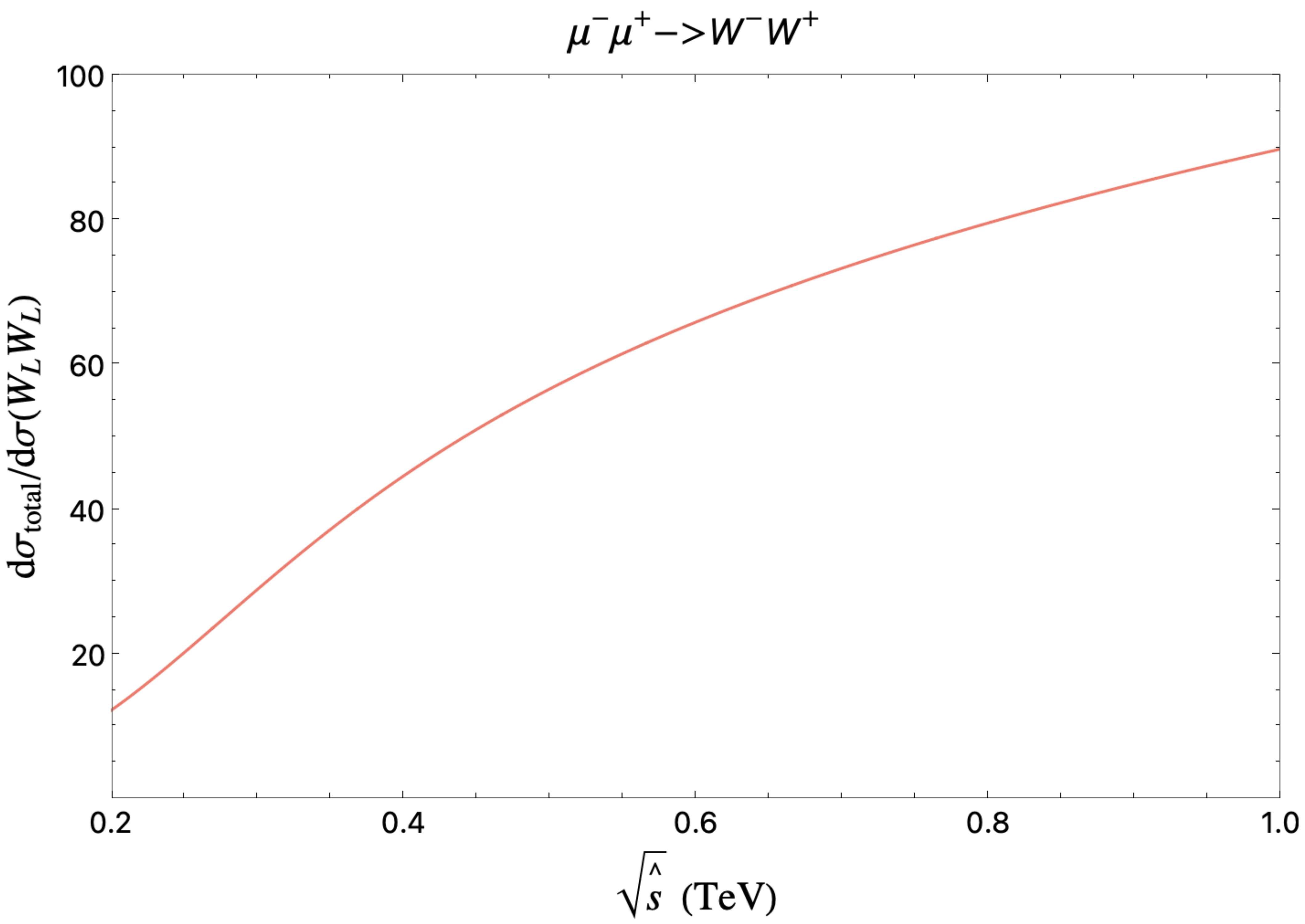


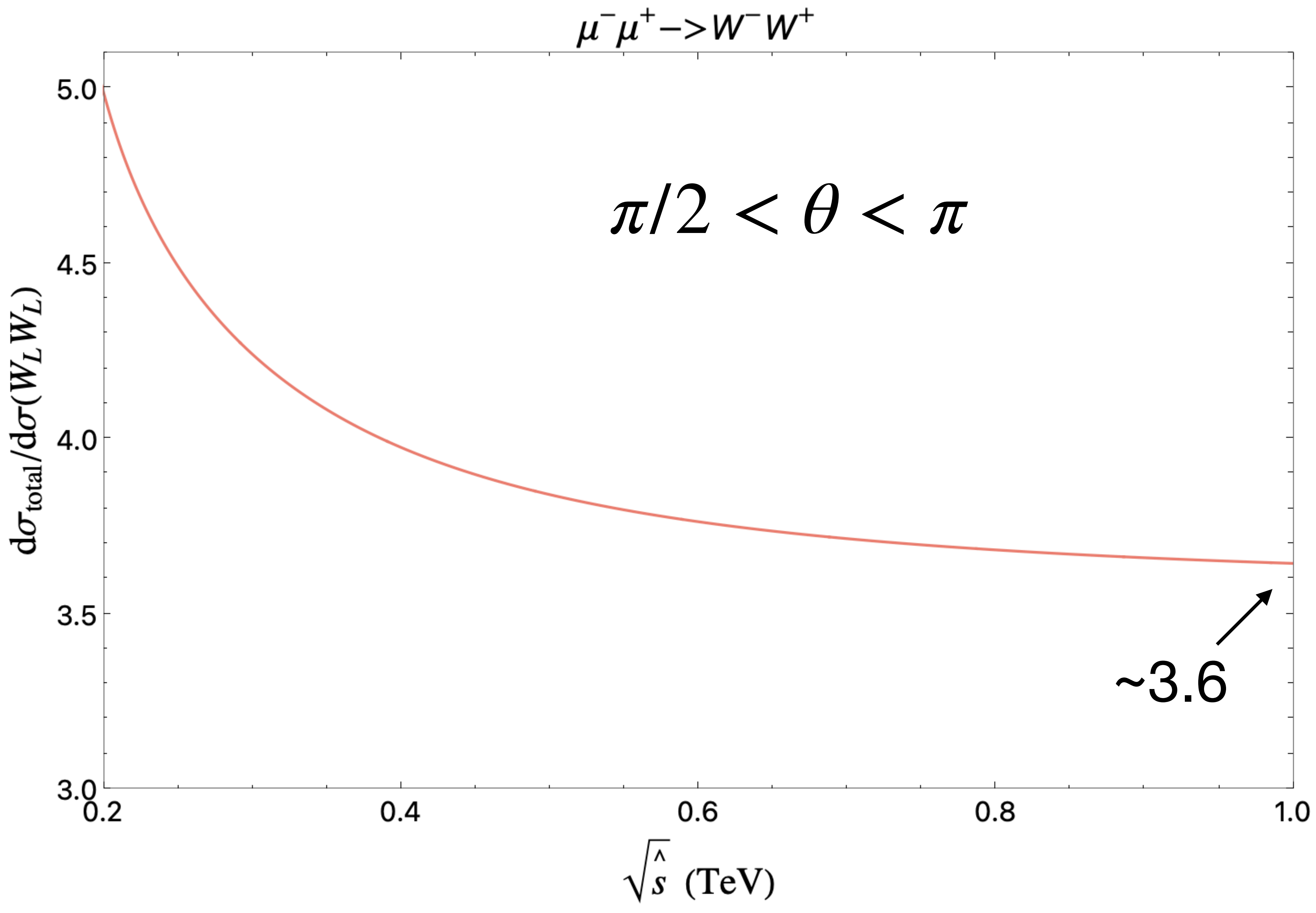




Test of EW Restoration

$$\mu^- \mu^+ \to W^- W^+$$





We can test for convergence after forward-angle cuts