

Optimizations of a Rectilinear Cooling Channel for a Future Muon Collider

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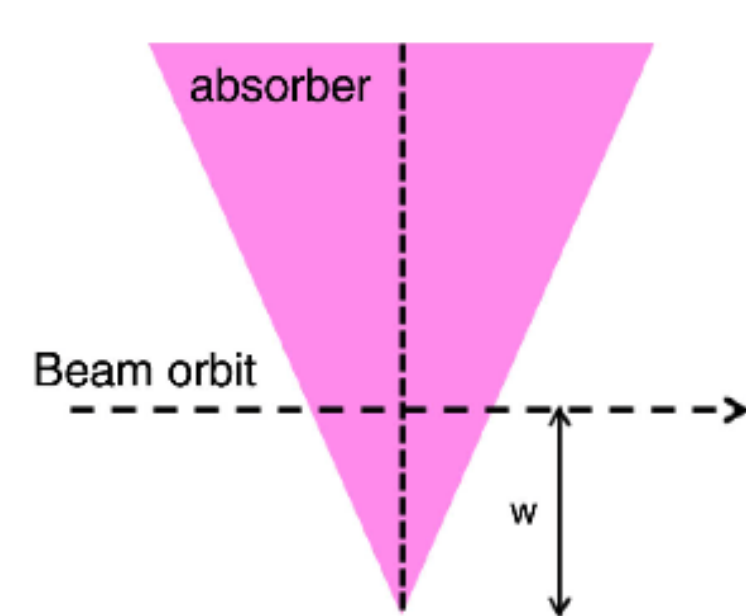
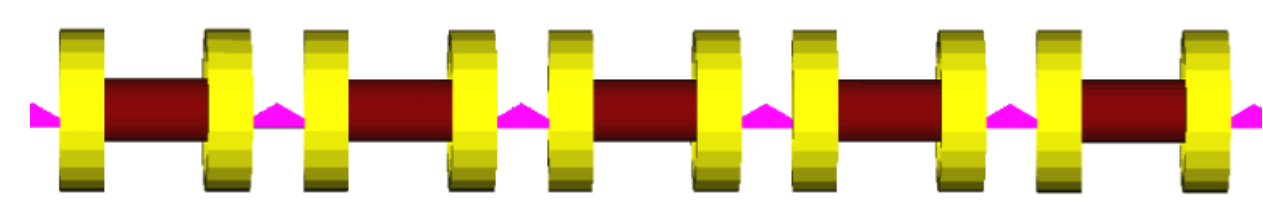
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Abstract

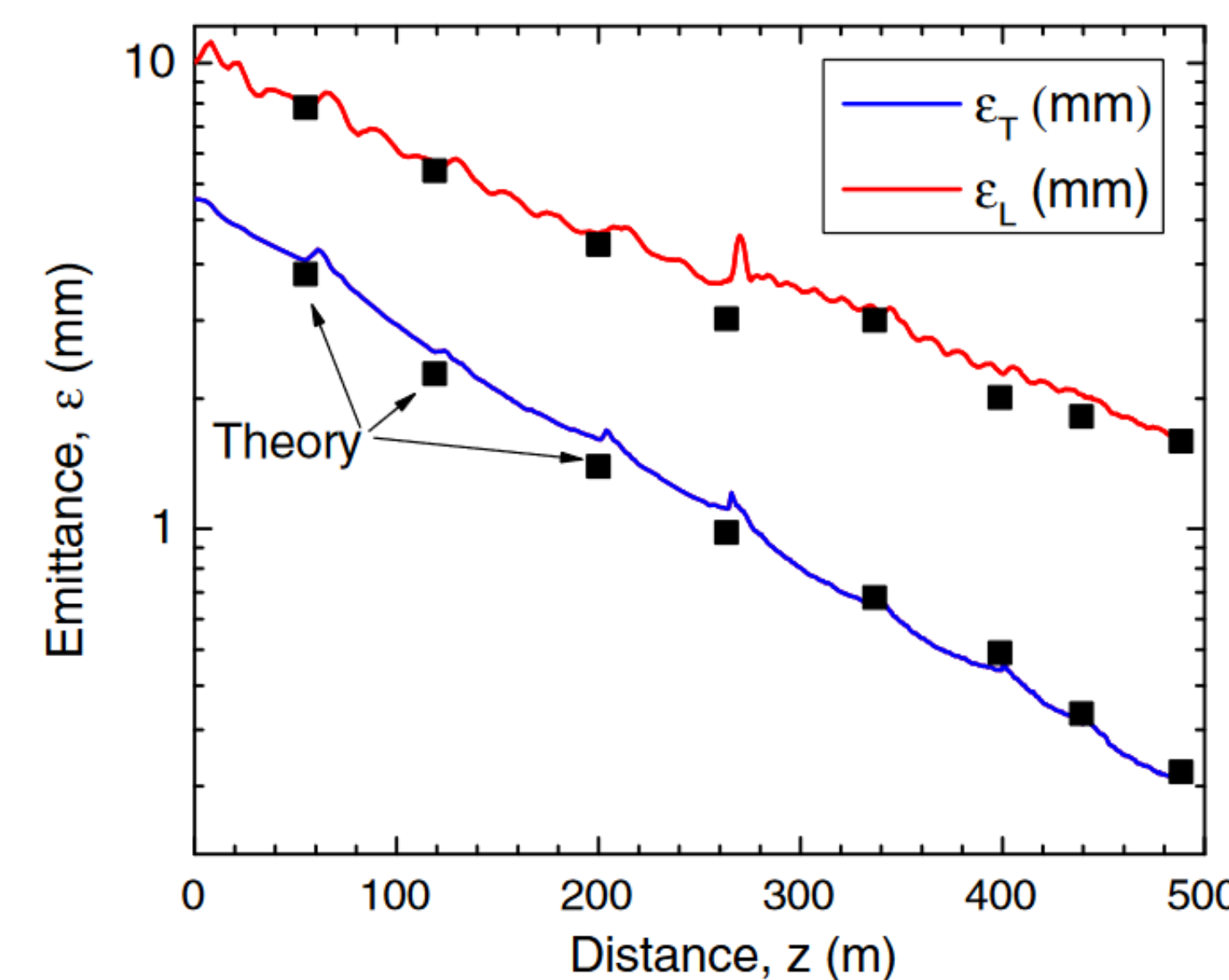
Muon colliders require significant beam cooling to achieve the luminosity needed for high-energy physics experiments. Ionization cooling has emerged as a promising solution. This study optimizes a rectilinear muon cooling channel using a **multi-objective optimization** framework that integrates beam dynamics simulations. We present novel optimizations confirming the theoretical trade-offs between transverse and longitudinal emittance, as well as brightness versus transmission. Our results surpass performance benchmarks reported in the literature, demonstrating an improved cooling efficiency. We also present results of multi-stage optimizations that push beyond the improvements achieved by optimizations of individual stages.

Realistic ionization muon cooling design

A rectilinear ionization cooling system consists of liquid hydrogen wedge-shaped absorbers, solenoid magnets, and RF cavities placed periodically in a linear arrangement. The solenoid magnets are used not only to focus the beam but also to create dispersion by being tilted, allowing for significant cooling in both the transverse and longitudinal directions.



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Optimization with genetic algorithms

Genetic algorithms use an evolutionary biology-inspired iterative approach to explore a solution space and find optimal combinations of parameter values. An initial population of candidate solutions (individuals) is randomly generated. Then, individuals with the best objective values (greatest fitness) are most likely to be chosen as parents of the next generation, who reproduce to yield children with improved objective values. This procedure repeats until the generations converge to a set of local extrema in the solution space.

Parameters to be tuned (decision variables):

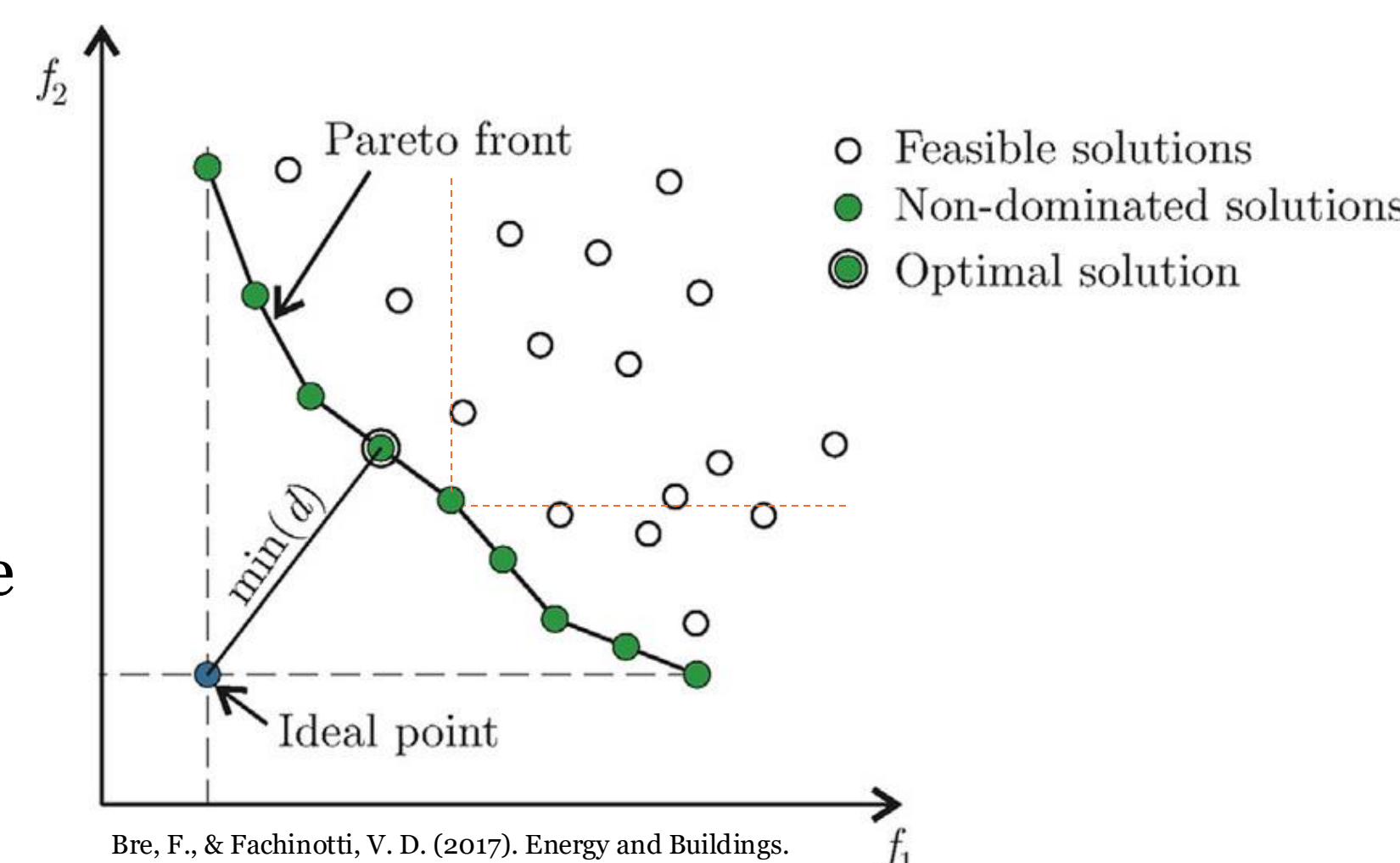
- Absorber angle and offset
- Cavity phase and amplitude
- Initial longitudinal momentum of beam
- Solenoid current and tilt

Objectives:

1. Minimize 6D emittance, maximize beam transmission
2. Minimize longitudinal emittance, minimize transverse emittance

Constraints:

- Transmission

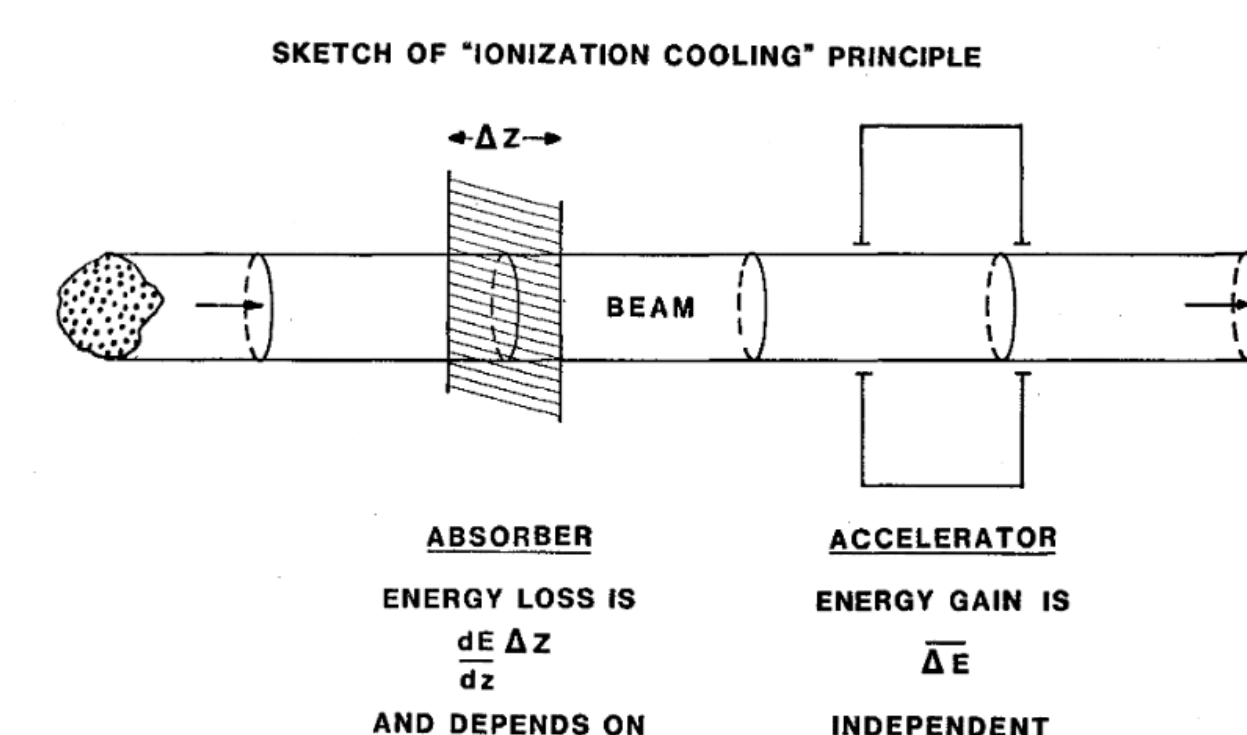


Bre, F., & Fachinotti, V. D. (2017). Energy and Buildings.

Transverse vs. longitudinal emittances

- **Transverse emittance:** Ionization cooling reduces beam momentum via energy loss in absorbers, with lost momentum restored only in the longitudinal direction by RF cavities. This reduces transverse emittance.
- **Longitudinal emittance:** To decrease longitudinal emittance, emittance exchange is employed, where a dispersive beam passes through an absorber, causing high-energy particles to lose more energy than low-energy ones.
- **Fundamental trade-off:** The cooling rates' opposing dependence on wedge geometry and dispersion creates inherent competition between transverse and longitudinal emittance.

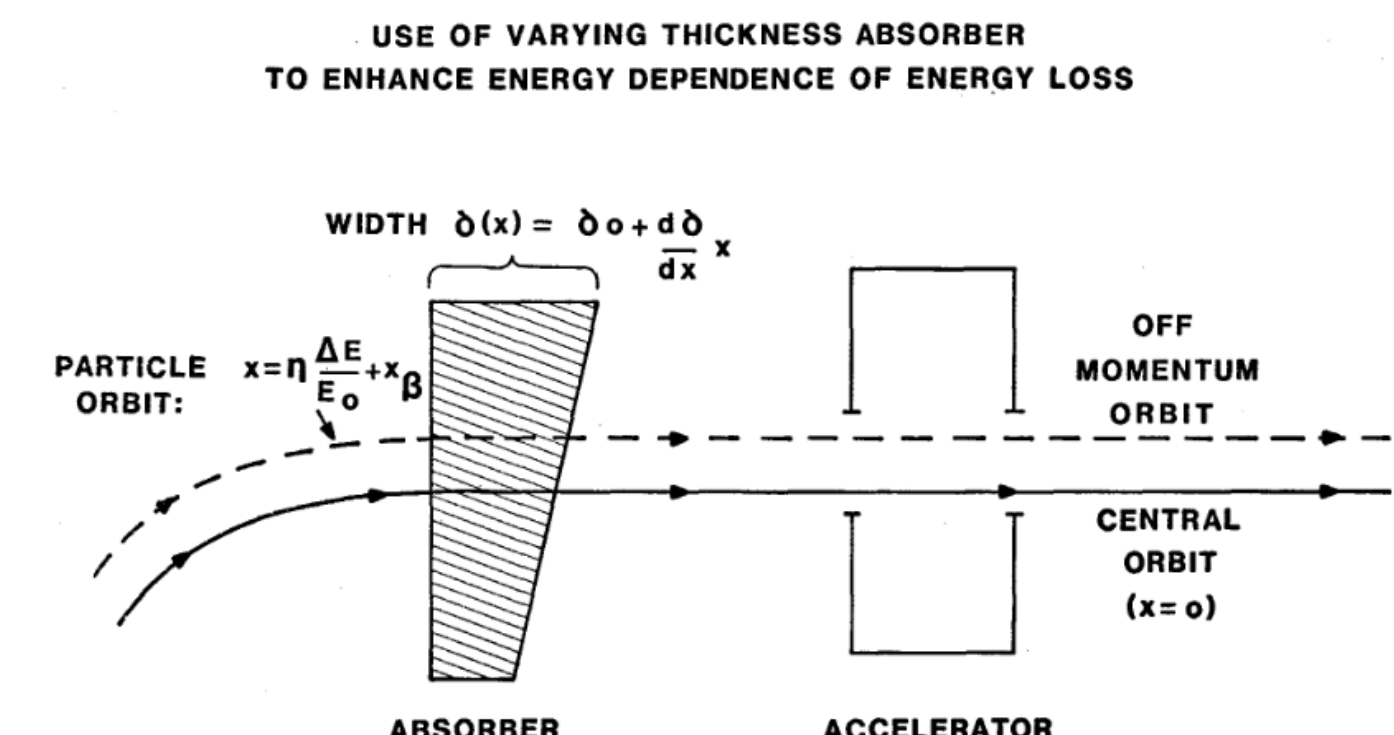
Transverse cooling:



$$\frac{d\epsilon_T}{ds} = -\frac{g_T}{\beta^2 E} \frac{dE}{ds} \epsilon_T + C_1$$

$$g_T = C_3 - \frac{D}{w}$$

Longitudinal cooling:

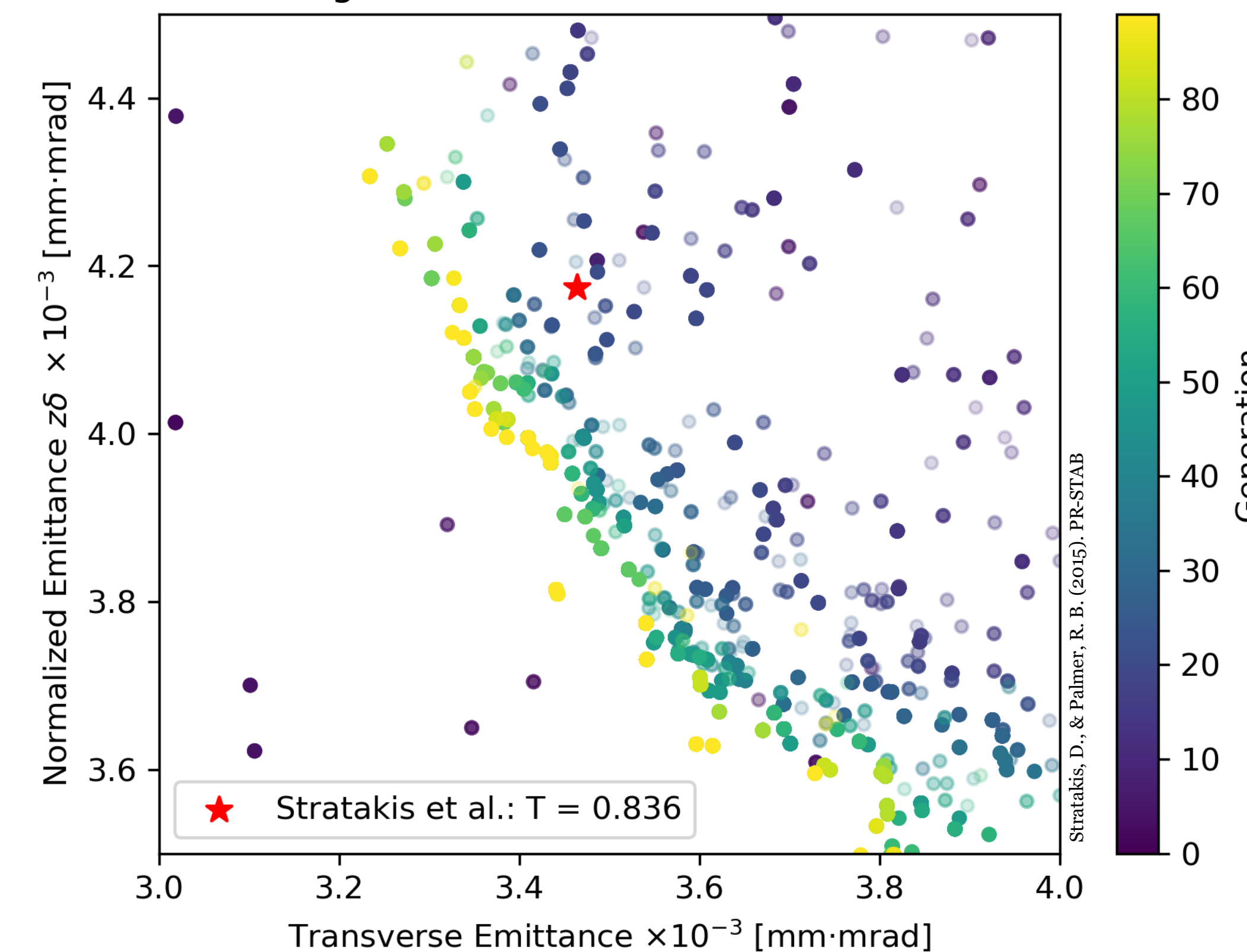


$$\frac{d\epsilon_L}{ds} = -\frac{g_L}{\beta^2 E} \frac{dE}{ds} \epsilon_L + C_2$$

$$g_L = C_4 + \frac{D}{w}$$

- We validate the theoretical relationship between transverse and longitudinal emittances, while also achieving a **4% increase** in luminosity with higher transmission.

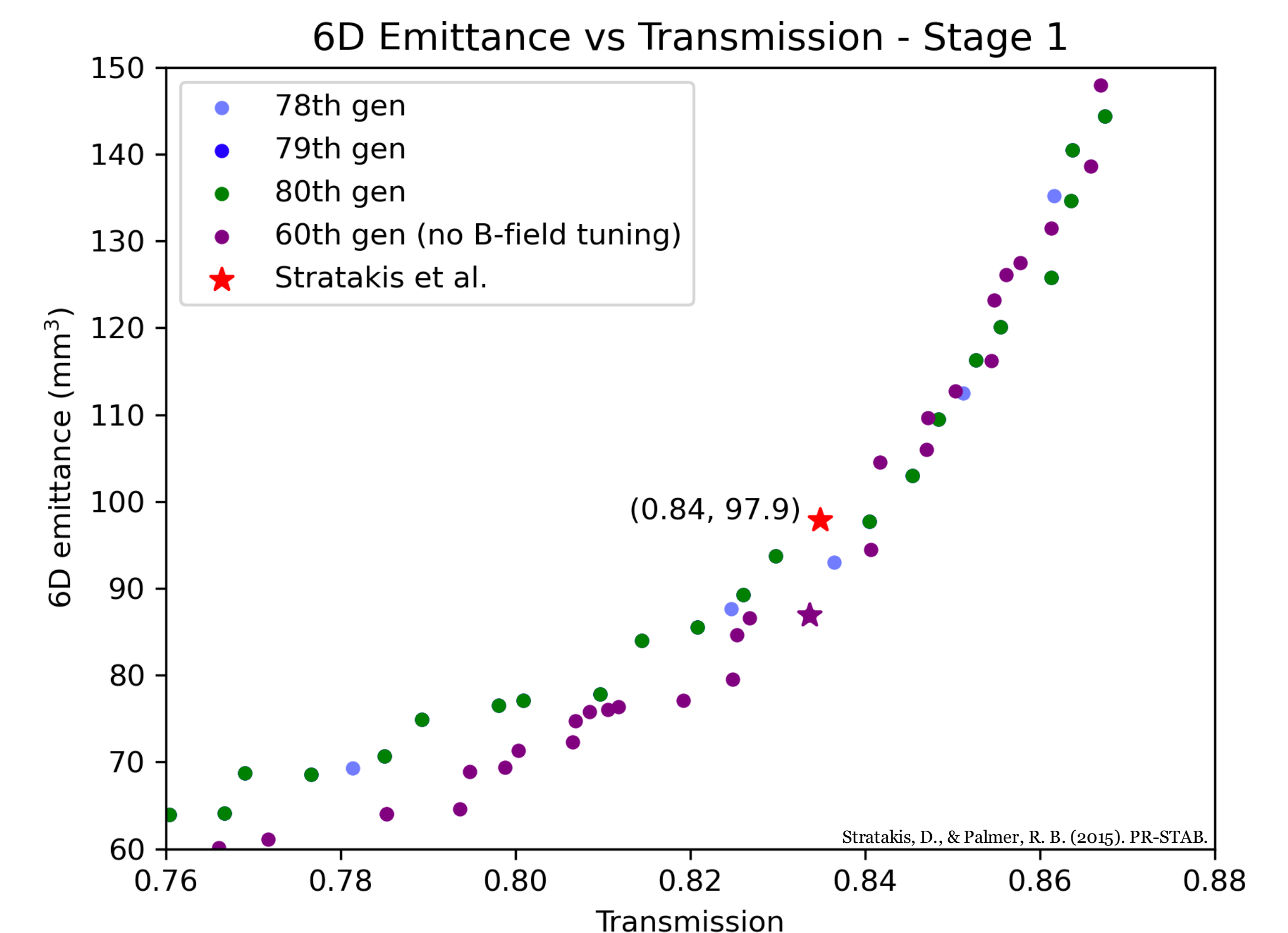
Transverse vs Longitudinal Emittance with Transmission > 83.6%



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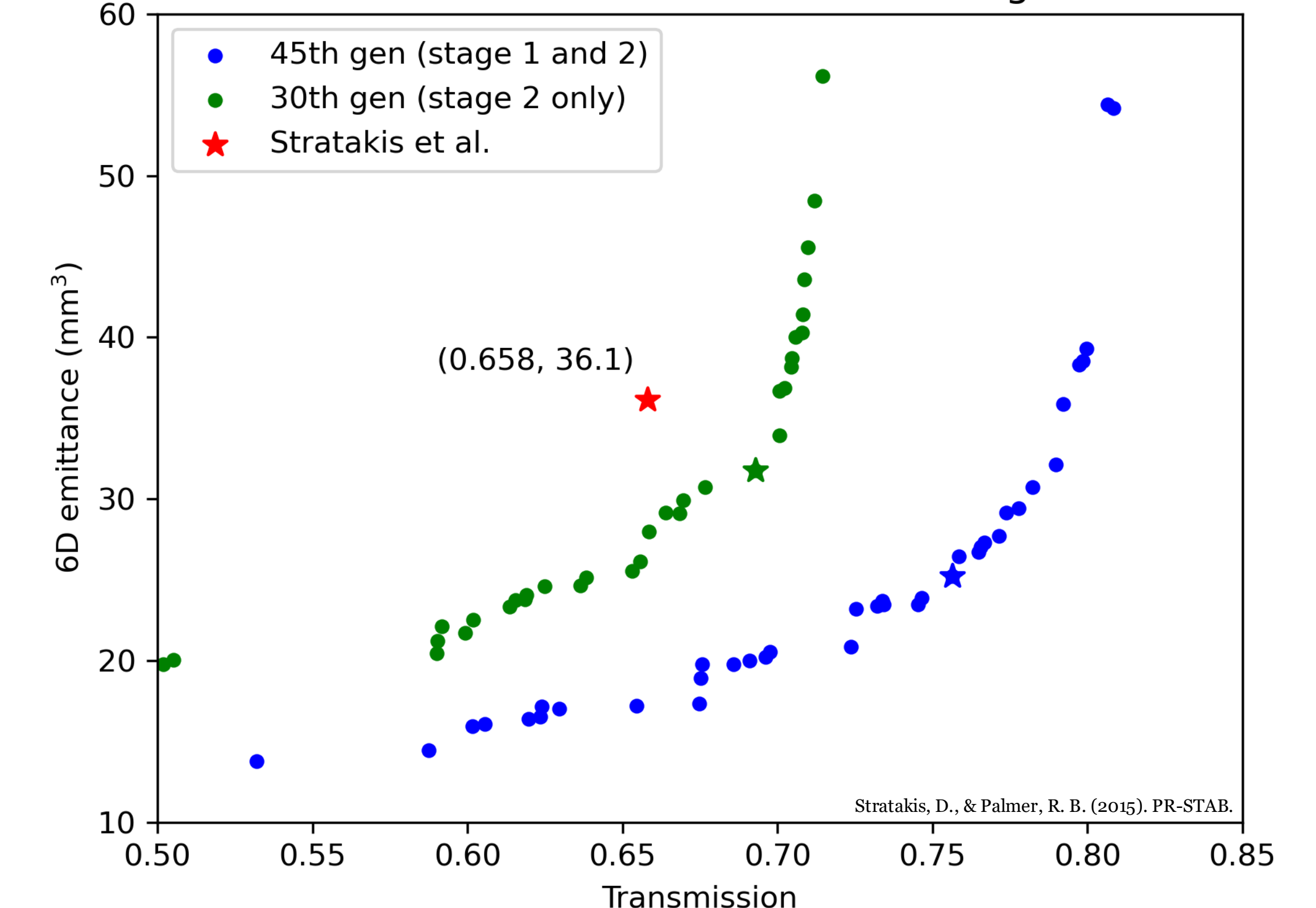
6D emittance vs. transmission

- **Stage 1:** 11% improvement in emittance without tuning B-field, similar transmission, 5680 input particles.
- **Stage 2:** 12% improvement in emittance, 5.3% improvement in transmission, 4895 input particles.
- **Stage 1 & 2:** 30% improvement in emittance, 15% improvement in transmission, 10k input particles.



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6D Emittance vs Transmission - Stage 2



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Conclusion

This study presents the **first multi-objective optimizations** of a rectilinear muon cooling channel, integrating numerical optimization with beam dynamics simulations to refine design parameters. Key trade-offs, such as transverse vs. longitudinal emittance and brightness vs. transmission, aligned with theoretical expectations, and our optimized parameters led to superior cooling performance over existing benchmarks while maintaining engineering feasibility. These optimizations bring us closer to a next-generation muon collider, presenting the potential of **increasing luminosity** by 25% over all the stages, **enhancing particle energy** by 4%, or **reducing overall costs** by 25%, and paving the way for more powerful and accessible high-energy physics experiments.

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